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## Admissible generalizations of examples as rules (2018)

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► **To cite this version:**

Philippe Besnard, Thomas Guyet, Véronique Masson. Admissible generalizations of examples as rules (2018). One-day Workshop on Machine Learning and Explainability, Christel Vrain, Université d'Orléans, Oct 2018, Orléans, France. hal-01940129

**HAL Id: hal-01940129**

**<https://inria.hal.science/hal-01940129>**

Submitted on 30 Nov 2018

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# Admissible generalizations of examples as rules

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Machine Learning and Explainability – 8 October 2018

# And now . . .

## Introduction

### Formalizing rule learning

- General formalism
- Notations

### Admissibility for generalization

- Formalizing admissibility
- Classes of choice functions

### Application to an analysis of CN2

## Conclusion

# Attribute-value rule learning

	(C) Price	(A <sub>1</sub> ) Area	(A <sub>2</sub> ) Rooms	(A <sub>3</sub> ) Energy	(A <sub>4</sub> ) Town	(A <sub>5</sub> ) District	(A <sub>6</sub> ) Exposure
1	low-priced	70	2	D	Toulouse	Minimes	
2	low-priced	75	4	D	Toulouse	Ranguel	
3	expensive	65	3		Toulouse	Downtown	
4	low-priced	32	2	D	Toulouse		SE
5	mid-priced	65	2	D	Rennes		SO
6	expensive	100	5	C	Rennes	Downtown	
7	low-priced	40	2	D	Betton		S

- ▶ Task: induce rules to predict the value of the class attribute (C)
- ▶ Rules extracted by Algorithm CN2

$$\pi_1^{CN2} : A_5 = \text{Downtown} \Rightarrow C = \text{expensive}$$

$$\pi_2^{CN2} : A_2 < 2.50 \wedge A_4 = \text{Toulouse} \Rightarrow C = \text{low-priced}$$

$$\pi_3^{CN2} : A_1 > 36.00 \wedge A_3 = D \Rightarrow C = \text{low-priced}$$

# Interpretability of rules and rulesets

- ▶ The logical structure of a rule can be easily interpreted by users

IF *conditions* THEN *class-label*

- ▶ Rule learning algorithms generate rules according to implicit or explicit principles<sup>1</sup>
    - ▶ are the generated rules the *interpretable* ones?
    - ▶ would it be possible to have different rulesets?
    - ▶ why a ruleset would be better than another one from the interpretability point of view?
- ⇒ We need ways to **analyze the interpretability of the outputs of rule learning algorithms**

---

<sup>1</sup>principles mainly based on statistical properties!

# Analyzing the interpretability of rules

## Analyzing the interpretativeness of ruleset

- ▶ Objective criteria on ruleset syntax [CZV13, BS15]
    - ▶ size of the rule (number of attributes)
    - ▶ size of the ruleset
  - ▶ Intuitiveness of rules through the effects of cognitive biases [KBF18]
- ⇒ Our approach formalizes **rule learning** and **some expected properties on rules** to shed light on properties of some extracted ruleset

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## In this talk

- ▶ We present the formalisation of rule learning, and we focus on the **generalization of examples as a rule**
- ▶ We introduce the notion of **admissible rule** that attempts to **capture an intuitive generalization of the examples**
- ▶ We develop the example of numerical attributes

# Impact of examples generalization on rule interpretability

	(C) <i>Price</i>	(A <sub>1</sub> ) <i>Area</i>	(A <sub>2</sub> ) <i>Rooms</i>	(A <sub>3</sub> ) <i>Energy</i>	(A <sub>4</sub> ) <i>Town</i>	(A <sub>5</sub> ) <i>District</i>	(A <sub>6</sub> ) <i>Exposure</i>
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## Toward the notion of admissibility

	(C) Price	(A <sub>1</sub> ) Area
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- ▶ Rote learning of a rule

$$A_1 = \{75\} \Rightarrow C = \text{low-priced}$$

- ▶ Most generalizing rule

$$A_1 = [32 : 75] \Rightarrow C = \text{low-priced}$$

- ▶ Would the following rule be better?

$$A_1 = [32 : 40] \cup [70 : 75] \Rightarrow C = \text{low-priced}$$

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The notion of admissibility has to capture an intuitive notion of generalization . . .

# And now . . .

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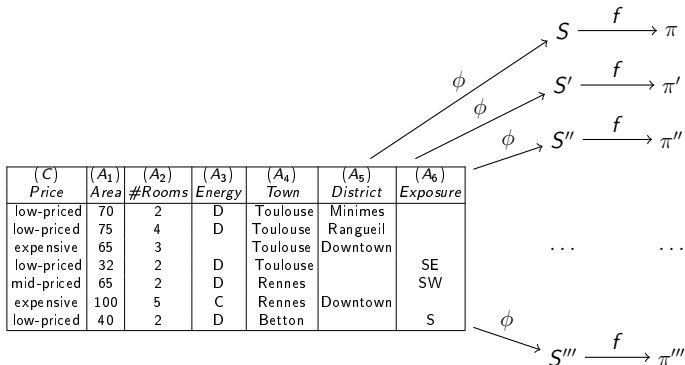
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# At a glance

Rule learning is formalized by two main functions

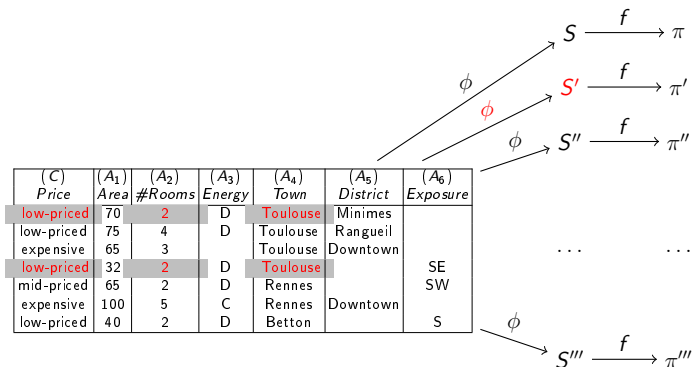
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- ▶  $f$ : generalizes examples as a rule (LearnOneRule process [Mit82])



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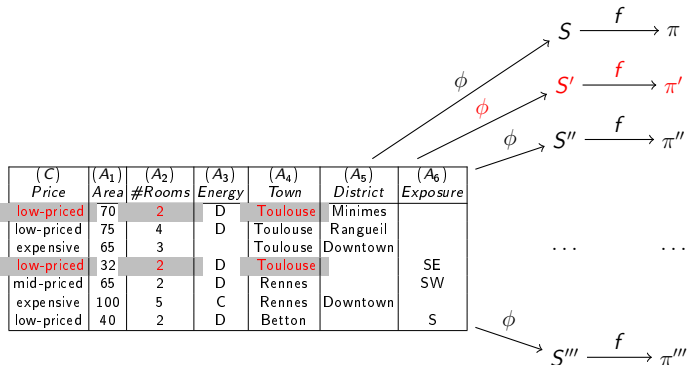
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# At a glance

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# The attribute-value model

	$A_1$	$A_2$	$A_3$	$A_4$	$A_5$	$A_6$	$A_7$	$\dots$	$A_n$
item 1	$a_{1,1}$	$a_{1,2}$	$a_{1,3}$	$a_{1,4}$	$a_{1,5}$	$a_{1,6}$	$a_{1,7}$	$\dots$	$a_{1,n}$
item 2	$a_{2,1}$	$a_{2,2}$	$a_{2,3}$	$a_{2,4}$	$a_{2,5}$	$a_{2,6}$	$a_{2,7}$	$\dots$	$a_{2,n}$
item 3	$a_{3,1}$	$a_{3,2}$	$a_{3,3}$	$a_{3,4}$	$a_{3,5}$	$a_{3,6}$	$a_{3,7}$	$\dots$	$a_{3,n}$
item 4	$a_{4,1}$	$a_{4,2}$	$a_{4,3}$	$a_{4,4}$	$a_{4,5}$	$a_{4,6}$	$a_{4,7}$	$\dots$	$a_{4,n}$
item 5	$a_{5,1}$	$a_{5,2}$	$a_{5,3}$	$a_{5,4}$	$a_{5,5}$	$a_{5,6}$	$a_{5,7}$	$\dots$	$a_{5,n}$
item 6	$a_{6,1}$	$a_{6,2}$	$a_{6,3}$	$a_{6,4}$	$a_{6,5}$	$a_{6,6}$	$a_{6,7}$	$\dots$	$a_{6,n}$
item 7	$a_{7,1}$	$a_{7,2}$	$a_{7,3}$	$a_{7,4}$	$a_{7,5}$	$a_{7,6}$	$a_{7,7}$	$\dots$	$a_{7,n}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
item $m$	$a_{m,1}$	$a_{m,2}$	$a_{m,3}$	$a_{m,4}$	$a_{m,5}$	$a_{m,6}$	$a_{m,7}$	$\dots$	$a_{m,n}$

Rows: items  $x_1, x_2, \dots, x_m$   
Columns: attributes  $A_1, A_2, \dots, A_n$   $\left. \vphantom{\begin{matrix} \text{Rows:} \\ \text{Columns:} \end{matrix}} \right] \forall i, j \ a_{j,i} \in \text{Rng } A_i$

$\text{Rng } A_i$  denotes the set of possible values for attribute  $A_i$

# Subsets of data to generalize

	$A_1$	$A_2$	$A_3$	$A_4$	$A_5$	$A_6$	$A_7$	$A_8$	$A_9$
item 1	$a_{1,1}$	$a_{1,2}$	$a_{1,3}$	$a_{1,4}$	$a_{1,5}$	$a_{1,6}$	$a_{1,7}$	$a_{1,8}$	$a_{1,9}$
item 2	$a_{2,1}$	$a_{2,2}$	$a_{2,3}$	$a_{2,4}$	$a_{2,5}$	$a_{2,6}$	$a_{2,7}$	$a_{2,8}$	$a_{2,9}$
item 3	$a_{3,1}$	$a_{3,2}$	$a_{3,3}$	$a_{3,4}$	$a_{3,5}$	$a_{3,6}$	$a_{3,7}$	$a_{3,8}$	$a_{3,9}$
item 4	$a_{4,1}$	$a_{4,2}$	$a_{4,3}$	$a_{4,4}$	$a_{4,5}$	$a_{4,6}$	$a_{4,7}$	$a_{4,8}$	$a_{4,9}$
item 5	$a_{5,1}$	$a_{5,2}$	$a_{5,3}$	$a_{5,4}$	$a_{5,5}$	$a_{5,6}$	$a_{5,7}$	$a_{5,8}$	$a_{5,9}$
item 6	$a_{6,1}$	$a_{6,2}$	$a_{6,3}$	$a_{6,4}$	$a_{6,5}$	$a_{6,6}$	$a_{6,7}$	$a_{6,8}$	$a_{6,9}$
item 7	$a_{7,1}$	$a_{7,2}$	$a_{7,3}$	$a_{7,4}$	$a_{7,5}$	$a_{7,6}$	$a_{7,7}$	$a_{7,8}$	$a_{7,9}$
item 8	$a_{8,1}$	$a_{8,2}$	$a_{8,3}$	$a_{8,4}$	$a_{8,5}$	$a_{8,6}$	$a_{8,7}$	$a_{8,8}$	$a_{8,9}$
item 9	$a_{9,1}$	$a_{9,2}$	$a_{9,3}$	$a_{9,4}$	$a_{9,5}$	$a_{9,6}$	$a_{9,7}$	$a_{9,8}$	$a_{9,9}$

⇒ "Square" = selection of rows and columns in the data

# Rules

A rule  $\pi$  expresses constraints (for a generic item  $x$ ) which lead to conclusion  $C(x)$  (class which the item belongs to)

$$\pi : A_1(x) \in v_1^\pi \wedge \cdots \wedge A_n(x) \in v_n^\pi \rightarrow C(x) \in v_0^\pi \quad (*)$$

where  $\begin{cases} v_i^\pi \subseteq \text{Rng } A_i \text{ pour } i = 1, \dots, n, \\ v_0^\pi \subseteq \text{Rng } C. \end{cases}$

attributes  $i = 1, \dots, n$  without constraints are such that  $v_i^\pi = \text{Rng } A_i$ .

# And now . . .

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# Eliciting a rule

- ▶  $S$  being a square is supposed to capture a rule  $\pi$  **requires that every item of  $S$  satisfies  $\pi$** 
  - generalisation does not capture the statistical representativeness of dataset, but only elicits a rule generalizing all its items

(C) Price	(A <sub>1</sub> ) Area	(A <sub>2</sub> ) #Rooms	(A <sub>3</sub> ) Energy	(A <sub>4</sub> ) Town	(A <sub>5</sub> ) District	(A <sub>6</sub> ) Exposure
low-priced	70	2	D	Toulouse	Minimes	
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mid-priced	65	2	D	Rennes		SW
expensive	100	5	C	Rennes	Downtown	
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$f$  ↓

$$A_0 = 2 \wedge A_4 = \text{Toulouse} \Rightarrow C = \text{low-priced}$$

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$f$  ↓

$$A_0 \in [2, 4] \Rightarrow C \in \{\text{low-priced}, \text{expensive}\}$$

## Eliciting a rule ( $f$ function)

- ▶ For every attribute  $A_i$ ,  $S_i$  is the set of values of  $A_i$  in items of  $S$

	$(A_0)$ <i>Price</i>	$(A_1)$ <i>Area</i>	$(A_2)$ <i>Rooms</i>
1	low-priced	70	2
2	low-priced	75	4
4	low-priced	32	2
7	low-priced	40	2

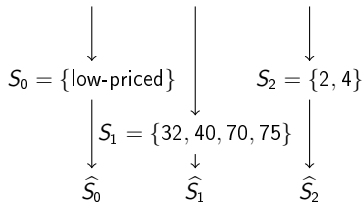
$S_0 = \{\text{low-priced}\}$

$S_1 = \{32, 40, 70, 75\}$

$S_2 = \{2, 4\}$

## Eliciting a rule ( $f$ function)

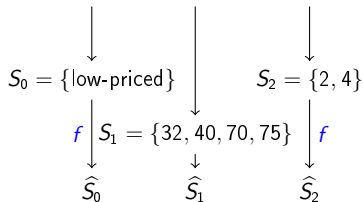
	( $A_0$ ) Price	( $A_1$ ) Area	( $A_2$ ) Rooms
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- ▶ For every attribute  $A_i$ ,  $S_i$  is the set of values of  $A_i$  in items of  $S$
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- ▶ The generalisation process thus consists in selecting **one** of these supersets:
  - $f$  **choice function** that is given as input a collection of supersets of  $S_i$  and picks **one**

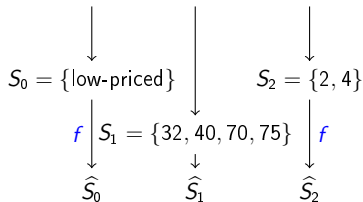
We are looking for an appropriate  $\hat{\cdot}$  for (§) i.e.

$$A_1(x) \in \hat{S}_1 \wedge \dots \wedge A_n(x) \in \hat{S}_n \rightarrow C(x) \in \hat{S}_0 \quad (§)$$



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Generalization of  $S_i$ :  $\hat{S}_i = f(\{Y \mid S_i \subseteq Y \subseteq \text{Rng } A_i\})$

## Notion of admissibility: propositions

Generalization of  $S_i$ :  $\widehat{S}_i = f(\{Y \mid S_i \subseteq Y \subseteq \text{Rng } A_i\})$

What collection  $\mathcal{X} = \{\widehat{S}_i \mid S_i \subseteq \text{Rng } A_i\}$  would do?

What choice function(s) can in practice capture these expected algebraic properties?

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**What collection  $\mathcal{X} = \{\hat{S}_i \mid S_i \subseteq \text{Rng } A_i\}$  would do?**

- (i)  $\text{Rng } A_i \in \mathcal{X}$
- (ii) if  $X$  and  $Y$  are in  $\mathcal{X}$  then so  $X \cap Y$ .
  - ▶  $\mathcal{X}$  is a closure system upon  $\text{Rng } A_i$ .
  - ▶  $\hat{\cdot}$  is an operation enjoying weaker properties than closure operators; alternatives looked at:
    - ▶ pre-closure operator
    - ▶ capping operator

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**What choice function(s) can in practice capture these expected algebraic properties?**

- ▶ Proposal for some classes of choice functions generating specific types of operators
- ▶ Concrete examples of such functions for numerical rules

# Weakening closure operators

- ▶ List of Kuratowski's axioms [Kur14] (closure system):

$$\begin{aligned}\widehat{\emptyset} &= \emptyset \\ S \subseteq \widehat{S} &\subseteq \text{Rng } A; \\ \widehat{\widehat{S}} &= \widehat{S} \\ \widehat{S \cup S'} &= \widehat{S} \cup \widehat{S'} \quad (\text{pre-closure})\end{aligned}$$

- ▶ Actually, we downgrade Kuratowski's axioms as follows

$$\begin{aligned}\widehat{S} \subseteq \widehat{S'} &\text{ whenever } S \subseteq S' && (\text{closure}) \\ \widehat{S} = \widehat{S'} &\text{ whenever } S \subseteq S' \subseteq \widehat{S} && (\text{cumulation}) \\ \widehat{S \cup S'} \subseteq \widehat{S} &\text{ whenever } S' \subseteq \widehat{S} && (\text{capping})\end{aligned}$$

**Lemma:** Kuratowski  $\Rightarrow$  closure  $\Rightarrow$  cumulation  $\Rightarrow$  capping

## Class of choice functions satisfying pre-closure

**Theorem.** Given a set  $Z$ , let  $f : 2^{2^Z} \rightarrow 2^Z$  be a function st for every upward closed  $\mathcal{X} \subseteq 2^Z$  and every  $\mathcal{Y} \subseteq 2^Z$ :

1.  $f(2^Z) = \emptyset$
2.  $f(\mathcal{X}) \in \mathcal{X}$
3.  $f(\mathcal{X} \cap \mathcal{Y}) = f(\mathcal{X}) \cup f(\mathcal{Y})$   
whenever  $\bigcup \min(\mathcal{X} \cap \mathcal{Y}) = \bigcup \min \mathcal{X} \cup \bigcup \min \mathcal{Y}$

Then,  $\hat{\cdot} : 2^Z \rightarrow 2^Z$  as defined by

$$\hat{X} \stackrel{\text{def}}{=} f(\{Y \mid X \subseteq Y \subseteq Z\})$$

is a pre-closure operator upon  $Z$ .

*Intuition:*  $Z$  is  $\text{Rng } A_i$

$\mathcal{X}$  (and  $\mathcal{Y}$ , too) is a collection of intervals over  $\text{Rng } A_i$ ;  
moreover,  $\mathcal{X}$  is a collection containing all super-intervals  
of an interval belonging to the collection

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whenever  $\bigcup \min(\mathcal{X} \cap \mathcal{Y}) = \bigcup \min \mathcal{X} \cup \bigcup \min \mathcal{Y}$

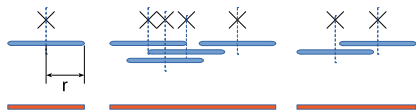
Then,  $\widehat{\cdot} : 2^Z \rightarrow 2^Z$  as defined by

$$\widehat{X} \stackrel{\text{def}}{=} f(\{Y \mid X \subseteq Y \subseteq Z\})$$

is a pre-closure operator upon  $Z$ .

Numerical attributes: principle of **single point ( $u$ ) interpolation**

$$A_i(x) \in [u - r : u + r] \rightarrow C(x) = c.$$



## Class of choice functions satisfying capping

**Theorem.** Given a set  $Z$ , let  $f : 2^{2^Z} \rightarrow 2^Z$  be a function st for every  $\mathcal{X} \subseteq 2^Z$  such that  $\bigcap \mathcal{X} \in \mathcal{X}$  and for every  $\mathcal{Y} \subseteq 2^Z$

1.  $f(\mathcal{X}) \in \mathcal{X}$
2. if  $\mathcal{Y} \subseteq \mathcal{X}$  and  $\exists W \in \mathcal{Y}, W \subseteq f(\mathcal{X})$  then  $f(\mathcal{Y}) \subseteq f(\mathcal{X})$

Then,  $\hat{\cdot} : 2^Z \rightarrow 2^Z$  as defined by

$$\hat{X} \stackrel{\text{def}}{=} f(\{Y \mid X \subseteq Y \subseteq Z\})$$

is a capping operator upon  $Z$ .

*Intuition:*  $Z$  is Rng  $A_i$ ;

$\mathcal{X}$  (and  $\mathcal{Y}$ , too) is a collection of intervals over Rng  $A_i$ ;

moreover,  $\mathcal{X}$  is a collection whose intersection

is itself a member of the collection



# Class of choice functions satisfying capping

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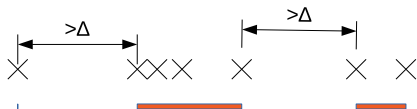
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is a capping operator upon  $Z$ .

Numerical attributes: principle of pairwise point interpolation



# And now . . .

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# Illustrations of the behaviour of CN2

Generation of synthetic data:

- ▶ Data with 2 dimensions: a numerical attribute and a symbolic class attribute
- ▶ Data with two classes (green and blue)

Objective:

- ▶ Illustrate the behaviour of the rule learning algorithm in terms of characteristics of generalisation of examples
  - ▷ based on a pre-closure (*interpolation over single points*)
  - ▷ based on a capping (*interpolation over pairs of points*)

Using Algorithm CN2 [CN89]

# Separating interesting intervals

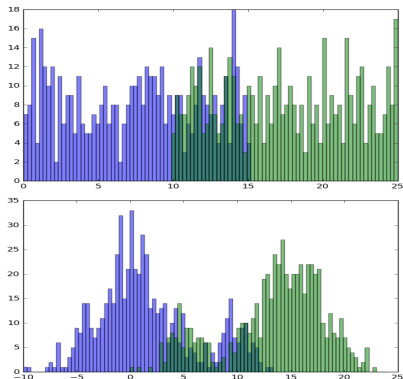


Figure: Distributions of the data for the classes blue and green.

Comparison of rules obtained out of uniform distributions vs. normal distributions

- ▶ distance between two successive values are small wrt the range of the attribute
- ▶ mixing normal distributions causes disparate average distances (pairwise distance between examples)
- ▶ the second dataset can be viewed as a super set of the first dataset (add of examples in between examples)

# Separating interesting intervals

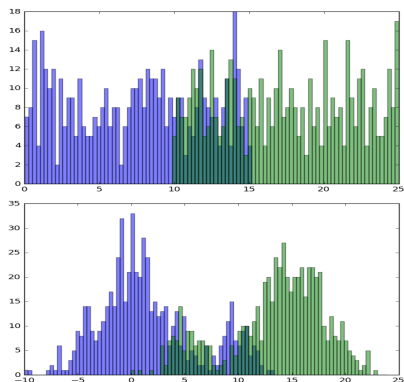


Figure: Distributions of the data for the classes blue and green.

Expected rules **assuming capping or pre-closure** for each class:

- topmost dataset:
  - $v \in [-\infty : 15] \Rightarrow A_0 = \text{blue}$
  - $v \in [10 : +\infty] \Rightarrow A_0 = \text{green}$
- bottom dataset:
  - $v \in [-\infty : 15] \Rightarrow A_0 = \text{blue}$
  - $v \in [0 : +\infty] \Rightarrow A_0 = \text{green}$

# Separating interesting intervals

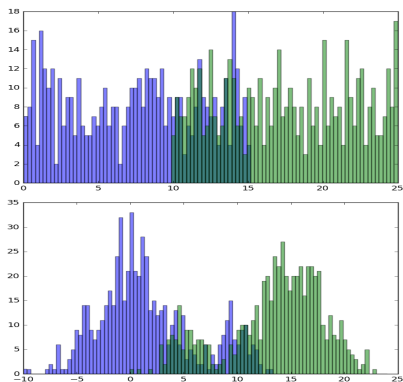


Figure: Distributions of the data for the classes blue and green.

Rules learned by CN2, topmost dataset:

- $v \in [-\infty : 10.03] \Rightarrow A_0 = \text{blue}$
- $v \in [12.73 : 14.83] \Rightarrow A_0 = \text{blue}$
- $v \in [10.65 : 12.81] \Rightarrow A_0 = \text{green}$
- $v \in [15.01 : +\infty] \Rightarrow A_0 = \text{green}$

Rules learned by CN2, bottom dataset:

- $v \in [-\infty : 0.96] \Rightarrow A_0 = \text{blue}$
- $v \in [0.97 : 2.57] \Rightarrow A_0 = \text{blue}$
- $v \in [3.09 : 10.04] \Rightarrow A_0 = \text{blue}$
- $v \in [3.50 : 7.18] \Rightarrow A_0 = \text{green}$
- $v \in [11.55 : 13.14] \Rightarrow A_0 = \text{green}$
- $v \in [13.15 : +\infty] \Rightarrow A_0 = \text{green}$

# Does density influence the choice of boundaries?

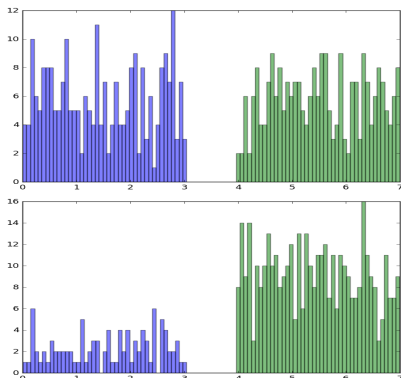


Figure: Distributions of the data for the classes blue and green.

Comparison of rules obtained out of well-separated uniform distributions, for two similar situations

- ▶ topmost dataset: same number of examples in both classes
- ▶ bottom dataset: the blue class is under-represented as compared to the green class

# Does density influence the choice of boundaries?

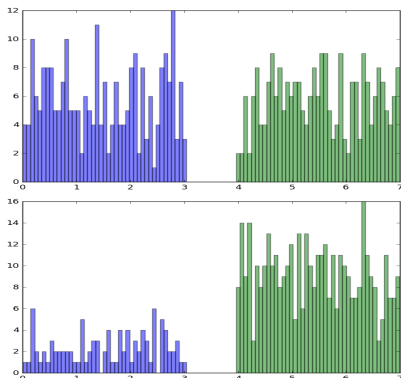


Figure: Distributions of the data for the classes blue and green.

Observed behaviour:

- ▶ no difference between the extracted rules for either dataset
- ▶ CN2 systematically chooses the boundary to be the middle of the limits in between the two classes



# Does density influence the choice of boundaries?

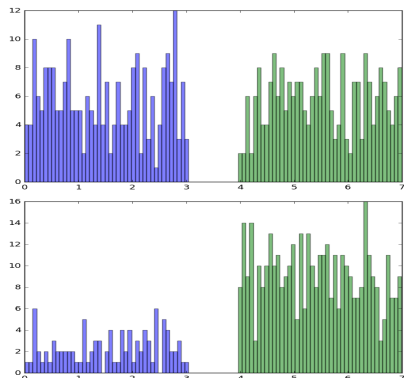
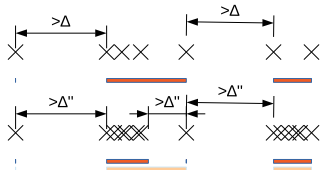


Figure: Distributions of the data for the classes blue and green.

Behaviour from *capping* :

- ▶ adding extra examples can alter boundaries



- ⇒ to be insensitive to density of examples corresponds to a *cumulation* operator

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## Conclusion (1)

- ▶ The logical structure of rules makes them easy to read  
**but ...**
- ▶ The interpretability of rules learned from examples requires, in particular, to take care of the way examples are generalized
  - ▶ Example of numerical attributes, but also symbolic attributes with structures (e.g. orders)
- ▶ Qualifying the interpretable nature of rule learning outputs is challenging

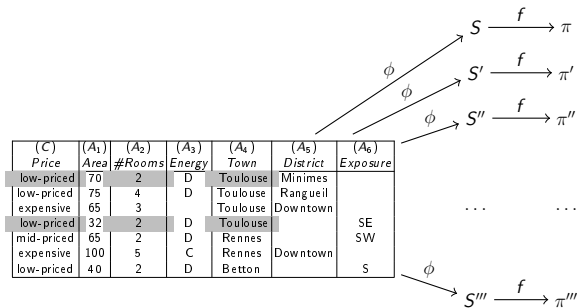
## What can our approach do for rule interpretability

- ▶ Our work contributes by giving a way to do such analysis
  - ▶ A proposal of a general framework for rule learning
  - ▶ A topological study of *admissible generalisations* of examples

# Conclusion (2)

## Formalisation of rule learning

- ▶  $\phi$ : selects possible subsets of data
  - ▶  $\hat{\cdot}$ : elicits the rule
- offer a framework for analysing rule learning algorithms



## Conclusion (3)

### Admissible generalisation of examples

- ▶ Admissible generalisations resulting of a choice among the supersets of the examples
- ▶ Proposed topological property of the choice: closure-like operators (pre-closure, capping)
- ▶ Definition of classes of choice functions
  - ▶ Proposal of concrete choice functions upon numerical attributes
  - ▶ Can be generalized to symbolic attributes, including attributes with structure (e.g. total order)

# Perspectives

- ▶ Long term objective: study the characteristics of extracted rulesets
  - ▶ Comparing the *set* of rules extracted by machine learning
- ▶ Need a formalism to represent a set of rules
  - ▶ A formalism that enables to represent
    - rules actually extracted by machine learning algorithms (e.g., Ripper, CN2, etc)
    - rules selected using a selection criteria (interestingness measures, etc)
  - ▶ Formalize essential notions of rule learning
  - ▶ The formalism will be a way to reason about the machine learning algorithms

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