

Detection and modeling of the propagation regimes in fatigue crack propagation

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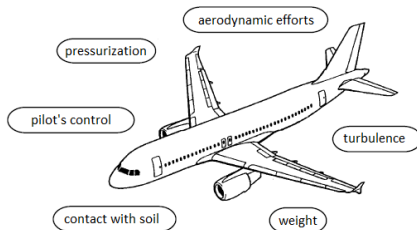
- 1 Industrial context
- 2 Model estimation
- 3 Simulations
- 4 Perspectives

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Engine parts failure

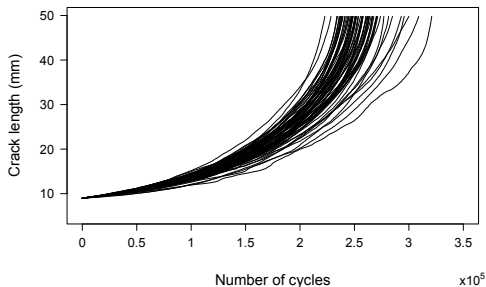
Risks of engine parts failure are accentuated by the extreme environment



Stress accumulation \Rightarrow initiation and crack propagation of the engine parts

Behavior of the crack propagation

Study of the crack length a according to the number of cycles N



Virkler data, Journal of Engineering Materials and Technology, 1979

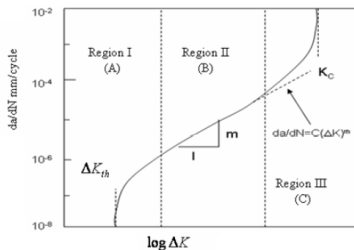
Mechanical standard representation of the crack

The mechanical standard representation of the crack propagation involves:

- The speed (also named crack growth rate) da/dN in $m/cycle$: determined across the test results (a, N) ;
- The stress intensity factor ΔK in $MPa\sqrt{m}$: function of
 - The crack length;
 - The geometry of the specimen;
 - The constraint applied.

State of the art

Empirical curve of the crack propagation law



- Regime I: crack initiation;
- Regime II: linear regime;
- Regime III: acceleration and rupture.

Industrial questions

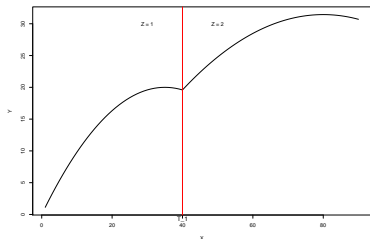
- Is the number of regimes exactly three?
- How to precisely identify the beginning of the regimes?
- What is the dynamics of each regime?

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Piecewise-polynomial model

Main assumption: each regime is polynomial



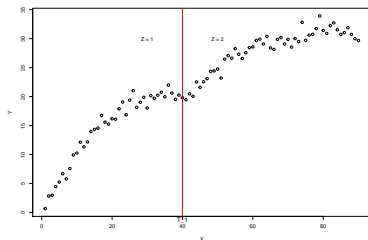
- Regime variable $Z \in \{1, \dots, K\}$, K is unknown;
- $K - 1$ Times of transition between regimes (T_1, \dots, T_{K-1}) ;
- Y is a function of X

$$Y = p_k^d(X), \text{ if } T_{k-1} < X \leq T_k$$

d is the degree of p_k^d ;

- Y is continuous over the whole interval of X .

Observed data



- X_i : measurement times variable. The X_1, \dots, X_n are fixed;
- $Y_i = p_k^d(X_i) + \epsilon_i$
 $\epsilon_i \sim N(0, \sigma_{Z_i}^2)$.

Industrial questions

- Is the number of regimes exactly three?
 - Estimation of K ;
- How to precisely identify the beginning of the regimes?
 - Estimation of $(T_k)_k$;
- What is the dynamics of each regime?
 - Estimation of $(p_k^d)_{k,d}$.

Estimation of the model (K and d are fixed)

The vector of parameters:

$$\theta = ((\alpha_k^\delta)_{k,\delta}, (\sigma_k^2)_k, (T_k)_k)^t.$$

$$\left\{ \begin{array}{ll} (\alpha_k^\delta)_{k,\delta} & \text{coefficients of } X^\delta \text{ in } p_k^d; \\ (\sigma_k^2)_k & \text{variance of the noise in each regime;} \\ (T_k)_k & \text{the } K - 1 \text{ times of transition.} \end{array} \right.$$

All the coefficients of θ are linked because of the continuity constraint

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All the coefficients of θ are linked because of the continuity constraint

We want to estimate θ by maximising the complete log likelihood under the constraint $p_{k-1}^d(T_k)_k = p_k^d(T_k)_k$:

$$L((Y_i, Z_i)_{1 \leq i \leq n}, \theta | (X_i)_{1 \leq i \leq n}) = \sum_{i=1}^n \log(\omega_\theta(Y_i | Z_i, X_i) P_\theta(Z_i | X_i)).$$

Estimation of the model (K and d are fixed)

- $P_\theta(Z|X)$: conditional probability function of Z given X ;
 Z is deterministic and depends on X and T by this relation

$$P_\theta(Z = k|X) = \mathbb{1}_{\{T_{k-1} \leq X < T_k\}}$$

- $\omega_\theta(Y|Z, X)$: the gaussian density of Y given Z and X .

Decomposition of the problem into two parts

We want to estimate θ by:

$$\max_{\substack{\theta \\ \text{continuity constraint}}} \mathcal{L}(\theta, (T_k)_k) = \max_{(T_k)_k} \max_{\tilde{\theta}} \mathcal{L}(\tilde{\theta}, (T_k)_k)$$

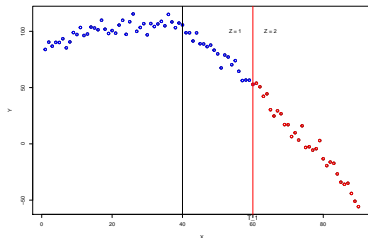
continuity constraint

where,

$$\tilde{\theta} = (\alpha_k^\delta, \sigma_k^2)^t$$

We first do the maximization in $\tilde{\theta}$ by assuming the $(T_k)_k$ are fixed.

$P_\theta(Z|X)$ implies some rigidity

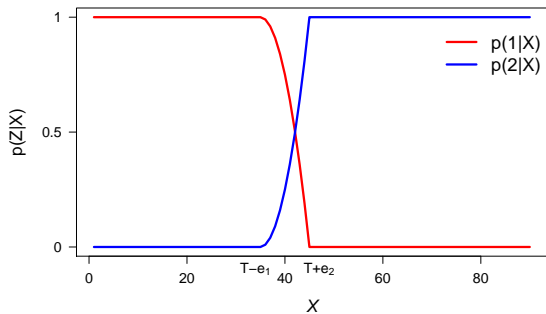


Extreme rigidity given by the indicators functions coming from the

$$P_\theta(Z|X) = \mathbb{1}_{\{T_{k-1} \leq X < T_k\}};$$

Modification of $P_\theta(Z|X)$ to approximate the likelihood by smoothing the indicator function.

Modified $P_{\theta}(Z|X)$



Conditionnaly to the times of transition, Z will not be completely observed \Rightarrow EM algorithm

EM algorithm

The l^{th} iteration of the algorithm breaks down in two steps:

- *E – step*: considering $\hat{\theta}_{l-1}$ we compute the conditional expected value given the observations $\mathcal{L}(Y; \tilde{\theta}, \hat{\theta}_{l-1})$;
- *M – step*: we find the parameters $\hat{\theta}_l$ that maximizes $\mathcal{L}(Y; \tilde{\theta}, \hat{\theta}_{l-1})$ in $\tilde{\theta}$.

$\mathcal{L}(Y; \tilde{\theta}, \hat{\theta}_{l-1})$, the conditional log-likelihood at the l^{th} iteration is given by:

$$\begin{aligned}\mathcal{L}(Y; \tilde{\theta}, \hat{\theta}_{l-1}) &= E(\log(L(Y, Z, \tilde{\theta}|X))|X) \\ &= \sum_{i=1}^n \int g_{\hat{\theta}_{l-1}}(z|Y_i, X_i) \log(h_{\tilde{\theta}}(Y_i, z|X_i)) dz.\end{aligned}$$

EM algorithm

- $g_{\tilde{\theta}}(Z|Y, X)$: posterior probability distribution of Z given X and Y given by

$$g_{\tilde{\theta}}(Z|Y, X) = \frac{h_{\tilde{\theta}}(Y, Z|X)}{f_{\tilde{\theta}}(Y|X)} = \frac{\omega_{\tilde{\theta}}(Y|Z, X)P_{\tilde{\theta}}(Z|X)}{\sum_{k=1}^2 \omega_{\tilde{\theta}}(Y|Z, X)P_{\tilde{\theta}}(Z|X)}.$$

- $h_{\tilde{\theta}}(Y, Z|X)$: conditional probability function of Y and Z given X defined by

$$h_{\tilde{\theta}}(Y, Z|X) = \omega_{\tilde{\theta}}(Y|Z, X)P_{\tilde{\theta}}(Z|X)$$

Estimation of $\tilde{\theta}$ (K and $(T_k)_k$ are fixed)

Under the continuity constraint $\tilde{\theta}$ satisfy the linear equation:

$$A\tilde{\theta} = b$$

where A is a matrix of size $k(d+1) + (k-1) \times k(d+1) + (k-1)$

Each step of the EM algorithm reduces to inverse the matrix A .

Estimation of the times of transition and K

$$\max_{\substack{\theta \\ \text{continuity constraint}}} \mathcal{L}(\theta, (T_k)_k) = \max_{(T_k)_k} \max_{\substack{\tilde{\theta} \\ \text{continuity constraint}}} \mathcal{L}(\tilde{\theta}, (T_k)_k)$$

Estimation of $(T_k)_k$:

- maximization in $(T_k)_k$;
- optimization method: exhaustive search, gradient algorithm ...

Estimation of K :

- K is estimated by a BIC.

$$BIC = -2 \log(L(\theta, (T_k)_k)) + n \log(N)$$

where,

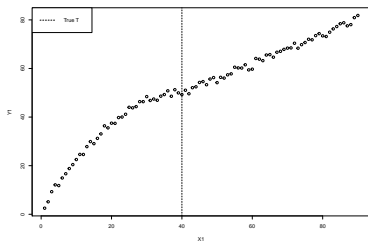
n the number of free parameters;

N The number of observations.

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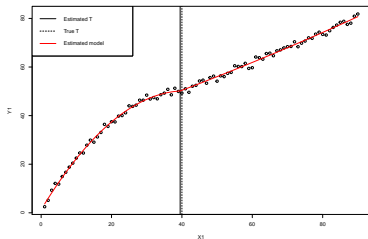
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Data simulated on two regimes



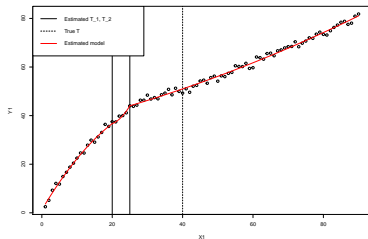
- Find $K = 2$;
- Find $T = 40$;
- Find the polynomial regression parameters.

Estimation results



(a) 1 time of transition

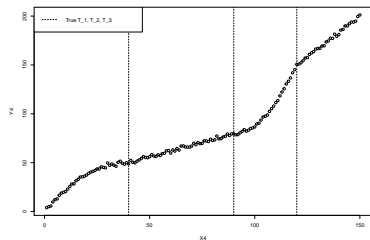
BIC = 270.8



(b) 2 times of transition

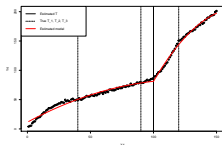
BIC = 273.5

Data simulated on four regimes

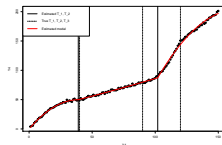


- Find $K = 4$;
- Find $T_1 = 40$, $T_2 = 90$,
 $T_3 = 120$;
- Find the polynomial regression parameters.

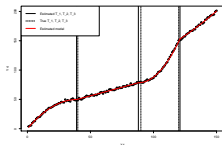
Estimation results



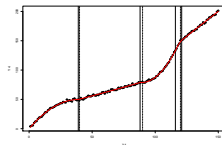
(a) 1 time of transition, BIC = 810



(b) 2 times of transition, BIC = 598



(c) 3 times of transition, BIC = 527



(d) 4 times of transition, BIC = 515

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Perspectives

- Find another criterion to select the good K ;
- Assess the method implemented;
- Test a stronger constraint (derivability);
- Test the method on real data;
- Use the method to discover the impact of the mechanical covariables.