



POLYNOMIAL SPACE-TIME TREFFTZ-DG METHOD FOR ELASTO-ACOUSTICS

MATHIAS'18 | Computational Science Engineering & Data Science by TOTAL

CONTEXT

TREFFTZ-DG METHODS

BIBLIOGRAPHY

Frequency domain:

Farhat, Tezaur, Harari, Hetmaniuk (2003 - 2006), Gabard (2007), Badics (2014), Hiptmair, Moiola, Pirugia (2011 - 2016), Barucq, Bendali, Diaz, Tordeux, Fares, Mattesi (2017)

Time domain:

Tsukerman, Egger, Kretzschmar, Schnepf, Weiland (2014 - 2015), Wang, Tezaur, Farhat (2014), Moiola, Pirugia (2016 - 2018), Banjai, Lijoka (2017)

DISCONTINUOUS GALERKIN METHODS

PROS AND CONS

- Adapted to the complex geometries
- High-order accuracy and hp-adaptivity
- Explicit semi-discrete form
- Conservation laws

Higher number of degrees of freedom, compared to the methods with continuous approximation

TREFFTZ METHOD

E. TREFFTZ, 1926

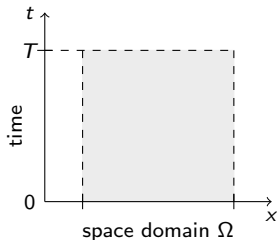
Basis functions are **local solutions of the initial PDEs**

MATHEMATICAL FORMULATION

ACOUSTIC SYSTEM

ACOUSTIC SYSTEM

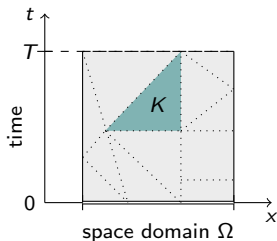
DOMAIN DESCRIPTION



$$\left\{ \begin{array}{ll} \frac{1}{c^2 \rho} \frac{\partial \mathbf{p}}{\partial t} + \operatorname{div} \mathbf{v} = f & \text{in } \Omega \times [0, T], \\ \rho \frac{\partial \mathbf{v}}{\partial t} + \nabla \mathbf{p} = 0 & \text{in } \Omega \times [0, T], \\ \mathbf{v}(\cdot, 0) = \mathbf{v}_0, \mathbf{p}(\cdot, 0) = p_0 & \text{in } \Omega, \\ \mathbf{v} \cdot \mathbf{n}^x = g_D & \text{in } \partial\Omega \times [0, T]. \end{array} \right.$$

ACOUSTIC SYSTEM

SPACE-TIME DISCRETIZATION

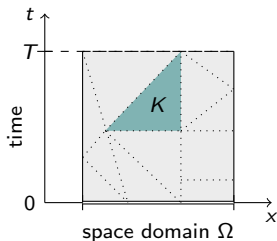


$$\mathcal{F}_h = \bigcup_{K \in \mathcal{T}_h} \partial K - \text{mesh skeleton}$$

- \mathcal{F}^I - internal
- \mathcal{F}^D - boundary
- == \mathcal{F}^0 - initial time
- \mathcal{F}^T - final time

ACOUSTIC SYSTEM

SPACE-TIME DISCRETIZATION



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..... \mathcal{F}^I - internal

— \mathcal{F}^D - boundary

== \mathcal{F}^0 - initial time

-- \mathcal{F}^T - final time

$$(\mathbf{v}_h, p_h) \in \mathbf{V}^h(\mathcal{T}_h)^d \times V^h(\mathcal{T}_h) \equiv \mathbf{V}^h(\mathcal{T}_h)$$

$$(\omega, q) \in V^h(\mathcal{T}_h)^d \times V^h(\mathcal{T}_h) \equiv \mathbf{V}^h(\mathcal{T}_h)$$

ACOUSTIC SYSTEM

SPACE-TIME DISCRETIZATION

$$\begin{aligned}
 & - \sum_K \int_K \mathbf{p}_h \left(\frac{1}{c^2 \rho} \frac{\partial \mathbf{q}}{\partial t} + \operatorname{div} \boldsymbol{\omega} \right) + \mathbf{v}_h \cdot \left(\rho \frac{\partial \boldsymbol{\omega}}{\partial t} + \nabla q \right) \\
 & + \sum_K \int_{\partial K} \left(\frac{1}{c^2 \rho} \check{\mathbf{p}}_h q + \check{\mathbf{v}}_h \cdot \boldsymbol{\omega} \right) \mathbf{n}_K^t + \left(q \hat{\mathbf{v}}_h + \hat{\mathbf{v}}_h \omega \right) \cdot \mathbf{n}_K^x = \sum_K \int_K f q
 \end{aligned}$$

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ACOUSTIC SYSTEM

SPACE-TIME DISCRETIZATION

$$\begin{aligned}
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 \end{aligned}$$

TREFFTZ SPACE

$$\mathbf{T}(\mathcal{T}_h) \equiv \left\{ (\omega, q) \in \mathbf{V}^h(\mathcal{T}_h) \text{ s.t. } \frac{1}{c^2 \rho} \frac{\partial q}{\partial t} + \operatorname{div} \omega = \rho \frac{\partial \omega}{\partial t} + \nabla q = 0, \quad \forall K \in \mathcal{T}_h \right\}$$

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SPACE-TIME DISCRETIZATION

$$\begin{aligned}
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SPACE-TIME DISCRETIZATION IN \mathbf{T}

$$\sum_K \int_{\partial K} \left(\frac{1}{c^2 \rho} \check{\rho}_h q + \check{\mathbf{v}}_h \cdot \omega \right) n_K^\dagger + \left(q \hat{\mathbf{v}}_h + \hat{\mathbf{v}}_h \omega \right) \cdot \mathbf{n}_K^x = \sum_K \int_K f q$$

ACOUSTIC SYSTEM

TREFFTZ-DG FORMULATION

$$\sum_K \int_{\partial K} \left(\frac{1}{c^2 \rho} \check{p}_h q + \check{v}_h \cdot \omega \right) n_K^t + \left(q \hat{v}_h + \hat{v}_h \omega \right) \cdot n_K^x = \sum_K \int_K f q$$

NUMERICAL FLUXES THROUGH THE ELEMENT FACES

$$\begin{aligned} \begin{pmatrix} \hat{v}_h \\ \hat{p}_h \end{pmatrix} &\equiv \begin{pmatrix} \{v_h\} + \beta [[p_h]]_x \\ \{p_h\} + \alpha [[v_h]]_x \end{pmatrix} && \text{on } \mathcal{F}^I \\ \begin{pmatrix} \check{v}_h \\ \check{p}_h \end{pmatrix} &\equiv \begin{pmatrix} \{v_h\} \\ \{p_h\} \end{pmatrix} && \text{on } \mathcal{F}^I \end{aligned}$$

ACOUSTIC SYSTEM

TREFFTZ-DG FORMULATION

Seek $(\mathbf{v}_h, p_h) \in \mathbf{V}^h(\mathcal{T}_h)$ s.t. $\forall (\omega, q) \in \mathbf{T}(\mathcal{T}_h)$ it holds true:

$$\mathcal{A}_{TDG}\left((\mathbf{v}_h, p_h); (\omega, q)\right) = \ell_{TDG}(\omega, q)$$

ACOUSTIC SYSTEM

WELL POSEDNESS

Trefftz-DG formulations for the first order Acoustic System^{1,2}, Elastodynamic System², and Elasto-Acoustic System² are **well-posed.**

¹ A. Moiola & I. Perugia.

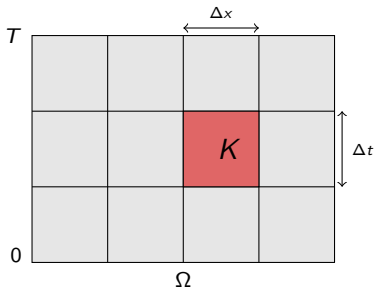
A space-time Trefftz- dG method for the acoustic wave equation in first-order formulation. (2018)
Numerische Mathematik, 138(2):389-435

² H. Barucq, H. Calandra, J. Diaz & Elvira Shishenina.

Space-Time Trefftz-Discontinuous Galerkin Approximation for Elasto-Acoustics. (2017)
<https://hal.inria.fr/hal-01614126/document>

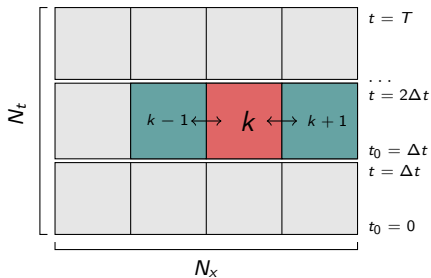
IMPLEMENTATION OF THE ALGORITHM

MESH AND ELEMENT NUMBERING



IMPLEMENTATION OF THE ALGORITHM

MESH AND ELEMENT NUMBERING



IMPLEMENTATION OF THE ALGORITHM

BILINEAR FORM

$$\mathcal{A}_{TDG} = \underbrace{\int_{\mathcal{F}^T} + \int_{\mathcal{F}^0}}_{\mathcal{A}_{TDG}^{\Omega}} + \underbrace{\int_{\mathcal{F}^I} + \int_{\mathcal{F}^D}}_{\mathcal{A}_{TDG}^I}$$

GLOBAL SPACE-TIME MATRIX

$$\mathbf{M} = \bar{\Delta}_x \mathbf{M}_{\Omega} + \bar{\Delta}_t \mathbf{M}_I$$

IMPLEMENTATION OF THE ALGORITHM

BILINEAR FORM

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\mathbf{M}_{Ω} - block-diagonal

IMPLEMENTATION OF THE ALGORITHM

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GLOBAL SPACE-TIME MATRIX

$$\mathbf{M} = \bar{\Delta}_x \mathbf{M}_{\Omega} + \bar{\Delta}_t \mathbf{M}_I$$

\mathbf{M}_{Ω} - block-diagonal \mathbf{M}_I - sparse

IMPLEMENTATION OF THE ALGORITHM

GLOBAL MATRIX INVERSION

$$\begin{aligned}\mathbf{M}^{-1} &\equiv \left[\bar{\Delta}_x \mathbf{M}_\Omega + \bar{\Delta}_t \mathbf{M}_I \right]^{-1} \\ &= \\ &\left[I + \frac{\bar{\Delta}_t}{\bar{\Delta}_x} (\mathbf{M}_\Omega^{-1} \mathbf{M}_I) \right]^{-1} \left[\bar{\Delta}_x \mathbf{M}_\Omega \right]^{-1}\end{aligned}$$

TAYLOR EXPANSION (SMALL ENOUGH $\frac{\bar{\Delta}_t}{\bar{\Delta}_x} |\mathbf{M}_\Omega^{-1} \mathbf{M}_I| \propto \frac{\Delta t}{\Delta x} |\mathbf{M}_\Omega^{-1} \mathbf{M}_I|$)

$$\mathbf{M}^{-1} \approx \left[\sum_{i=0}^n (-1)^i \left(\frac{\bar{\Delta}_t}{\bar{\Delta}_x} \right)^i (\mathbf{M}_\Omega^{-1} \mathbf{M}_I)^i \right] \left[\bar{\Delta}_x \mathbf{M}_\Omega \right]^{-1}$$

IMPLEMENTATION OF THE ALGORITHM

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block-diagonal

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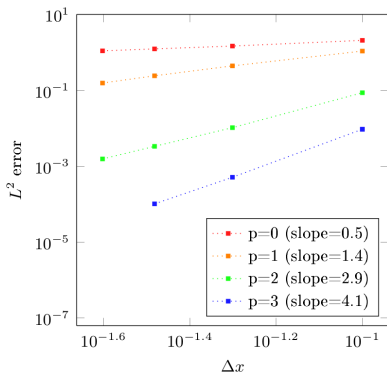
block-diagonal \times sparse

NUMERICAL RESULTS

2D ACOUSTIC AND ELASTODYNAMIC SYSTEMS

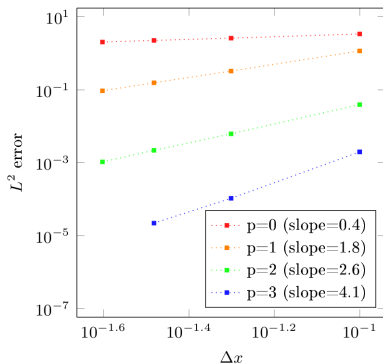
2D Acoustic domain.

Convergence of velocity v in function of cell size



2D Elastodynamic domain.

Convergence of velocity v in function of cell size



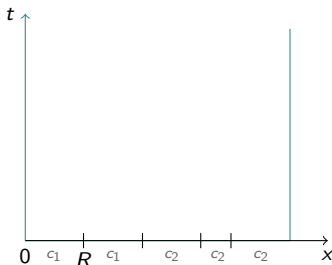
TENT PITCHING MESHES

APPLICATION TO TREFFTZ-DG METHOD

TENT PITCHING MESHES

1D WAVE EQUATION

$$\begin{cases} \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \Delta p = 0, \\ p(\cdot, 0) = p_0, \\ \frac{\partial p}{\partial t}(\cdot, 0) = p_1. \end{cases}$$



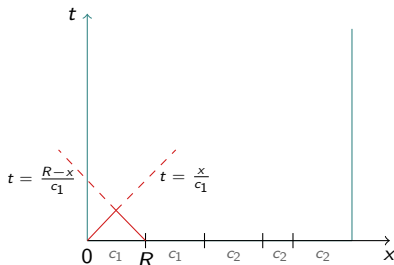
SOLUTION

$$p(x, t) = \frac{1}{2} \left(p_0(x - ct) + p_0(x + ct) \right) + \frac{1}{2c} \left(p_1(x - ct) + p_1(x + ct) \right)$$

TENT PITCHING MESHES

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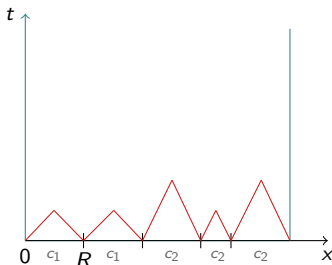
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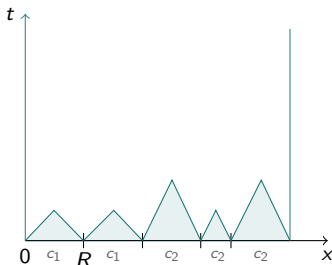
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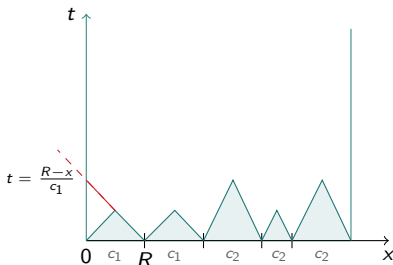
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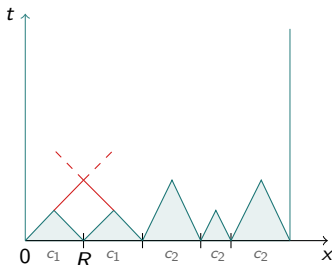
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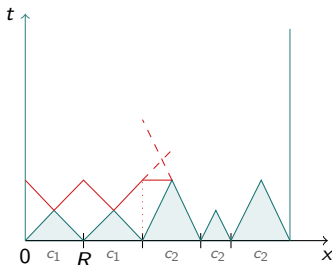
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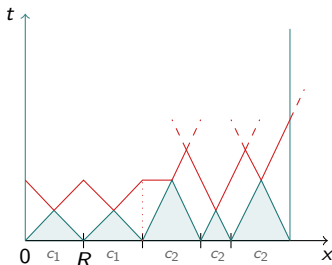
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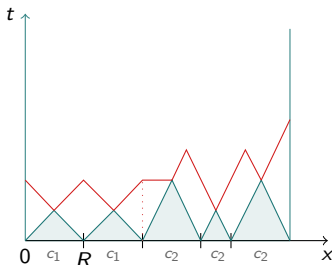
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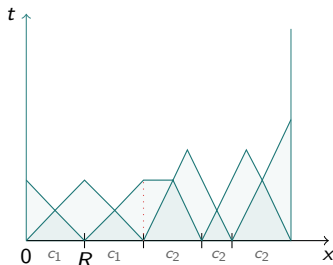
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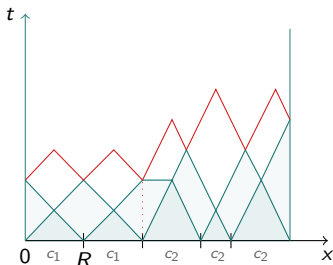
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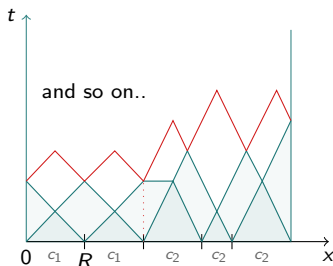
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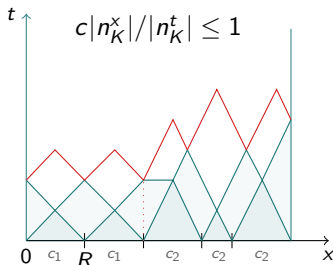
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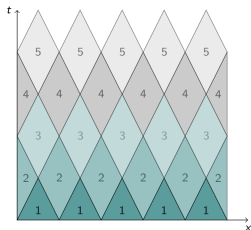


SOLUTION

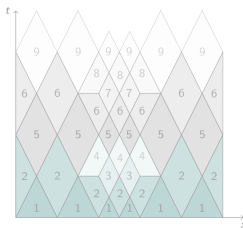
$$p(x, t) = \frac{1}{2} \left(p_0(x - ct) + p_0(x + ct) \right) + \frac{1}{2} \left(p_1(x - ct) + p_1(x + ct) \right)$$

TENT PITCHING MESHES

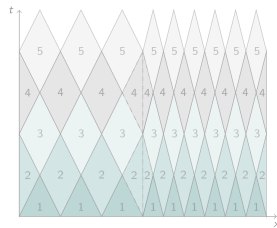
1D TENT PITCHING MESHES



Uniform tent mesh
(homogeneous medium)



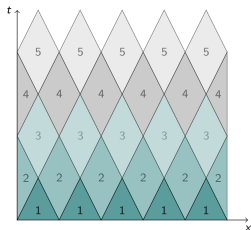
Non-uniform tent mesh
(homogeneous medium)



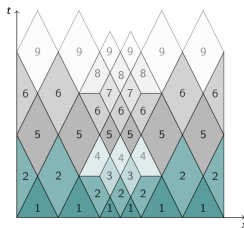
Non-uniform tent mesh
(heterogeneous medium)

TENT PITCHING MESHES

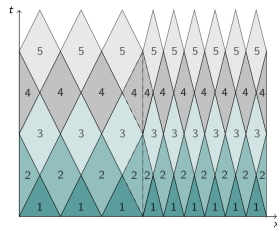
1D TENT PITCHING MESHES



Uniform tent mesh
(homogeneous medium)



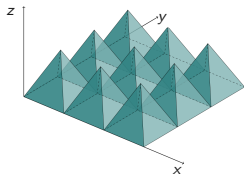
Non-uniform tent mesh
(homogeneous medium)



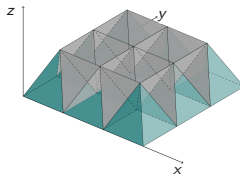
Non-uniform tent mesh
(heterogeneous medium)

TENT PITCHING MESHES

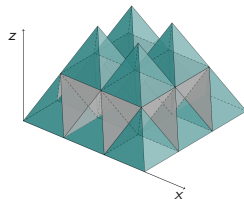
2D TENT PITCHING MESHES



1st tent



2nd tent



3rd tent

TENT PITCHING MESHES

ANALYTICAL STABILITY CONDITIONS ¹

$$c|n_K^x|/|n_K^t| \leq 1 \quad - \text{TDG acoustic formulation}$$

$$\sqrt{|\mathbf{C}|}|n_K^x|/\sqrt{\rho}|n_K^t| \leq 1 \quad - \text{TDG elastodynamic formulation}$$

¹ H. Barucq, H. Calandra, J. Diaz & Elvira Shishenina.

Space-Time Discretization of Elasto-Acoustic Wave Equation in Polynomial Trefftz-DG Bases. (2018)

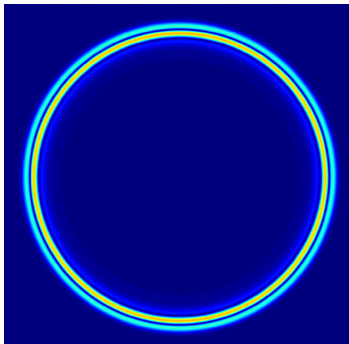
PhD thesis [under review]

TENT PITCHING MESHES

PROPAGATION OF NUMERICAL VELOCITY

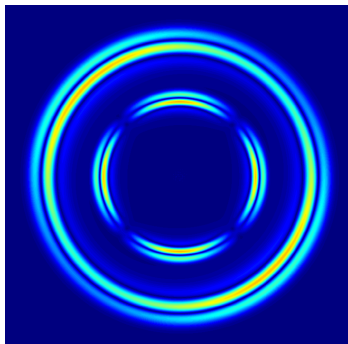
2D Acoustic domain.

L^2 norm of numerical velocity v



2D Elastodynamic domain.

L^2 norm of numerical velocity v

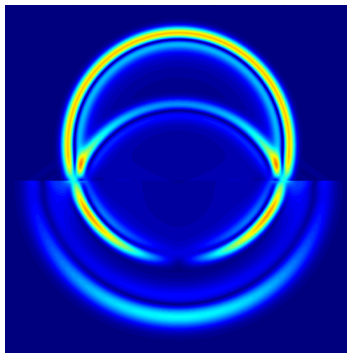


TENT PITCHING MESHES

PROPAGATION OF NUMERICAL VELOCITY

2D Elasto-Acoustic domain.

L^2 norm of numerical velocity v

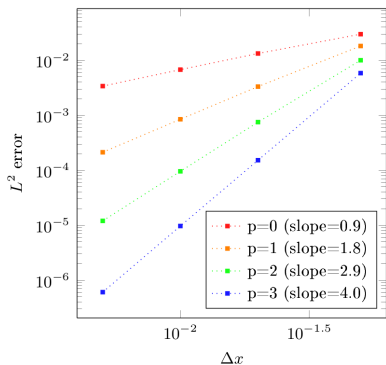


TENT PITCHING MESHES

CONVERGENCE CURVES

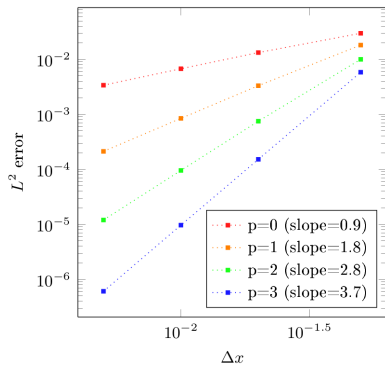
2D Acoustic domain.

Convergence of velocity v in function of cell size



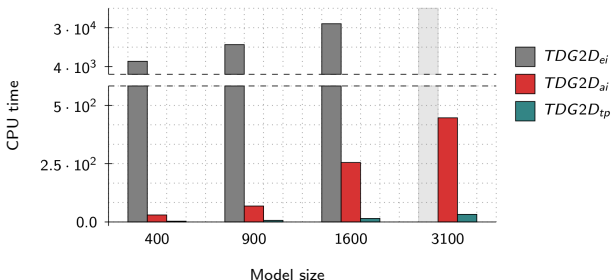
2D Elastodynamic domain.

Convergence of velocity v in function of cell size



TENT PITCHING MESHES

SPEEDUP FOR DIFFERENT MODEL SIZES



CONCLUSION

CONCLUSION

ON-GOING WORK AND PERSPECTIVES

Standard Trefftz-DG method:

- Development of 2D-3D hp propagators
- Analytical study of convergence, comparison with numerical results
- Study of CFL condition, comparison with standard DG method

Tent pitching Trefftz-DG method:

- Mesh generation study
- Implementation of the boundary conditions
- 3D case

THANK YOU!

IMPLEMENTATION OF THE ALGORITHM

POLYNOMIAL BASIS (ORDER 3)

p=0, ndof=2

$$\phi_1^v = 0$$

$$\phi_2^v = 1$$

$$\phi_1^p = -c_F$$

$$\phi_2^p = 0$$

p=1, ndof=4

$$\phi_3^v = x$$

$$\phi_4^v = c_F t$$

$$\phi_3^p = -c_F^2 t$$

$$\phi_4^p = -c_F x$$

p=2, ndof=6

$$\phi_5^v = -\frac{x^2}{2} - \frac{c_F^2 t^2}{2}$$

$$\phi_6^v = -c_F x t$$

$$\phi_5^p = c_F^2 x t$$

$$\phi_6^p = c_F \left(\frac{x^2}{2} + \frac{c_F^2 t^2}{2} \right)$$

p=3, ndof=8

$$\phi_7^v = -\frac{x^3}{6} - \frac{x c_F^2 t^2}{2}$$

$$\phi_8^v = -\frac{c_F^3 t^3}{6} - \frac{x^2 c_F t}{2}$$

$$\phi_7^p = c_F \left(\frac{c_F^3 t^3}{6} + \frac{x^2 c_F t}{2} \right)$$

$$\phi_8^p = c_F \left(\frac{x^3}{6} + \frac{x c_F^2 t^2}{2} \right)$$

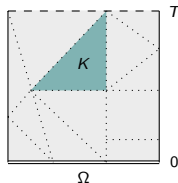
ACOUSTIC SYSTEM

TREFFTZ-DG FORMULATION

$$\sum_K \int_{\partial K} \left(\frac{1}{c^2 \rho} \check{p}_h q + \check{v}_h \cdot \omega \right) n_K^t + \left(q \hat{v}_h + \hat{v}_h \omega \right) \cdot n_K^x = \sum_K \int_K f q$$

NUMERICAL FLUXES THROUGH THE ELEMENT FACES

$$\begin{aligned} \begin{pmatrix} \hat{v}_h \\ \hat{p}_h \end{pmatrix} &\equiv \begin{pmatrix} \{v_h\} + \beta [p_h]_x \\ [p_h] + \alpha [v_h]_x \end{pmatrix} && \text{on } \mathcal{F}^I && \cdots \\ \begin{pmatrix} \check{v}_h \\ \check{p}_h \end{pmatrix} &\equiv \begin{pmatrix} \{v_h\} \\ [p_h] \end{pmatrix} && \text{on } \mathcal{F}^I && \cdots \\ \begin{pmatrix} \hat{v}_h \cdot n_K^x \\ \hat{p}_h \end{pmatrix} &\equiv \begin{pmatrix} g_D \\ [p_h] + \alpha (v_h \cdot n_K^x - g_D) \end{pmatrix} && \text{on } \mathcal{F}^D && \text{---} \\ \begin{pmatrix} \check{v}_h \\ \check{p}_h \end{pmatrix} &\equiv \begin{pmatrix} v_h \\ p_h \end{pmatrix} && \text{on } \mathcal{F}^T && \text{-- --} \\ \begin{pmatrix} \check{v}_h \\ \check{p}_h \end{pmatrix} &\equiv \begin{pmatrix} v_0 \\ p_0 \end{pmatrix} && \text{on } \mathcal{F}^0 && \equiv \end{aligned}$$



REFERENCES

J.S.Hesthaven T.Warburton. *Nodal Discontinuous Galerkin Methods. Algorithms, analysis, and applications*. 2007.

F.Kretzschmar A.Moiola I.Perugia. *A Priori Error Analysis of Space-Time Trefftz Discontinuous Galerkin Methods for Wave Problems*. 2016.

O.C.Zienkiewicz. *Trefftz Type Approximation and the Generalized Finite Element Method - History and Developement*. 1997.

I.Herrera. *Trefftz Method: a General Theory*. 2000

A.Maciag. *Wave Polynomials in Elasticity Problems*. 2006