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The effects of deterioration models on the value of damage detection information

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ABSTRACT: This paper addresses the effects of the deterioration on the value of damage detection information. The quantification of the value of damage detection information for deteriorated structures is based on Bayesian pre-posterior decision analysis, comprising structural system performance models, consequence, benefit and costs models and damage detection information models throughout the service life of a structural system. The value of damage detection information accounts for the relevance and precision of the information to ensure the structural integrity and to reduce the potential structural system risks and expected costs throughout the service life before implementing damage detection system. With the developed approach, the value of damage detection information for a statically determinate Pratt truss bridge girder subjected to different deterioration models is calculated. The analysis shows the impact of the deterioration model parameters on the value of damage detection information. The results can be used to develop optimal maintenance strategies before implementation of the damage detection system.

1 INTRODUCTION

Evaluation of the structural system performance with Damage Detection System (DDS) information has recently been introduced. An approach encompassing DDS and algorithms is developed by (Döhler and Thöns, 2016) and the reliability of a structural system of two components with DDS information is updated. Building upon the developed approach, the value of DDS information for a deteriorating Pratt truss bridge system is quantified by (Long et al., 2017). However, these studies contained only a general and generic deterioration model. This paper addresses the impact of various deterioration rates corresponding to different deterioration environment on the cumulative probability of bridge failure over time, which results in different service life benefits and value of DDS information.

The approach takes basis in the value of DDS information as the relevance for the reduction of the structural system risks and the expected cost of the system integrity management throughout the life cycle. The results can be used to identify how deterioration models influence the value of DDS information, to better predict the service life of the truss bridge girder, and to develop optimal lifetime reliability-based maintenance strategies for these bridges.

The description of structural system performance is developed in section 2. The general deterioration model subject to various deterioration types is introduced. The resistance degradation is explained and the deterioration model is coupled into the probabilistic models. In section 3, the DDS performance is described and the Bayesian approach to update the structural system reliability with DDS information is introduced. The method for the quantification of the value of information for structural system is presented in section 4. The approach is applied to a truss bridge girder to address the effects of different structural deterioration rate on the value of DDS information, which is developed in section 5. Section 6 is the conclusion.

2 STRUCTURAL SYSTEM PERFORMANCE

2.1 Deterioration model

The event of general deterioration failure of component i at time t can be expressed by a limit state function $g_{D_i}(t)$, which describes the deterioration model so that probability of component damage can be calculated as $P_{D.i} = P(g_{D_i}(t) \le 0)$ (Straub and Der Kiureghian, 2011, Thöns, 2018):

$$g_{D_i}(t) = \Delta_i - D_i(t) \tag{1}$$

where Δ_i is the damage limit of component i, $D_i(t)$ is the time-variant continuous damage state for component i at time t. The time variant damage state can be presented in terms of three parameters (Enright

and Frangopol, 1998b, Straub and Der Kiureghian, 2011):

$$D_i(t) = \alpha (t - T_i)^{\beta} \tag{2}$$

 α is annual deterioration rate, β is the deterioration type, T_j is the deterioration initiating time at time j for element i. For β =1, this corresponds to most applied corrosion models and to the Palmgren-Miner fatigue model with a stationary stress process; for β =0.5, the model is representative of diffusion controlled deterioration; and for β =2, the model approximates concrete deterioration owing to sulfate attack.

2.2 Resistance degradation

The structural resistance represents the internal material properties and geometrical characteristics of elements, which can be modeled with the reduction of the initial component resistance in dependency of the resistance degradation function and damage state.

$$R_i(t) = R_{i,0} \cdot g_{D_i}(t) \cdot f_R(\mathbf{D}) \tag{3}$$

 $R_i(t)$ is the time variant resistance for component i, $R_{i,0}$ is the initial resistance of element i and $g_{D_i}(t)$ is the deterioration limit state function from section 2.1, $f_R(\mathbf{D})$ is the degradation function between resistance and damage states, since resistance is continuously reduced due to the accumulated damage evolution.

2.3 Stiffness degradation

With the increase of damage states, the degradation of resistance of each component will lead to the loss of element stiffness, which can be detected through Structural Health Monitoring (SHM) techniques. The generic stiffness model is independent of initial stiffness, stiffness degradation function and damage state.

$$k_i(t) = k_{i,0} \cdot g_{D_i}(t) \cdot f_k(\mathbf{D}) \tag{4}$$

Where $k_i(t)$ is the time variant stiffness for component i, $k_{i,0}$ is the initial resistance of element i and $f_k(\mathbf{D})$ is the degradation function between stiffness and damage states.

2.4 Probabilistic model of system

For any structural model, failure occurs when the external load S exceeds internal resistance $R_i(t)$ due to increase of damage and degradation. Considering the resistance model uncertainties M_R , and the loading model uncertainty M_S , the probability of series-parallel system failure can be written as:

$$P(F_S) = P\left(\bigcap_{i=1}^{n_i} \bigcup (M_R R_i(t) - M_S S_i) \le 0\right)$$
 (5)

Let $g_i(\mathbf{X}, \mathbf{D})$ denote the limit state function, shown in Equation 6, such that $g_i(\mathbf{X}, \mathbf{D}) \leq 0$ represents the condition of failure of structure component. The vector of the system performance random variables \mathbf{X} then comprises the resistance model uncertainties M_R , the time dependent component resistances $R_i(t)$, the loading model uncertainty M_s and the component loading S. The vector of the system degradation random variables \mathbf{D} contains the collection of the deterioration states for all components. Monte Carlo simulation can be used to find the cumulative probability of system failure throughout the life cycle.

$$g_i(\mathbf{X}, \mathbf{D}) = M_R R_{i,0} \cdot g_{D_i}(t) \cdot f_R(\mathbf{D}) - M_s S_i$$
 (6)

3 DDS INFORMATION UPDATING

3.1 Damage detection systems

The stiffness changes in the elements of the structure can be detected with a damage detection method. Based on ambient vibration measurements from a (healthy) reference state and measurements from the current system, e.g. the Stochastic Subspace Damage Detection (SSDD) method computes a hypothesis test statistic that compares both states (Döhler et al., 2014). This test statistic results in a χ^2 -distributed damage indicator, having a central χ^2 distribution in the reference state and a non-central χ^2 distribution in the damaged state.

A threshold is set up in the distribution of the reference state for a desired type I error for a decision between reference and damaged states. Based on such a threshold and the theoretical properties of the χ^2 distribution for any given damage, the desired probability of indication of such a damage can be calculated without using measurement data (Döhler and Thöns, 2016).

3.2 Bayesian updating

Based on the damage state, the DDS can provide the indication or no indication of the probability of damage. The updated probability of failure if given no indication of damage $P(F_s|\mathbf{D},\bar{I})$ can be calculated through Bayesian updating (Hong, 1997):

$$P(F_{S}|\mathbf{D},\bar{I}) = \frac{P(\bar{I}|F_{S},\mathbf{D})P(F_{S},\mathbf{D})}{P(\bar{I})}$$
$$= \frac{P(F_{S}|\mathbf{D}\cap\bar{I})}{P(\bar{I})}$$
(7)

 $P(\bar{I})$ is the probability of no indication, $P(\bar{I}|F_s, \mathbf{D})$ is the probability of no indication given damage failure. $P(F_s, \mathbf{D})$ is the probability of damage failure. To solve this equation, the joint function of two limit states is computed. The limit state function of system failure $g_s \leq 0$ can refer to Equation 6.

$$P(F_{S}|\bar{I}, \mathbf{D}) = \frac{P(g_{S} \le 0 \cap g_{u} \le 0)}{P(g_{u} \le 0)}$$
(8)

The probability of no indication of detecting damage $P(\bar{I})$ can be calculated by integrating in the region which is defined with the limit state function $g_U \leq 0$. The limit state function g_U is defined as the difference between the probability of indication given damage $P(I|\mathbf{D})$ and a uniformly distributed random variable u (Thöns and Döhler, 2012). $P(I|\mathbf{D})$ can be calculated through realization of damage state from section 3.1.

$$g_U = P(I|\mathbf{D}) - u \tag{9}$$

4 VALUE OF INFORMATION

The quantification of the value of monitoring information for deteriorated structures is based on Bayesian (pre-)posterior decision analysis, comprising decision rules, structural system performance models, probabilistic reliability models, consequences analysis as well as benefit and costs analysis associated with monitoring results over their life cycle. The value of DDS information is quantified by calculating the difference of the service life utilities with and without damage detection information, show in Equation 10,11 and 12, where \mathbf{i} is the choice of information strategy, \mathbf{Z} is the possible outcomes, \mathbf{a} is the choice of the action and $\mathbf{\theta}$ is the system states, u_0^* is the prior utility, u_i^* is the (pre-)posterior utility, E is the expected value (Thöns, 2018).

$$VoI_i = u_i^* - u_0^* (10)$$

$$u_0^* = \max_{\mathbf{a}} E_{\mathbf{\theta}}'[u(\mathbf{a}, \mathbf{\theta})] \tag{11}$$

$$u_i^* = \max_{\mathbf{i}} E_{\mathbf{Z}|\mathbf{i}} \left[\max_{\mathbf{a}} E_{\mathbf{\theta}|\mathbf{Z}}^{"}[u(\mathbf{i}, \mathbf{Z}, \mathbf{a}, \mathbf{\theta})] \right]$$
(12)

5 EXAMPLE

With the developed approach, the value of damage detection information for a statically determinate Pratt truss bridge girder subjected to different deteriorating rate is calculated. The service life of the truss bridge girder is assumed to be 50 years.

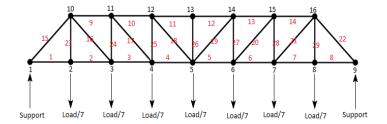


Figure 1. Pratt truss.

5.1 Deterioration model

The Pratt truss bridge girder is composed with 29 components, shown in Figure 1. This paper focusses on the corrosion and fatigue deterioration on the steel truss bridge girder, so that β =1 is chosen. Assume the girder is under different severity of corrosion or fatigue. The deterioration model is chosen following Equation 13. Once deterioration has been initiated, the cross-section area of the components decreases with time at a different rate. As indicated in Table 1, three cases were considered. They are associated with assumptions of low, medium and high deterioration corresponding to different environment or structural materials.

$$D_i(t) = \alpha(t - T_i) \tag{13}$$

Table 1. Deterioration parameters

Table 1. Deterioration parameters							
Degrada-	$E(T_i)$	Distribu-	Mean	Standard			
tion rate α	(yr)	tion		deviation			
Low	15	Lognor- mal	0.000013	0.001			
Medium	10	Lognor- mal	0.000076	0.001			
High	5	Lognor- mal	0.000254	0.001			

When it is under low deterioration, it initiates from year 15 with a mean of deterioration rate of 0.000013, for instance when it is in an industrial environment. When in medium deterioration, it starts from year 10 with a mean deterioration rate of 0.000076. When it is highly deteriorated, it will start from year 5 with mean rate of 0.000254 such as the bridge is in the marine environment (Tinnea, 2012, Enright and Frangopol, 1998a).

5.2 Resistance Degradation

The resistance is degraded following Equation 14. The description of structural reliability model of the truss bridge girder is summarized in Table 2. The expected value of resistance degradation E(R(t)) in three

cases is shown in Figure 2, which gives a linear degradation.

$$R_i(t) = R_{i,0} \left(\Delta_i - \alpha(t - T_j) \right) \tag{14}$$

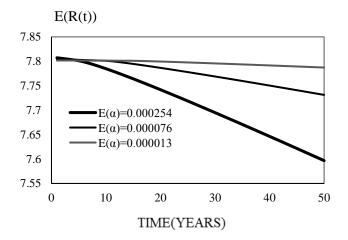


Figure 2. Expected resistance degradation in service life.

5.3 Prior probability failure

The structural system performance of the Pratt truss bridge girder builds upon a series system and is coupled with time-variant damage models describing continuously the deterioration process and structural resistance degradation throughout the service life:

$$P(F_S) = P\left(\bigcup_{i=1}^{n_i} (M_R R_i(t) - M_S S_i) \le 0\right)$$
 (15)

For the series system, the system limit state function is the minimum of the components limit state function:

$$g_s = \min_{i=1} \left(M_R \ R_{i,0} \left(\Delta_i - \alpha (t - T_j) \right) - M_s S_i \right)$$
 (16)

The mean of the initial resistance $R_{j,0}$ is calibrated to a probability of system failure of 10^{-6} disregarding any damage. The initial probability of failure is associated to large consequence of failure and small relative cost of safety measures (JCSS, 2006).

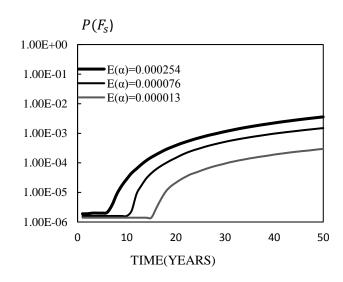


Figure 3. Prior probability of system failure in service life.

The probability of system failure is calculated by Monte Carlo simulations. The probability of truss failure will increase with time due to deterioration damage and it is shown in Figure 3. With a higher expected value of the deterioration rate, the resistance of the bridge will degrade faster and the truss bridge will have a higher probability of failure at the end of service life.

Table 2. Deterioration parameters

Random variable	Distribu-	Mean	Standard
	tion		deviation
Loading S_i	WBL	3.50	0.1
Model uncertainty M_s	LN	1.00	0.1
Component re-	LN	Cali-	0.1
sistances in undam-		brated	
aged state $R_{i,0}$			
Model uncertainty M_R	LN	1.00	0.1
Damage limit Δ_i	LN	1.00	0.3
Coefficient resistance	Determin-	0.9	
correlation ρ _R	istic	0.9	
Coefficient damage	Determin-	0.9	
correlation ρ _D	istic	0.7	

5.4 DDS and probability of damage indication

The damage detection system (DDS) is detecting the stiffness loss of each element of the structure, which is expressed as relative changes of ratio of the initial axial stiffness:

$$dk_i = 1 - \frac{k_i}{k_{i,0}} \tag{17}$$

When the truss is under tensile, stiffness has a relation with cross section area, length and Young's modulus, $k_i = E * A_i(t)/L$. The cross section is reduced due to the increase of damage states:

$$A_{i}(t) = A_{i,0} - h(D_{i}(t))$$
(18)

 $A_{i,0}$ is the initial cross section area, $A_i(t)$ is the cross-sectional area at time t, h is the function between damage state and the cross-section area. So that the relation between damage state and stiffness loss can be expressed as:

$$dk_i = 1 - \frac{A_i(t)}{A_{i,0}} = 1 - \frac{A_{i,0} - h(D_i(t))}{A_{i,0}}$$
 (19)

$$dk_i = \gamma \cdot h(D_i(t)) \tag{20}$$

 γ is the correction factor, in which $\gamma = 1/A_{i,0}$

The damage detection system is modelled with the acceleration sensors located in node 12, 13, 14 of the truss in Y-direction recording the response using the SSDD algorithm. Based on the dynamic structural system model, a reference dataset of length N = 10000 at a sampling frequency of 50 Hz is simulated in the undamaged state. Ambient excitation (white noise) is assumed at all degrees of freedom whose covariance is the identity matrix. Measurement noise is added on the resulting accelerations with standard deviation at each sensor of 5% of the standard deviation of the signal. The type I error for the indication threshold is 1%.

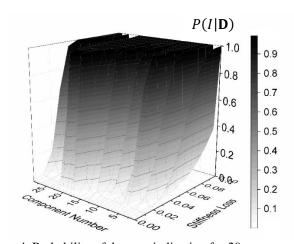


Figure 4. Probability of damage indication for 29 components in dependency of stiffness loss.

The probability of indication of damage independency of discrete damage states (represented by the stiffness loss) for the 29 individual components is shown in Figure 4. It should be noted that the joint probability of indication is not depicted but accounted for. The probability of damage indication for continuously damage state for one component is calculated based on Equation 20 and the results for three cases are shown in Figure 5, where correction factor is 1 and $dk_i = D_i(t)$. With higher expected value of deterioration rate, the damage will have accumulated

faster and the truss bridge will have higher probability of damage indication.

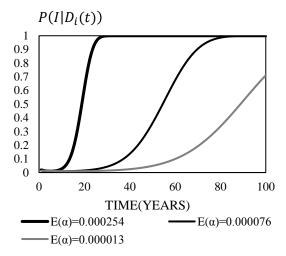


Figure 5. Probability of damage indication for one component during service life.

5.5 Bayesian updating

The updated probability of system failure given damage detection information is computed utilizing Bayesian updating theorem. If implementing the DDS in a specific year and updating the results of probability of no indication of damage, the (pre-)posterior probability of system failure is calculated based on the Equations 8, 9 and 16. The results when implementing DDS at year 14 if expected deterioration rate is 0.000254, at year 18 if expected deterioration rate is 0.000076, at year 30 if expected deterioration rate is 0.000013 are shown in Figure 6.

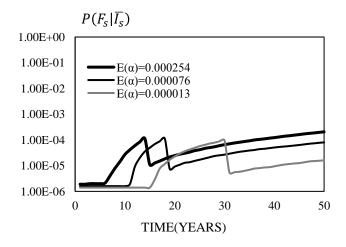


Figure 6. (Pre-)posterior probability of system failure if implementing DDS at certain year.

It can be seen from the figure that the (pre-)posterior of probability of failure when given no indication of damage will be reduced significantly.

5.6 Value of information

The bridge manager wants to carry out a repair plan but he is not sure whether or when to implement damage detection system as the bridge is observed to experience an unusually high deterioration. Therefore, a value of information analysis is made to provide the decision basis. The illustration of decision tree is shown in Figure 7.

The investment cost of the bridge is C_L The decision nodes are implementing DDS or no DDS and repair or do nothing. When implementing DDS, there will be a cost of DDS, C_{DDS} ; when performing a repair action, there will be a repair cost, C_R ; when DDS is providing a damage indication of the bridge, there will be a damage localization cost, C_{loc} . The state of bridge is either safe or failure. If it is failed, there will be a failure cost, C_F , accounting for the direct and indirect consequences. Considering the discount rate, the costs model is shown in Table 3.

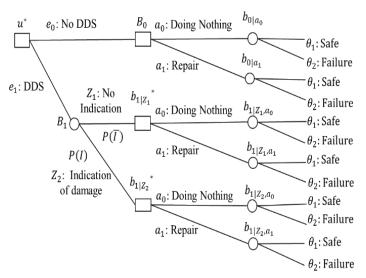


Figure 7. Illustration of decision tree combining a prior decision analysis and a pre-posterior decision analysis.

The cost of repair is increased with time due to the fact of increased severity of damage, which followed the Equation 21 (Higuchi, 2008). When the bridge is repaired, it performs as a new one with the same probabilistic characteristics as originally, in which probability of system failure after repair equals to 10^{-6} . The system is required to take repair action when probability of failure is exceeded of 10^{-4} according to the same target reliability class with high costs of safety measures.

$$C_R = \frac{C_I}{T_{SL} + 2 - t} \tag{21}$$

Table 3. Costs model

Discount	Invest-	Failure	Localiza-	DDS cost
rate <i>r</i>	ment	$cost C_F$	tion cost	C_{DDS}
	$cost C_I$		C_{loc}	
0.02	10	1000	0.1	0.1

Figures 8, 9 and 10 show the repair plan of the bridge during service life when installing DDS at different years under specific deterioration conditions. If without DDS, the system will need to be repaired three times when deterioration is large, to be repaired two times when deterioration is medium and be repaired one time when the deterioration is small.

If utilizing DDS at certain year, the repair times of the system can be reduced, for example, reduced to two times if implementing DDS at year 14 when deterioration is large. When the deterioration is medium and small, if carrying out DDS separately at year 18 and 30, the bridge will not need to be repaired during service life.

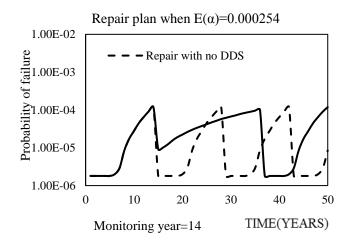


Figure 8. Repair plan during service life if implementing DDS at year 14 when expected mean deterioration rate is $E(\alpha)=0.000254$.

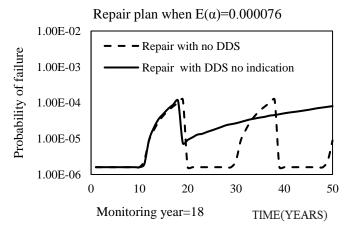


Figure 9. Repair plan during service life if implementing DDS at year 18 when expected mean deterioration rate is $E(\alpha)=0.000073$.

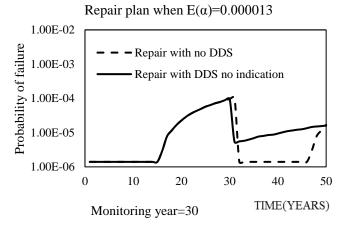


Figure 10. Repair plan during service life if implementing DDS at year 30 when expected mean deterioration rate is $E(\alpha)=0.000013$.

When the deterioration rate is large, it will deteriorate from year 5. From Figure 3, it is shown that the probability of failure will reach the threshold 10^{-4} in year 14. After year 14, it will always take two repair actions and the value of DDS information may be low. Therefore, the VoI analysis is only focused on the period from year 6 to year 14. When installing DDS at year 6 and 7, the bridge needs to be repaired two times if given no indication of damage. If installing from year 8 to year 11, the repair times can be reduced to only one time, which will lead to a higher value of DDS due to lower sum of repair costs. However, when implementing DDS from year 12 due to shorter impact on the risk reduction, the bridge needs to be repaired two times again, which will result in a negative value of information (VoI), shown in Figure 11. The maximum VoI will be in year 8, with 24% of relative value of DDS information compared with benefits of not doing DDS.

When the deterioration rate is medium, it will deteriorate from year 10. From Figure 3, it is shown that the probability of failure will reach to the threshold 10^{-4} in year 18. After year 18, it will always take repair action, the DDS information will have no impact on the decision. Therefore, the VoI analysis only focuses on period from year 11 to year 18 when deterioration is 0.000076. If implementing DDS at year 11 and 12, the system will need to be repaired one time given no indication of damage, which results in a higher repair costs in the late of life cycle and a negative VoI, shown in Figure 12. However, since year 13, the bridge will no longer need to be repaired if given no damage indication due to a large reduction of risk. The Value of DDS Information is decreasing in the consecutive years as the period for which the DDS information provide a risk reduction becomes shorter. When the deterioration rate is medium, implementing DDS at year 13 can get the maximum VoI with 28% of relative of value of information.

When the deterioration rate is small, it will deteriorate from year 15. From Figure 3, the probability of failure will reach to the threshold 10^{-4} in year 30. There will be no need to implement DDS after year 30 because the system will always require one repair action. Therefore, the low deterioration VoI analysis focuses on period from year 16 to year 30. The implementation of DDS will always have a positive effect leading to no repair costs during service life because of the large reduction of the risks (Figure 13). The Value of DDS Information is increasing at the beginning due to reduction of accumulated failure risk and decreasing in the consecutive years as the period for which the DDS information provide a risk reduction becomes shorter. The maximum VoI will be in year 20 when expected deteriorating rate is 0.000013 with 67% of relative value of DDS information.

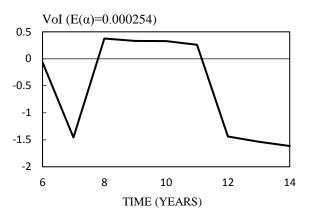


Figure 11. Value of DDS information independency of the DDS implement year when expected deterioration rate is $E(\alpha)=0.000254$.

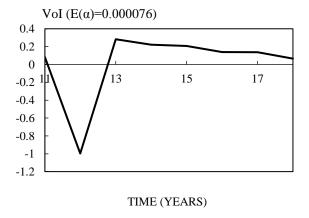


Figure 12. Value of DDS information independency of the DDS implement year when expected deterioration rate is $E(\alpha)=0.000076$.

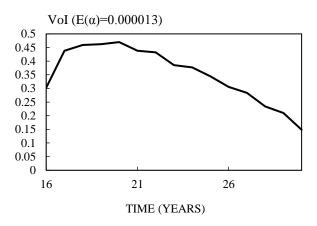


Figure 13. Value of DDS information independency of the DDS implement year when expected deterioration rate is $E(\alpha)$ =0.000013.

6 CONCLUSION

It has been demonstrated that the structural deterioration has an effect on the value of damage detection information. The analysis shows the impact of various deterioration rates corresponding to different types of deteriorating environments on the cumulative probability of bridge failure over time, which results in different service life benefits and values of damage detection information.

The example of deteriorating truss bridge girder illustrates that the value of information is highly depending on how much the risk and the accumulated repair costs can be reduced during the service life. The maximum relative value of DDS information will be higher if the deterioration rate is smaller. The results can provide decision basis to develop optimal lifetime maintenance strategies before implementation of the damage detection system for bridges under different deterioration processes.

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