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# Foldy-Lax Model for the Scattering Problem in Electromagnetism

Justine Labat, Victor Péron, Sébastien Tordeux

PhD student in Applied Mathematics

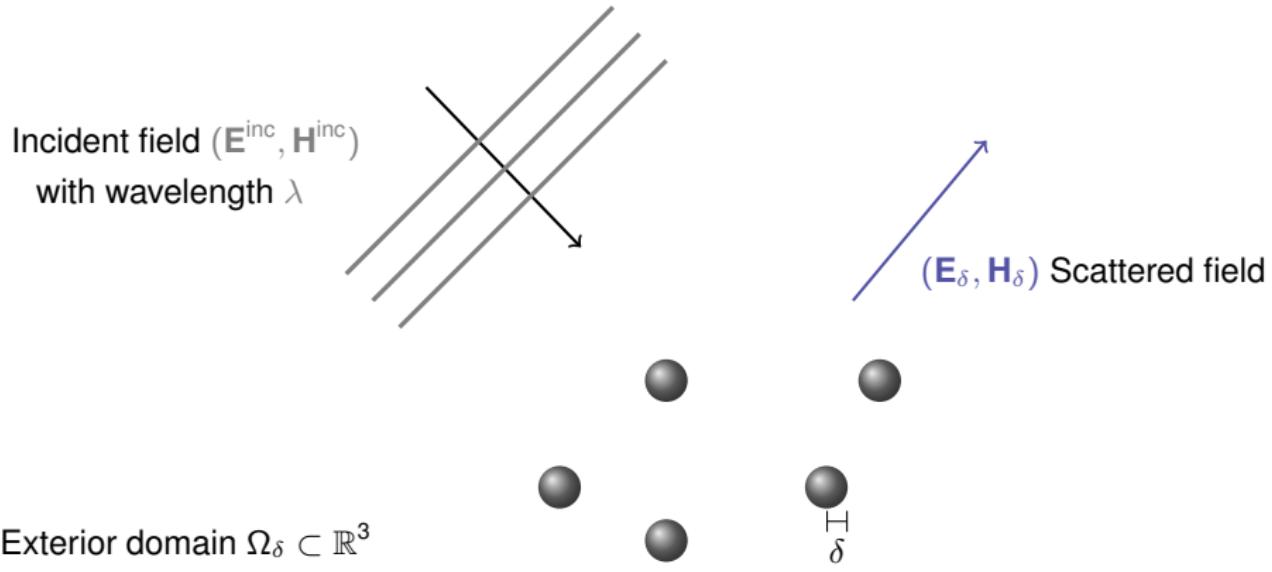
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5th International Congress on Multiphysics, Multiscale and Optimization Problems

Bilbao, 24th May 2018

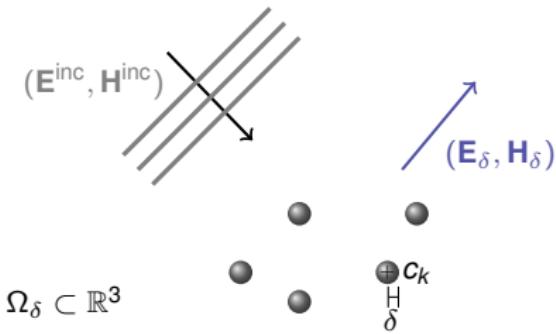


# 3D-Multiple Scattering Problem by Small Spheres



Asymptotic assumption :  $\delta \ll \lambda$

# Model Problem



$$\Omega_\delta \subset \mathbb{R}^3$$

- Time-harmonic domain
- Homogeneous & isotropic medium  $\Omega_\delta$
- Perfect conductors  $\mathcal{B}(c_k, \delta)$ ,  $k = 1 \dots N_{\text{obs}}$

Time-harmonic Maxwell equations

$$\begin{cases} \operatorname{curl} \mathbf{E}_\delta - i\kappa \mathbf{H}_\delta = 0 & \text{in } \Omega_\delta \\ \operatorname{curl} \mathbf{H}_\delta + i\kappa \mathbf{E}_\delta = 0 & \text{in } \Omega_\delta \end{cases}$$

$$\text{with } \kappa^2 = \omega^2 \mu(\epsilon + \frac{i\sigma}{\omega}), \Im(\kappa) \geq 0$$

Boundary condition

$$\mathbf{n} \times \mathbf{E}_\delta = -\mathbf{n} \times \mathbf{E}^{\text{inc}} \quad \text{on } \partial\Omega_\delta$$

Silver-Müller radiation condition

$$r(\mathbf{H}_\delta \times \hat{x} - \mathbf{E}_\delta) \xrightarrow[r \rightarrow \infty]{} 0 \quad \text{unif. in } \hat{x} = \frac{\mathbf{x}}{r}$$

Mathematical well-posedness [Phillips (1973)]

For any  $\mathbf{E}^{\text{inc}} \in \mathbf{H}_{\text{loc}}(\operatorname{curl}, \Omega_\delta)$  there exist a unique solution  $(\mathbf{E}_\delta, \mathbf{H}_\delta) \in \mathbf{H}_{\text{loc}}(\operatorname{curl}, \Omega_\delta)^2$  to the exterior Maxwell problem.

# Approaches and Objectives

## Asymptotic models

Born approximation

Foldy-Lax approximation



H. Ammari, M.S. Vogelius, D. Volkov (2001)

Asymptotic formulas for perturbations in the electromagnetic fields due to the presence of inhomogeneities of small diameter II. The full Maxwell equations



D.P. Challa, G. Hu, M. Sini (2013)

Multiple scattering of electromagnetic waves by finitely many point-like obstacles

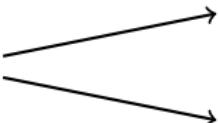


D.V. Korikov, B.A. Plamenevskii (2017)

Asymptotics of solutions for stationary and nonstationary Maxwell systems in a domain with small holes

# Approaches and Objectives

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## Numerical solution



High-order spectral method



M. Ganesh, S.C. Hawkins (2009)

A high-order algorithm for multiple electromagnetic scattering in three dimensions



H. Barucq, J. Chabassier, H. Pham, S. Tordeux (2017)

Numerical robustness of single-layer method with Fourier basis for multiple obstacle acoustic scattering in homogeneous media

# Motivations

- ✗ **High computational cost** with classical numerical methods
- ✗ Mesh **refinement**
- ✓ High-order **spectral method**

## Numerical Solution

# Motivations

- ✗ **High computational cost** with classical numerical methods
- ✗ Mesh **refinement**
- ✓ High-order **spectral method**

## Numerical Solution

Superposition  
principle

## Born approximation

- ✗ **High computational cost**
  - ✗ volumical methods
  - ~ boundary element methods
- ✓ High-order **spectral method**

# Motivations

- ✗ High computational cost with classical numerical methods
- ✗ Mesh refinement
- ✓ High-order spectral method

## Numerical Solution

Superposition principle

### Born approximation

- ✗ High computational cost
  - ✗ volumical methods
  - ~ boundary element methods
- ✓ High-order spectral method

## Approximation

### Asymptotic models

- ✗ Restricted to small obstacles
- ✓ Low computational cost
- ✓ Meshless method

# Motivations

- ✗ **High computational cost** with classical numerical methods
- ✗ Mesh **refinement**
- ✓ High-order **spectral method**

- ✓ Interactions taken into account
- ✓ Low computational cost
- ✓ Meshless method

## Numerical Solution

Superposition principle

## Born approximation

- ✗ **High computational cost**
  - ✗ volumical methods
  - ~ boundary element methods
- ✓ High-order **spectral method**

## Foldy-Lax model

Multiple scattering

## Approximation

## Asymptotic models

- ✗ Restricted to small obstacles
- ✓ Low computational cost
- ✓ Meshless method

# Outline

## 1. Asymptotic expansions for the single electromagnetic scattering

- First terms of the asymptotics
- Numerical results

## 2. Multiple electromagnetic scattering by small spheres

- Born approximation
- Foldy-Lax approximation
- Preliminary numerical results

## 3. Conclusions and perspectives

# Outline

## 1. Asymptotic expansions for the single electromagnetic scattering

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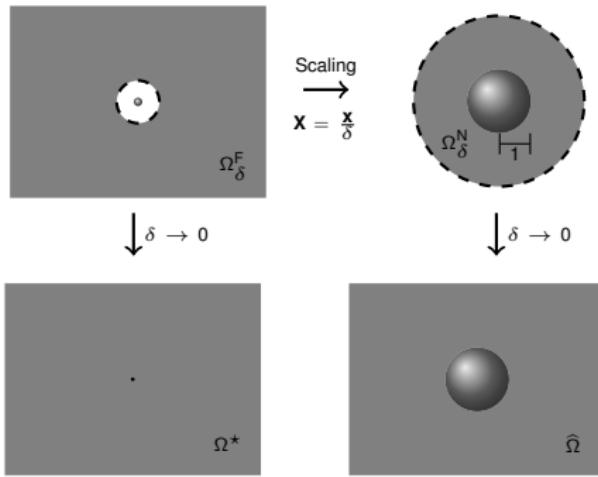
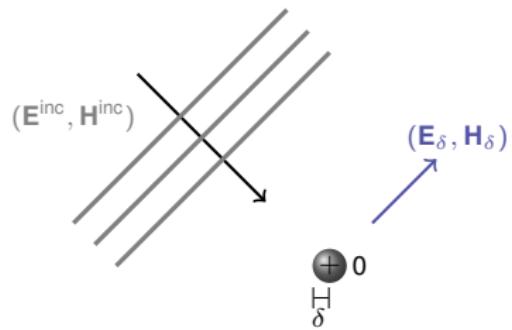
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# Approximation for Single Scattering

## Method of matched asymptotic expansions

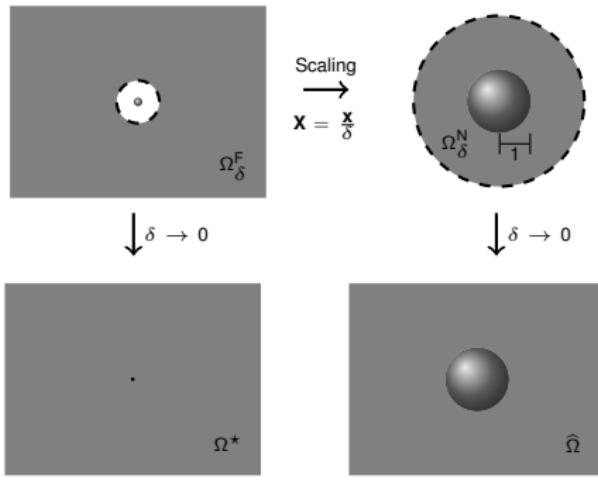
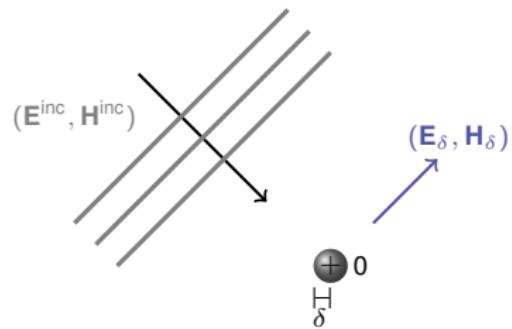
- Domain decomposition
- Local approximations
- Matching procedure



# Approximation for Single Scattering

## Method of matched asymptotic expansions

- Domain decomposition
- Local approximations
- Matching procedure



## Asymptotic expansions

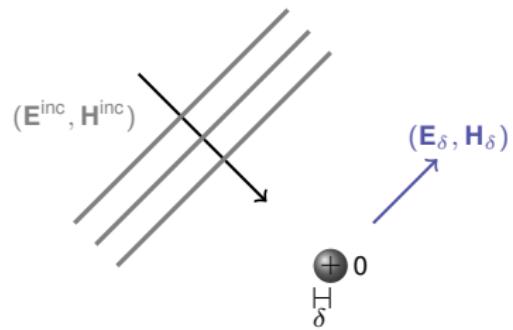
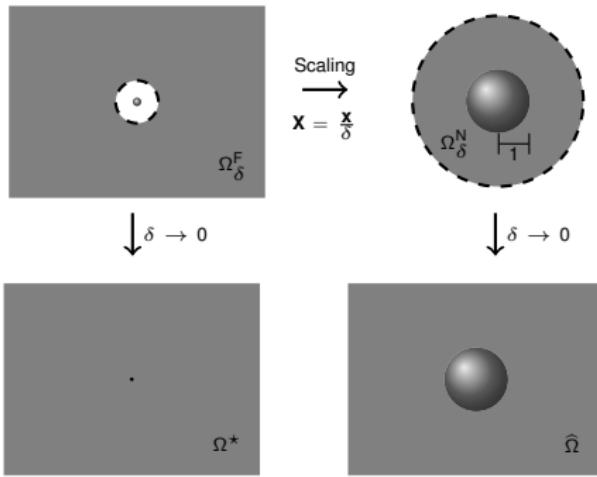
**Far field** expansion in  $\Omega^* = \mathbb{R}^3 \setminus \{0\}$

**Near field** expansion in  $\widehat{\Omega} = \mathbb{R}^3 \setminus \overline{\mathcal{B}(0, 1)}$

# Approximation for Single Scattering

## Method of matched asymptotic expansions

- Domain decomposition
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- Matching procedure



## Asymptotic expansions

**Far field** expansion in  $\Omega^* = \mathbb{R}^3 \setminus \{0\}$

**Near field** expansion in  $\widehat{\Omega} = \mathbb{R}^3 \setminus \overline{\mathcal{B}(0, 1)}$

Near field  $\implies$  Far field

# Near Field Expansion I

$$\mathbf{E}_\delta(\delta \mathbf{X}) = \widehat{\mathbf{E}}_0(\mathbf{X}) + \delta \widehat{\mathbf{E}}_1(\mathbf{X}) + \delta^2 \widehat{\mathbf{E}}_2(\mathbf{X}) + \dots$$

$$\mathbf{H}_\delta(\delta \mathbf{X}) = \widehat{\mathbf{H}}_0(\mathbf{X}) + \delta \widehat{\mathbf{H}}_1(\mathbf{X}) + \delta^2 \widehat{\mathbf{H}}_2(\mathbf{X}) + \dots$$

**0-order** (static **dipole**)

$$\widehat{\mathbf{E}}_0(\mathbf{X}) = \frac{1}{|\mathbf{X}|^3} \left( 3(\mathbf{e}_r \cdot \mathbf{E}^{\text{inc}}(0)) \mathbf{e}_r - \mathbf{E}^{\text{inc}}(0) \right)$$

$$\widehat{\mathbf{H}}_0(\mathbf{X}) = -\frac{1}{2|\mathbf{X}|^3} \left( 3(\mathbf{e}_r \cdot \mathbf{H}^{\text{inc}}(0)) \mathbf{e}_r - \mathbf{H}^{\text{inc}}(0) \right)$$

**1-order** (static **quadrupole** + static **dipole**)

$$\widehat{\mathbf{E}}_1(\mathbf{X}) = -\frac{1}{|\mathbf{X}|^4} \gamma_t \left( \mathbf{J}_{\mathbf{E}^{\text{inc}}}^{\text{sym}}(0) \mathbf{e}_r \right) + \frac{3}{2|\mathbf{X}|^4} (\mathbf{e}_r \cdot \mathbf{J}_{\mathbf{E}^{\text{inc}}}(0) \mathbf{e}_r) \mathbf{e}_r - \frac{i\kappa}{2|\mathbf{X}|^2} \mathbf{H}^{\text{inc}}(0) \times \mathbf{e}_r$$

$$\widehat{\mathbf{H}}_1(\mathbf{X}) = \frac{2}{3|\mathbf{X}|^4} \gamma_t \left( \mathbf{J}_{\mathbf{H}^{\text{inc}}}^{\text{sym}}(0) \mathbf{e}_r \right) - \frac{1}{|\mathbf{X}|^4} (\mathbf{e}_r \cdot \mathbf{J}_{\mathbf{H}^{\text{inc}}}(0) \mathbf{e}_r) \mathbf{e}_r + \frac{i\kappa}{|\mathbf{X}|^2} \mathbf{E}^{\text{inc}}(0) \times \mathbf{e}_r$$

with  $\gamma_t \mathbf{u} = \mathbf{u} - (\mathbf{e}_r \cdot \mathbf{u}) \mathbf{e}_r$  and  $\mathbf{J}^{\text{sym}} = \frac{1}{2} (\mathbf{J} + \mathbf{J}^\top)$  symmetric Jacobian

## Near Field Expansion II

2-order (static **octupole** + static **quadrupole** + static **dipole**)

$$\begin{aligned}\widehat{\mathbf{E}}_2(\mathbf{X}) = & \frac{1}{|\mathbf{X}|^5} \left\{ \frac{2}{3} (\mathbf{e}_r \cdot \mathbf{e}_r^\top \mathbb{H}_{\mathbf{E}^{\text{inc}}}(0) \mathbf{e}_r) \mathbf{e}_r + \frac{2\kappa^2}{15} (\mathbf{e}_r \cdot \mathbf{E}^{\text{inc}}(0)) \mathbf{e}_r - \frac{1}{2} \gamma_t (\mathbf{e}_r^\top \mathbb{H}_{\mathbf{E}^{\text{inc}}}(0) \mathbf{e}_r) \right. \\ & - \frac{i\kappa}{3} \mathbf{e}_r \times \mathbb{J}_{\mathbf{H}^{\text{inc}}}^{\text{sym}}(0) \mathbf{e}_r - \frac{\kappa^2}{5} \gamma_t (\mathbf{E}^{\text{inc}}(0)) \Big\} + \frac{i\kappa}{3|\mathbf{X}|^3} \mathbf{e}_r \times \mathbb{J}_{\mathbf{H}^{\text{inc}}}^{\text{sym}}(0) \mathbf{e}_r \\ & - \frac{3\kappa^2}{10|\mathbf{X}|^3} \left\{ \mathbf{E}^{\text{inc}}(0) - 3(\mathbf{e}_r \cdot \mathbf{E}^{\text{inc}}(0)) \mathbf{e}_r \right\} + \frac{\kappa^2}{2|\mathbf{X}|} \left\{ \mathbf{E}^{\text{inc}}(0) + (\mathbf{e}_r \cdot \mathbf{E}^{\text{inc}}(0)) \mathbf{e}_r \right\}\end{aligned}$$

$$\begin{aligned}\widehat{\mathbf{H}}_2(\mathbf{X}) = & \frac{1}{|\mathbf{X}|^5} \left\{ -\frac{1}{2} (\mathbf{e}_r \cdot \mathbf{e}_r^\top \mathbb{H}_{\mathbf{H}^{\text{inc}}}(0) \mathbf{e}_r) \mathbf{e}_r - \frac{\kappa^2}{10} (\mathbf{e}_r \cdot \mathbf{H}^{\text{inc}}(0)) \mathbf{e}_r + \frac{3}{8} \gamma_t (\mathbf{e}_r^\top \mathbb{H}_{\mathbf{H}^{\text{inc}}}(0) \mathbf{e}_r) \right. \\ & - \frac{i\kappa}{4} \mathbf{e}_r \times \mathbb{J}_{\mathbf{E}^{\text{inc}}}^{\text{sym}}(0) \mathbf{e}_r + \frac{3\kappa^2}{20} \gamma_t (\mathbf{H}^{\text{inc}}(0)) \Big\} + \frac{i\kappa}{2|\mathbf{X}|} \mathbf{e}_r \times \mathbb{J}_{\mathbf{E}^{\text{inc}}}^{\text{sym}}(0) \mathbf{e}_r \\ & + \frac{3\kappa^2}{10|\mathbf{X}|^3} \left\{ 3(\mathbf{e}_r \cdot \mathbf{H}^{\text{inc}}(0)) \mathbf{e}_r - \mathbf{H}^{\text{inc}}(0) \right\} - \frac{\kappa^2}{4|\mathbf{X}|} \left\{ \mathbf{H}^{\text{inc}}(0) + (\mathbf{e}_r \cdot \mathbf{H}^{\text{inc}}(0)) \mathbf{e}_r \right\}\end{aligned}$$

# Far Field Expansion I

$$\mathbf{E}_\delta(\mathbf{x}) = \delta^3 \tilde{\mathbf{E}}_3(\mathbf{x}) + \delta^5 \tilde{\mathbf{E}}_5(\mathbf{x}) + \dots \quad \mathbf{H}_\delta(\mathbf{x}) = \delta^3 \tilde{\mathbf{H}}_3(\mathbf{x}) + \delta^5 \tilde{\mathbf{H}}_5(\mathbf{x}) + \dots$$

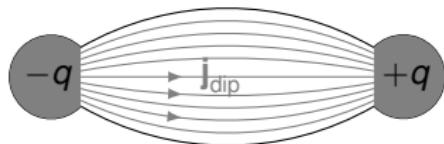
**3-order** ( time-harmonic **dipole**)

$$\tilde{\mathbf{E}}_3(\mathbf{x}) = -\frac{\kappa^3}{2} h_1^{(1)}(\kappa r) \mathbf{e}_r \times \mathbf{H}^{\text{inc}}(0) - \kappa^3 \frac{\tilde{h}_1^{(1)}(\kappa r)}{i\kappa r} \gamma_t \mathbf{E}^{\text{inc}}(0) - 2\kappa^3 \frac{h_1^{(1)}(\kappa r)}{i\kappa r} (\mathbf{e}_r \cdot \mathbf{E}^{\text{inc}}(0)) \mathbf{e}_r$$

$$\tilde{\mathbf{H}}_3(\mathbf{x}) = -\kappa^3 h_1^{(1)}(\kappa r) \mathbf{e}_r \times \mathbf{E}^{\text{inc}}(0) + \frac{\kappa^3}{2} \frac{\tilde{h}_1^{(1)}(\kappa r)}{i\kappa r} \gamma_t \mathbf{H}^{\text{inc}}(0) + \kappa^3 \frac{h_1^{(1)}(\kappa r)}{i\kappa r} (\mathbf{e}_r \cdot \mathbf{H}^{\text{inc}}(0)) \mathbf{e}_r$$

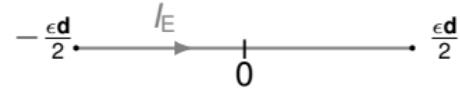
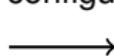
with  $h_n^{(1)}$  spherical Hankel of 1st kind and  $\tilde{h}_1^{(1)}(z) = h_1^{(1)}(z) + zh_1^{(1)'}(z)$

# Time-Harmonic Electromagnetic Dipole



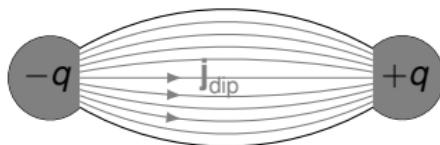
$\mathbf{j}_{\text{dip}}$  electric current

Ideal configuration

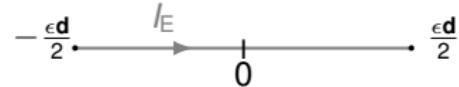


$\mathbf{d} \in \mathbb{R}^3$ ,  $\epsilon$  small parameter

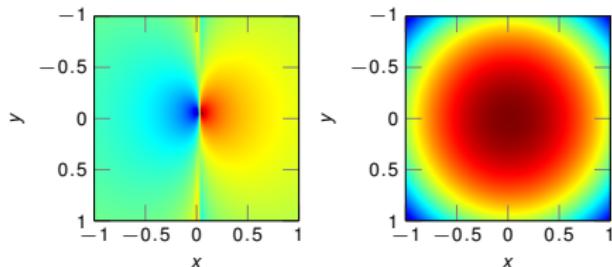
# Time-Harmonic Electromagnetic Dipole



Ideal configuration



$j_{\text{dip}}$  electric current



Time-harmonic electric potential  $V$  and magnetic potential  $\mathbf{A}$  ( $x$ -component) in the  $xy$ -plane

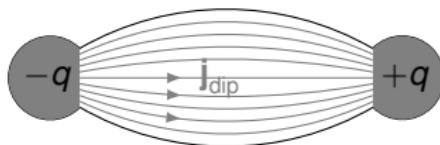
$\epsilon \rightarrow 0$



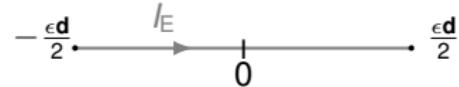
$\mathbf{d} \in \mathbb{R}^3$ ,  $\epsilon$  small parameter

- Ponctual charges with  $q = \frac{1}{\epsilon}$
- Filiform current ( $\mathbf{d}, I_E$ )
- Charge conservation  $I_E = -i\omega q$

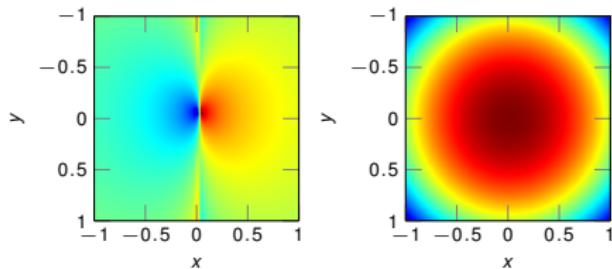
# Time-Harmonic Electromagnetic Dipole



Ideal configuration



$\mathbf{j}_{\text{dip}}$  electric current



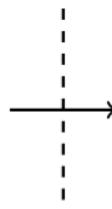
Time-harmonic electric potential  $V$  and magnetic potential  $\mathbf{A}$  ( $x$ -component) in the  $xy$ -plane

$$\mathcal{E}_{\text{dip}}^{\text{elec}}[\mathbf{d}] = -\nabla V + i\kappa \mathbf{A}$$

$$\mathcal{H}_{\text{dip}}^{\text{elec}}[\mathbf{d}] = \text{curl } \mathbf{A}$$

$$\epsilon \rightarrow 0$$

- Ponctual charges with  $q = \frac{1}{\epsilon}$
- Filiform current  $(\mathbf{d}, I_E)$
- Charge conservation  $I_E = -i\omega q$



Magnetic dipole

$$\mathcal{H}_{\text{dip}}^{\text{mag}}[\mathbf{d}] = \mathcal{E}_{\text{dip}}^{\text{elec}}[\mathbf{d}]$$

$$\mathcal{E}_{\text{dip}}^{\text{mag}}[\mathbf{d}] = -\mathcal{H}_{\text{dip}}^{\text{elec}}[\mathbf{d}]$$



## Far Field Expansion II

$$\mathbf{E}_\delta(\mathbf{x}) = \delta^3 \tilde{\mathbf{E}}_3(\mathbf{x}) + \delta^5 \tilde{\mathbf{E}}_5(\mathbf{x}) + \dots$$

**3-order** ( time-harmonic **dipole**)

$$\tilde{\mathbf{E}}_3 = \mathcal{E}_{\text{dip}}^{\text{elec}} [4\pi \mathbf{E}^{\text{inc}}(0)] + \mathcal{E}_{\text{dip}}^{\text{mag}} [-2\pi \mathbf{H}^{\text{inc}}(0)]$$

**5-order** ( time-harmonic **dipole** + time-harmonic **quadrupole**)

$$\begin{aligned}\tilde{\mathbf{E}}_5(\mathbf{x}) &= \frac{3\kappa^2}{10} \mathcal{E}_{\text{dip}}^{\text{elec}} [4\pi \mathbf{E}^{\text{inc}}(0)](\mathbf{x}) - \frac{3\kappa^2}{5} \mathcal{E}_{\text{dip}}^{\text{mag}} [-2\pi \mathbf{H}^{\text{inc}}(0)](\mathbf{x}) \\ &\quad - \frac{\kappa^4}{9} h_2^{(1)}(\kappa r) \mathbf{e}_r \times \mathbb{J}_{\mathbf{H}^{\text{inc}}}(0) \mathbf{e}_r - \frac{\kappa^4}{6} \frac{\tilde{h}_2^{(1)}(\kappa r)}{i\kappa r} \gamma_t \left( \mathbb{J}_{\mathbf{E}^{\text{inc}}}(0) \mathbf{e}_r \right) - \frac{\kappa^4}{2} \frac{h_2^{(1)}(\kappa r)}{i\kappa r} (\mathbf{e}_r \cdot \mathbb{J}_{\mathbf{E}^{\text{inc}}}(0) \mathbf{e}_r) \mathbf{e}_r\end{aligned}$$

**Collected dipolar model**

$$\mathbf{E}_\delta^{\text{Col}}(\mathbf{x}) = \left( \delta^3 + \frac{3\kappa^2 \delta^5}{10} \right) \mathcal{E}_{\text{dip}}^{\text{elec}} [4\pi \mathbf{E}^{\text{inc}}(0)](\mathbf{x}) + \left( \delta^3 - \frac{3\kappa^2 \delta^5}{5} \right) \mathcal{E}_{\text{dip}}^{\text{mag}} [-2\pi \mathbf{H}^{\text{inc}}(0)](\mathbf{x})$$

# Far Field Expansion III

$$\mathbf{H}_\delta(\mathbf{x}) = \delta^3 \tilde{\mathbf{H}}_3(\mathbf{x}) + \delta^5 \tilde{\mathbf{H}}_5(\mathbf{x}) + \dots$$

**3-order** ( time-harmonic **dipole**)

$$\tilde{\mathbf{H}}_3 = \mathcal{H}_{\text{dip}}^{\text{elec}}[4\pi \mathbf{E}^{\text{inc}}(0)] + \mathcal{H}_{\text{dip}}^{\text{mag}}[-2\pi \mathbf{H}^{\text{inc}}(0)]$$

**5-order** ( time-harmonic **dipole** + time-harmonic **quadrupole**)

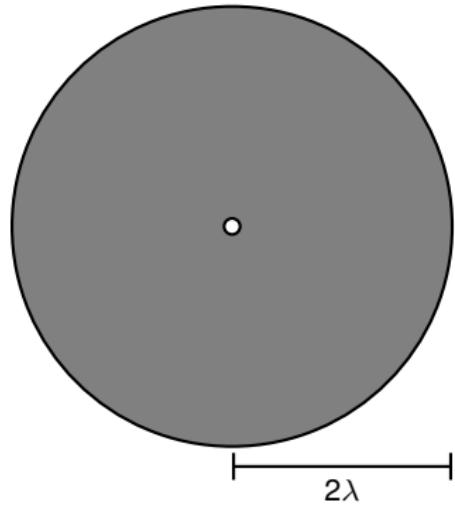
$$\begin{aligned}\tilde{\mathbf{H}}_5(\mathbf{x}) &= \frac{3\kappa^2}{10} \mathcal{H}_{\text{dip}}^{\text{elec}}[4\pi \mathbf{E}^{\text{inc}}(0)](\mathbf{x}) - \frac{3\kappa^2}{5} \mathcal{H}_{\text{dip}}^{\text{mag}}[-2\pi \mathbf{H}^{\text{inc}}(0)](\mathbf{x}) \\ &\quad - \frac{\kappa^4}{6} h_2^{(1)}(\kappa r) \mathbf{e}_r \times \mathbb{J}_{\mathbf{E}^{\text{inc}}}(0) \mathbf{e}_r + \frac{\kappa^4}{9} \frac{\tilde{h}_2^{(1)}(\kappa r)}{i\kappa r} \gamma_t \left( \mathbb{J}_{\mathbf{H}^{\text{inc}}}(0) \mathbf{e}_r \right) + \frac{\kappa^4}{3} \frac{h_2^{(1)}(\kappa r)}{i\kappa r} (\mathbf{e}_r \cdot \mathbb{J}_{\mathbf{H}^{\text{inc}}}(0) \mathbf{e}_r) \mathbf{e}_r\end{aligned}$$

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# Simulation Description

3D Whole domain



Small parameter

$$\delta \in \left\{ \frac{1}{10^p}, p = 0.5 : 0.1 : 4 \right\}$$

Physical parameters

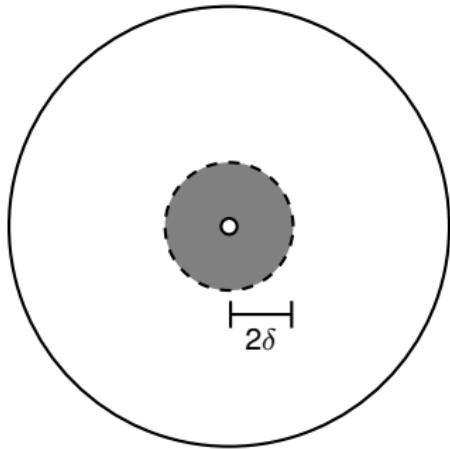
- $\varepsilon = \mu = 1.0, \sigma = 0$
- $\lambda = 5.0,$   
 $\omega = \kappa = \frac{2\pi}{5.0} \approx 1.26$

Incident plane wave

- $\mathbf{E}^{\text{inc}} = \vec{p} \exp(-i\vec{k} \cdot \mathbf{x})$
- $\mathbf{H}^{\text{inc}} = \frac{1}{i\kappa} \mathbf{curl} \mathbf{E}^{\text{inc}}$
- $\vec{p} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \vec{k} = \begin{pmatrix} 0 \\ 0 \\ \kappa \end{pmatrix}$

# Simulation Description

## Near field domain



Small parameter

$$\delta \in \left\{ \frac{1}{10^p}, p = 0.5 : 0.1 : 4 \right\}$$

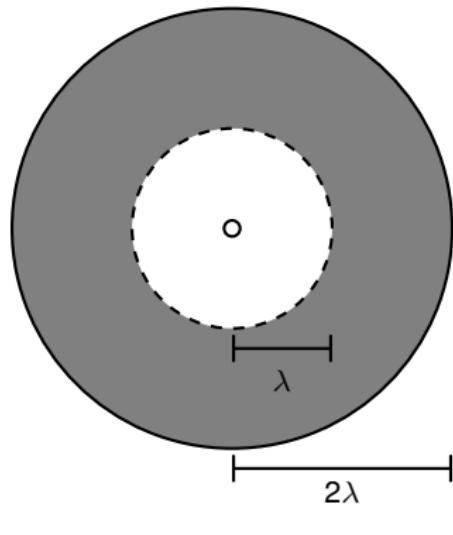
Physical parameters

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- $\lambda = 5.0,$   
 $\omega = \kappa = \frac{2\pi}{5.0} \approx 1.26$

Incident plane wave

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# Simulation Description



Small parameter

$$\delta \in \left\{ \frac{1}{10^p}, p = 0.5 : 0.1 : 4 \right\}$$

Physical parameters

- $\varepsilon = \mu = 1.0, \sigma = 0$
- $\lambda = 5.0,$   
 $\omega = \kappa = \frac{2\pi}{5.0} \approx 1.26$

Incident plane wave

- $\mathbf{E}^{\text{inc}} = \vec{p} \exp(-i\vec{k} \cdot \mathbf{x})$
- $\mathbf{H}^{\text{inc}} = \frac{1}{i\kappa} \mathbf{curl} \mathbf{E}^{\text{inc}}$
- $\vec{p} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \vec{k} = \begin{pmatrix} 0 \\ 0 \\ \kappa \end{pmatrix}$

# Numerical Convergence I

## Near field approximations

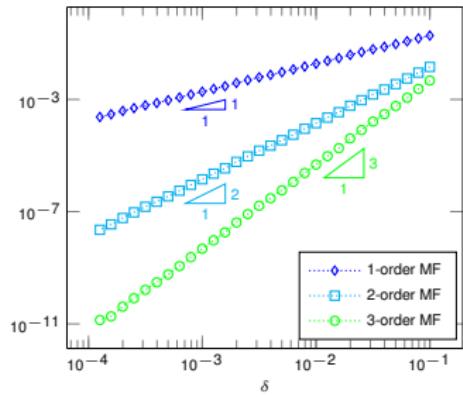
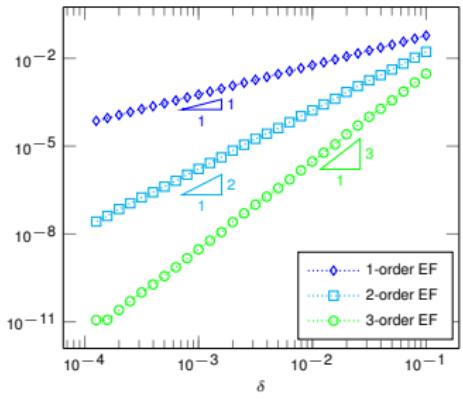
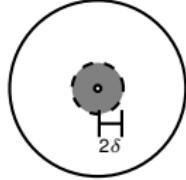


Figure: Relative  $L^2(\Omega_\delta^{2\delta})$ -errors for electric near field (left) magnetic near field (right)



$$\text{1-order EF} : \|\mathbf{E}_\delta - \widehat{\mathbf{E}}_0\left(\frac{\cdot}{\delta}\right)\|_{L^2(\Omega_\delta^{2\delta})} / \|\mathbf{E}_\delta\|_{L^2(\Omega_\delta^{2\delta})}$$

$$\text{2-order EF} : \|\mathbf{E}_\delta - \widehat{\mathbf{E}}_0\left(\frac{\cdot}{\delta}\right) - \delta \widehat{\mathbf{E}}_1\left(\frac{\cdot}{\delta}\right)\|_{L^2(\Omega_\delta^{2\delta})} / \|\mathbf{E}_\delta\|_{L^2(\Omega_\delta^{2\delta})}$$

$$\text{3-order EF} : \|\mathbf{E}_\delta - \widehat{\mathbf{E}}_0\left(\frac{\cdot}{\delta}\right) - \delta \widehat{\mathbf{E}}_1\left(\frac{\cdot}{\delta}\right) - \delta^2 \widehat{\mathbf{E}}_2\left(\frac{\cdot}{\delta}\right)\|_{L^2(\Omega_\delta^{2\delta})} / \|\mathbf{E}_\delta\|_{L^2(\Omega_\delta^{2\delta})}$$

# Numerical Convergence II

## Far field approximations

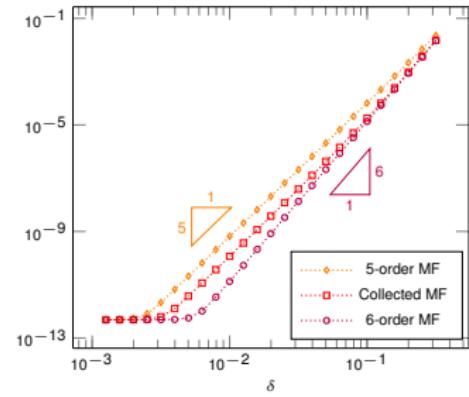
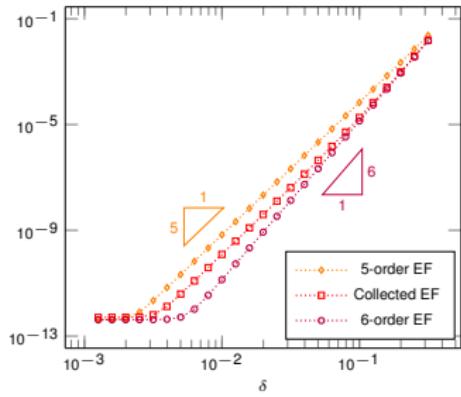
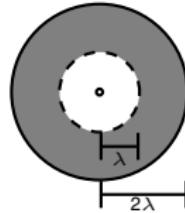


Figure: Absolute  $L^2(\Omega_2^{2\lambda})$ -errors for electric far field (left) magnetic far field (right)



$$5\text{-order EF} : \|\mathbf{E}_\delta - \delta^3 \tilde{\mathbf{E}}_3\|_{L^2(\Omega_2^{2\lambda})}$$

$$6\text{-order EF} : \|\mathbf{E}_\delta - \delta^3 \tilde{\mathbf{E}}_3 - \delta^5 \tilde{\mathbf{E}}_5\|_{L^2(\Omega_2^{2\lambda})}$$

# Outline

## 1. Asymptotic expansions for the single electromagnetic scattering

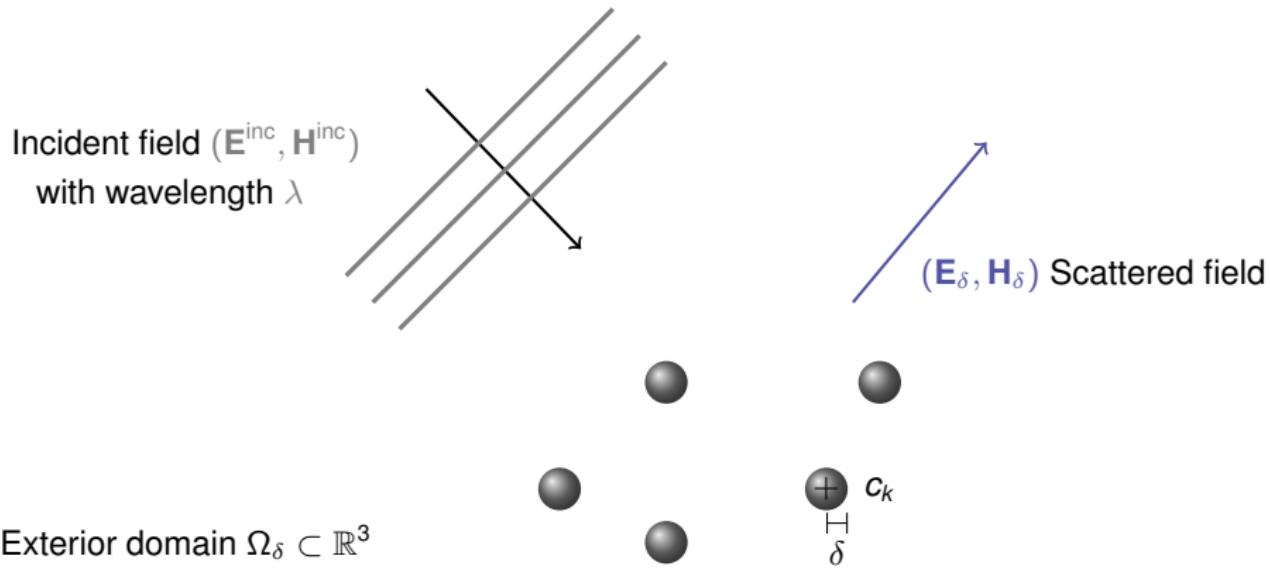
- First terms of the asymptotics
- Numerical results

## 2. Multiple electromagnetic scattering by small spheres

- Born approximation
- Foldy-Lax approximation
- Preliminary numerical results

## 3. Conclusions and perspectives

# 3D-Multiple Scattering Problem by Small Spheres



$$\Omega_\delta = \mathbb{R}^3 \setminus \bigcup_{k=1}^{N_{\text{obs}}} \overline{\mathcal{B}(c_k, \delta)}$$

# Born Approximation

$$\mathbf{E}_\delta^{\text{Born}}(\mathbf{x}) = \sum_{k=1}^{N_{\text{obs}}} \mathbf{E}_{\delta,k}(\mathbf{x})$$

$$\mathbf{H}_\delta^{\text{Born}}(\mathbf{x}) = \sum_{k=1}^{N_{\text{obs}}} \mathbf{H}_{\delta,k}(\mathbf{x})$$

- Each obstacle is modeled as a **dipolar source** around  $c_k$

$$\mathbf{E}_{\delta,k}(\mathbf{x}) = \mathcal{E}_{\text{dip}}^{\text{elec}}[\mathbf{d}_{\delta,k}^E](\mathbf{x} - \mathbf{c}_k) + \mathcal{E}_{\text{dip}}^{\text{mag}}[\mathbf{d}_{\delta,k}^H](\mathbf{x} - \mathbf{c}_k)$$

$$\mathbf{H}_{\delta,k}(\mathbf{x}) = \mathcal{H}_{\text{dip}}^{\text{elec}}[\mathbf{d}_{\delta,k}^E](\mathbf{x} - \mathbf{c}_k) + \mathcal{H}_{\text{dip}}^{\text{mag}}[\mathbf{d}_{\delta,k}^H](\mathbf{x} - \mathbf{c}_k)$$

- The directions  $\mathbf{d}_{\delta,k}^E$  and  $\mathbf{d}_{\delta,k}^H$  depend on the **nature** of the obstacles

# Born Approximation

$$\mathbf{E}_\delta^{\text{Born}}(\mathbf{x}) = \sum_{k=1}^{N_{\text{obs}}} \mathbf{E}_{\delta,k}(\mathbf{x})$$

$$\mathbf{H}_\delta^{\text{Born}}(\mathbf{x}) = \sum_{k=1}^{N_{\text{obs}}} \mathbf{H}_{\delta,k}(\mathbf{x})$$

- Each obstacle is modeled as a **dipolar source** around  $c_k$

$$\mathbf{E}_{\delta,k}(\mathbf{x}) = \mathcal{E}_{\text{dip}}^{\text{elec}}[\mathbf{d}_{\delta,k}^E](\mathbf{x} - c_k) + \mathcal{E}_{\text{dip}}^{\text{mag}}[\mathbf{d}_{\delta,k}^H](\mathbf{x} - c_k)$$

$$\mathbf{H}_{\delta,k}(\mathbf{x}) = \mathcal{H}_{\text{dip}}^{\text{elec}}[\mathbf{d}_{\delta,k}^E](\mathbf{x} - c_k) + \mathcal{H}_{\text{dip}}^{\text{mag}}[\mathbf{d}_{\delta,k}^H](\mathbf{x} - c_k)$$

- The directions  $\mathbf{d}_{\delta,k}^E$  and  $\mathbf{d}_{\delta,k}^H$  depend on the **nature** of the obstacles

## Case of perfectly conducting spheres

- 3-order approximation

$$\mathbf{d}_{\delta,k}^E = 4\pi\delta^3 \mathbf{E}^{\text{inc}}(c_k) \quad \mathbf{d}_{\delta,k}^H = -2\pi\delta^3 \mathbf{H}^{\text{inc}}(c_k)$$

- Corrected dipolar approximation

$$\mathbf{d}_{\delta,k}^E = 4\pi\delta^3 \left(1 + \frac{3(\kappa\delta)^2}{10}\right) \mathbf{E}^{\text{inc}}(c_k)$$

$$\mathbf{d}_{\delta,k}^H = -2\pi\delta^3 \left(1 - \frac{3(\kappa\delta)^2}{5}\right) \mathbf{H}^{\text{inc}}(c_k)$$

## Important restrictions

- Very small obstacles
- Small number of obstacles
- No interaction taken into account

# Foldy-Lax Approximation

$$\mathbf{E}_\delta^{\text{Foldy}}(\mathbf{x}) = \sum_{k=1}^{N_{\text{obs}}} \mathbf{E}_{\delta,k}(\mathbf{x})$$

$$\mathbf{H}_\delta^{\text{Foldy}}(\mathbf{x}) = \sum_{k=1}^{N_{\text{obs}}} \mathbf{H}_{\delta,k}(\mathbf{x})$$

- Each obstacle is modeled as a **dipolar source** around  $c_k$

$$\mathbf{E}_{\delta,k}(\mathbf{x}) = \mathcal{E}_{\text{dip}}^{\text{elec}}[\mathbf{d}_{\delta,k}^{\text{E}}](\mathbf{x} - c_k) + \mathcal{E}_{\text{dip}}^{\text{mag}}[\mathbf{d}_{\delta,k}^{\text{H}}](\mathbf{x} - c_k)$$

$$\mathbf{H}_{\delta,k}(\mathbf{x}) = \mathcal{H}_{\text{dip}}^{\text{elec}}[\mathbf{d}_{\delta,k}^{\text{E}}](\mathbf{x} - c_k) + \mathcal{H}_{\text{dip}}^{\text{mag}}[\mathbf{d}_{\delta,k}^{\text{H}}](\mathbf{x} - c_k)$$

## Case of perfectly conducting spheres

- 3-order approximation

$$\mathbf{d}_{\delta,k}^{\text{E}} = 4\pi\delta^3 \left( \mathbf{E}^{\text{inc}}(c_k) + \sum_{\substack{\ell=1 \\ \ell \neq k}}^{N_{\text{obs}}} \mathbf{E}_{\delta,\ell}(c_k) \right) \quad \mathbf{d}_{\delta,k}^{\text{H}} = -2\pi\delta^3 \left( \mathbf{H}^{\text{inc}}(c_k) + \sum_{\substack{\ell=1 \\ \ell \neq k}}^{N_{\text{obs}}} \mathbf{H}_{\delta,\ell}(c_k) \right)$$

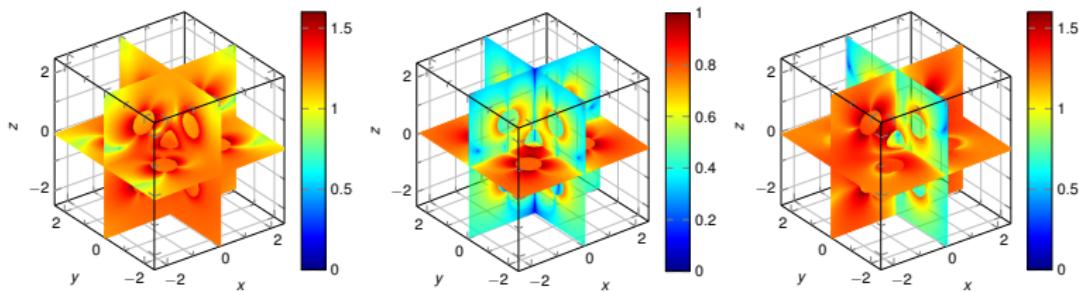
- Corrected dipolar approximation

$$\mathbf{d}_{\delta,k}^{\text{E}} = 4\pi\delta^3 \left( 1 + \frac{3(\kappa\delta)^2}{10} \right) \left( \mathbf{E}^{\text{inc}}(c_k) + \sum_{\substack{\ell=1 \\ \ell \neq k}}^{N_{\text{obs}}} \mathbf{E}_{\delta,\ell}(c_k) \right) \quad \mathbf{d}_{\delta,k}^{\text{H}} = -2\pi\delta^3 \left( 1 - \frac{3(\kappa\delta)^2}{5} \right) \left( \mathbf{H}^{\text{inc}}(c_k) + \sum_{\substack{\ell=1 \\ \ell \neq k}}^{N_{\text{obs}}} \mathbf{H}_{\delta,\ell}(c_k) \right)$$

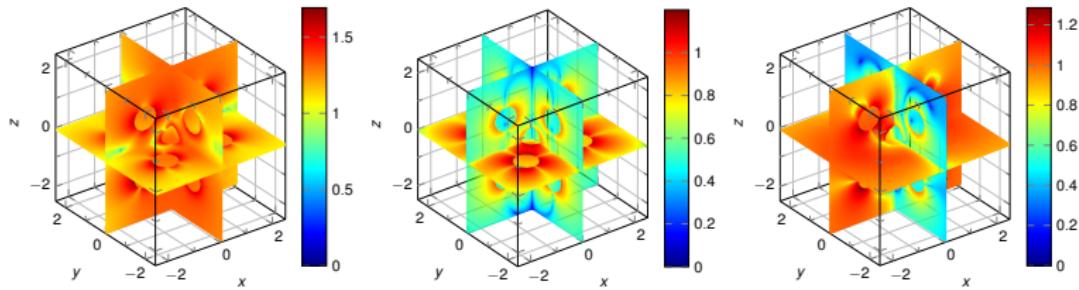
# First Numerical Results

13 obstacles ;  $d_{jk} \approx 1.0$  ;  $\delta = 0.4$

**Born** approximation



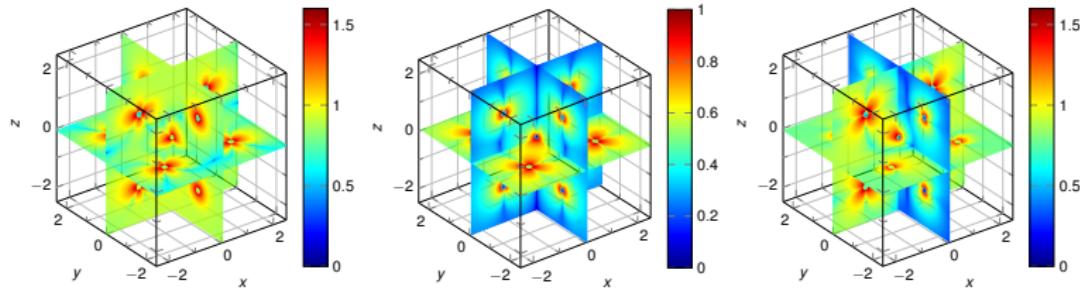
**Foldy-Lax** approximation



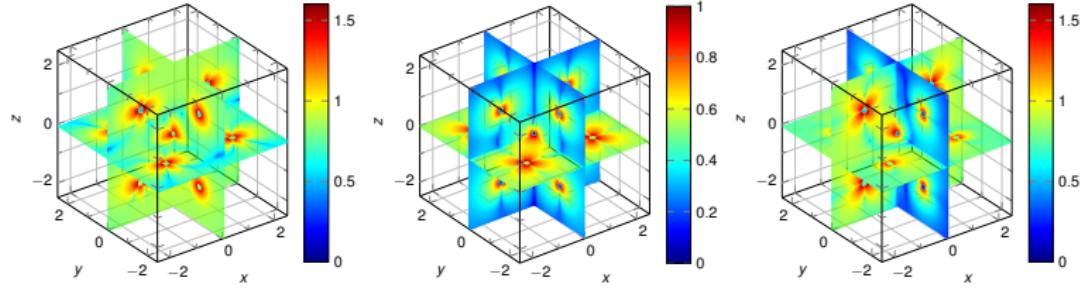
# First Numerical Results

13 obstacles ;  $d_{jk} \approx 1.25$  ;  $\delta = 0.1$

Born approximation



Foldy-Lax approximation



# Outline

## 1. Asymptotic expansions for the single electromagnetic scattering

- First terms of the asymptotics
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## 2. Multiple electromagnetic scattering by small spheres

- Born approximation
- Foldy-Lax approximation
- Preliminary numerical results

## 3. Conclusions and perspectives

# Conclusions and Perspectives

## Conclusions and on-going works

- Single scattering

- ✓ Derivation of first terms of asymptotics for one sphere
- ✓ Numerical validation



J. Labat, V. Péron, S. Tordeux (Inria Research Report n°9169, 2018)

Asymptotic Modeling of the Electromagnetic Scattering by Small Spheres Perfectly Conducting.

- Multiple scattering

- ✓ Born approximation
- ✓ Foldy-Lax approximation
- ~ High-order spectral method
- ~ Numerical validation of Born and Foldy-Lax models

## Perspectives

- Single scattering

- ✗ Justification of matched asymptotic expansions
- ✗ Extension to obstacles of arbitrary shape

- Multiple scattering

- ✗ Justification of Foldy-Lax approximation
- ✗ Justification of the high-order spectral method