

Foldy-Lax Model for the Scattering Problem in Electromagnetism

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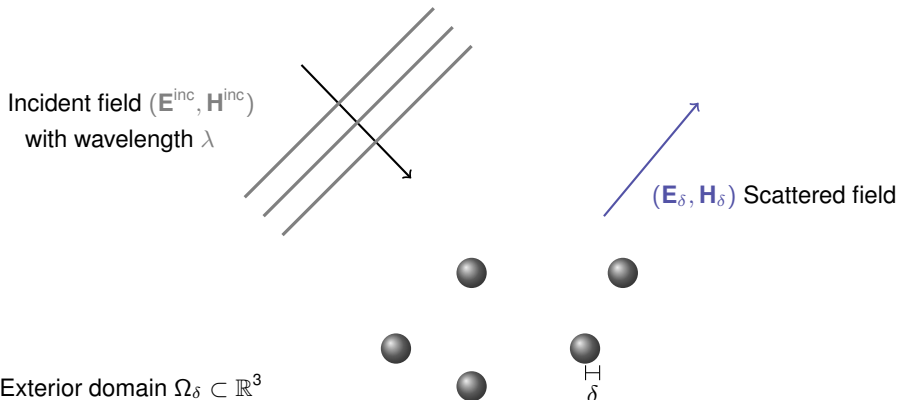
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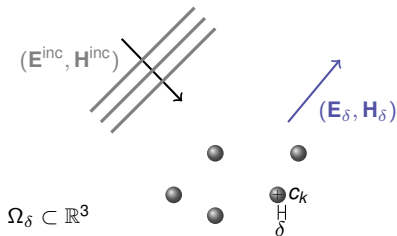


3D-Multiple Scattering Problem by Small Spheres



Asymptotic assumption : $\delta \ll \lambda$

Model Problem



- Time-harmonic domain
- Homogeneous & isotropic medium Ω_δ
- Perfect conductors $\mathcal{B}(c_k, \delta)$, $k = 1 \dots N_{\text{obs}}$

Time-harmonic Maxwell equations

$$\begin{cases} \mathbf{curl} \mathbf{E}_\delta - i\kappa \mathbf{H}_\delta = 0 & \text{in } \Omega_\delta \\ \mathbf{curl} \mathbf{H}_\delta + i\kappa \mathbf{E}_\delta = 0 & \text{in } \Omega_\delta \end{cases}$$

with $\kappa^2 = \omega^2 \mu (\epsilon + \frac{i\sigma}{\omega})$, $\Im(\kappa) \geq 0$

Boundary condition

$$\mathbf{n} \times \mathbf{E}_\delta = -\mathbf{n} \times \mathbf{E}^{\text{inc}} \quad \text{on } \partial\Omega_\delta$$

Silver-Müller radiation condition

$$r(\mathbf{H}_\delta \times \hat{\mathbf{x}} - \mathbf{E}_\delta) \xrightarrow[r \rightarrow \infty]{} 0 \quad \text{unif. in } \hat{\mathbf{x}} = \frac{\mathbf{x}}{r}$$

Mathematical well-posedness [Phillips (1973)]

For any $\mathbf{E}^{\text{inc}} \in \mathbf{H}_{\text{loc}}(\mathbf{curl}, \Omega_\delta)$ there exist a unique solution $(\mathbf{E}_\delta, \mathbf{H}_\delta) \in \mathbf{H}_{\text{loc}}(\mathbf{curl}, \Omega_\delta)^2$ to the exterior Maxwell problem.

Approaches and Objectives

Asymptotic models

Born approximation

Foldy-Lax approximation



H. Ammari, M.S. Vogelius, D. Volkov (2001)

Asymptotic formulas for perturbations in the electromagnetic fields due to the presence of inhomogeneities of small diameter II. The full Maxwell equations



D.P. Challa, G. Hu, M. Sini (2013)

Multiple scattering of electromagnetic waves by finitely many point-like obstacles



D.V. Korikov, B.A. Plamenevskii (2017)

Asymptotics of solutions for stationary and nonstationary Maxwell systems in a domain with small holes

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Numerical solution

High-order spectral method



M. Ganesh, S.C. Hawkins (2009)

A high-order algorithm for multiple electromagnetic scattering in three dimensions



H. Barucq, J. Chabassier, H. Pham, S. Tordeux (2017)

Numerical robustness of single-layer method with Fourier basis for multiple obstacle acoustic scattering in homogeneous media

Motivations

- ✗ **High computational cost** with classical numerical methods
- ✗ **Mesh refinement**
- ✓ High-order **spectral method**

Numerical Solution

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Numerical Solution

Superposition
principle

Born approximation

- ✗ **High computational cost**
 - ✗ volumical methods
 - ~ boundary element methods
- ✓ High-order **spectral method**

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Numerical Solution

Superposition
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Born approximation

Approximation

Asymptotic models

- ✗ **High computational cost**
 - ✗ volumical methods
 - ~ boundary element methods
- ✓ High-order **spectral method**

- ✗ Restricted to small obstacles
- ✓ Low computational cost
- ✓ Meshless method

Motivations

- ✗ **High computational cost** with classical numerical methods
- ✗ **Mesh refinement**
- ✓ High-order **spectral method**

Numerical Solution

Superposition principle

Born approximation

- ✗ **High computational cost**
 - ✗ volumical methods
 - ~ boundary element methods
- ✓ High-order **spectral method**

- ✓ Interactions taken into account
- ✓ Low computational cost
- ✓ Meshless method

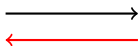
Foldy-Lax model

Multiple scattering

Asymptotic models

- ✗ Restricted to small obstacles
- ✓ Low computational cost
- ✓ Meshless method

Approximation



Outline

1. Asymptotic expansions for the single electromagnetic scattering

- First terms of the asymptotics
- Numerical results

2. Multiple electromagnetic scattering by small spheres

- Born approximation
- Foldy-Lax approximation
- Preliminary numerical results

3. Conclusions and perspectives

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1. Asymptotic expansions for the single electromagnetic scattering

- First terms of the asymptotics
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2. Multiple electromagnetic scattering by small spheres

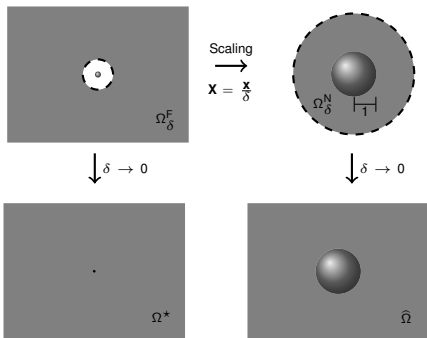
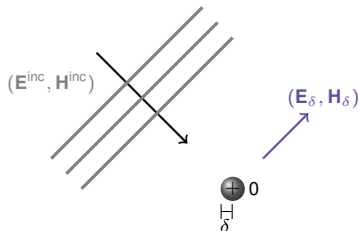
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3. Conclusions and perspectives

Approximation for Single Scattering

Method of **matched asymptotic expansions**

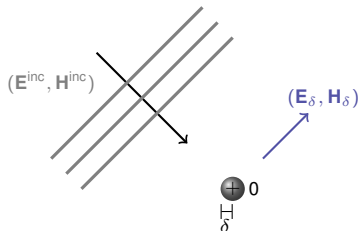
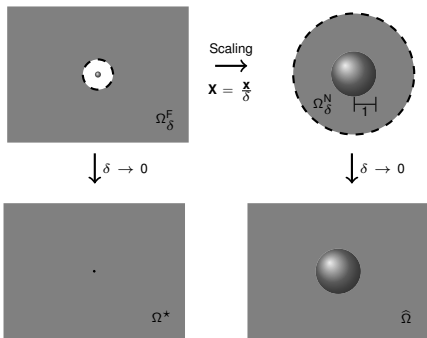
- **Domain decomposition**
- Local approximations
- Matching procedure



Approximation for Single Scattering

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Asymptotic expansions

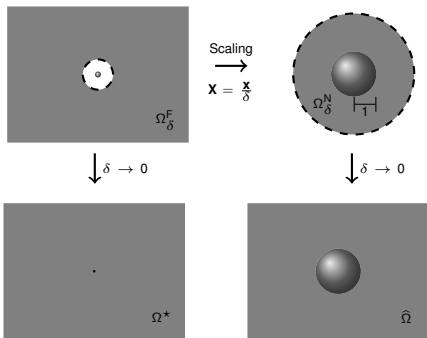
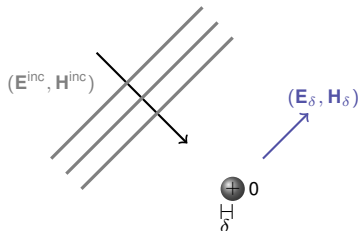
Far field expansion in $\Omega^* = \mathbb{R}^3 \setminus \{0\}$

Near field expansion in $\hat{\Omega} = \mathbb{R}^3 \setminus \overline{\mathcal{B}(0, 1)}$

Approximation for Single Scattering

Method of **matched asymptotic expansions**

- Domain decomposition
- Local approximations
- **Matching procedure**



Asymptotic expansions

Far field expansion in $\Omega^* = \mathbb{R}^3 \setminus \{0\}$

Near field expansion in $\hat{\Omega} = \mathbb{R}^3 \setminus \overline{\mathcal{B}(0, 1)}$

Near field \implies Far field

Near Field Expansion I

$$\mathbf{E}_\delta(\delta\mathbf{X}) = \widehat{\mathbf{E}}_0(\mathbf{X}) + \delta\widehat{\mathbf{E}}_1(\mathbf{X}) + \delta^2\widehat{\mathbf{E}}_2(\mathbf{X}) + \dots$$

$$\mathbf{H}_\delta(\delta\mathbf{X}) = \widehat{\mathbf{H}}_0(\mathbf{X}) + \delta\widehat{\mathbf{H}}_1(\mathbf{X}) + \delta^2\widehat{\mathbf{H}}_2(\mathbf{X}) + \dots$$

0-order (static **dipole**)

$$\widehat{\mathbf{E}}_0(\mathbf{X}) = \frac{1}{|\mathbf{X}|^3} \left(3(\mathbf{e}_r \cdot \mathbf{E}^{\text{inc}}(0))\mathbf{e}_r - \mathbf{E}^{\text{inc}}(0) \right)$$

$$\widehat{\mathbf{H}}_0(\mathbf{X}) = -\frac{1}{2|\mathbf{X}|^3} \left(3(\mathbf{e}_r \cdot \mathbf{H}^{\text{inc}}(0))\mathbf{e}_r - \mathbf{H}^{\text{inc}}(0) \right)$$

1-order (static **quadrupole** + static **dipole**)

$$\widehat{\mathbf{E}}_1(\mathbf{X}) = -\frac{1}{|\mathbf{X}|^4} \gamma_t \left(\mathbf{J}_{\mathbf{E}^{\text{inc}}}^{\text{sym}}(0)\mathbf{e}_r \right) + \frac{3}{2|\mathbf{X}|^4} (\mathbf{e}_r \cdot \mathbf{J}_{\mathbf{E}^{\text{inc}}}(0)\mathbf{e}_r)\mathbf{e}_r - \frac{i\kappa}{2|\mathbf{X}|^2} \mathbf{H}^{\text{inc}}(0) \times \mathbf{e}_r$$

$$\widehat{\mathbf{H}}_1(\mathbf{X}) = \frac{2}{3|\mathbf{X}|^4} \gamma_t \left(\mathbf{J}_{\mathbf{H}^{\text{inc}}}^{\text{sym}}(0)\mathbf{e}_r \right) - \frac{1}{|\mathbf{X}|^4} (\mathbf{e}_r \cdot \mathbf{J}_{\mathbf{H}^{\text{inc}}}(0)\mathbf{e}_r)\mathbf{e}_r + \frac{i\kappa}{|\mathbf{X}|^2} \mathbf{E}^{\text{inc}}(0) \times \mathbf{e}_r$$

with $\gamma_t \mathbf{u} = \mathbf{u} - (\mathbf{e}_r \cdot \mathbf{u})\mathbf{e}_r$ and $\mathbf{J}^{\text{sym}} = \frac{1}{2}(\mathbf{J} + \mathbf{J}^T)$ symmetric Jacobian

Near Field Expansion II

2-order (static **octupole** + static **quadrupole** + static **dipole**)

$$\begin{aligned} \hat{\mathbf{E}}_2(\mathbf{X}) = & \frac{1}{|\mathbf{X}|^5} \left\{ \frac{2}{3} (\mathbf{e}_r \cdot \mathbf{e}_r^\top \mathbf{H}_{\mathbf{E}^{\text{inc}}}(0) \mathbf{e}_r) \mathbf{e}_r + \frac{2\kappa^2}{15} (\mathbf{e}_r \cdot \mathbf{E}^{\text{inc}}(0)) \mathbf{e}_r - \frac{1}{2} \gamma_t (\mathbf{e}_r^\top \mathbf{H}_{\mathbf{E}^{\text{inc}}}(0) \mathbf{e}_r) \right. \\ & \left. - \frac{i\kappa}{3} \mathbf{e}_r \times \mathbf{J}_{\mathbf{H}^{\text{inc}}}(0) \mathbf{e}_r - \frac{\kappa^2}{5} \gamma_t (\mathbf{E}^{\text{inc}}(0)) \right\} + \frac{i\kappa}{3|\mathbf{X}|^3} \mathbf{e}_r \times \mathbf{J}_{\mathbf{H}^{\text{inc}}}(0) \mathbf{e}_r \\ & - \frac{3\kappa^2}{10|\mathbf{X}|^3} \left\{ \mathbf{E}^{\text{inc}}(0) - 3(\mathbf{e}_r \cdot \mathbf{E}^{\text{inc}}(0)) \mathbf{e}_r \right\} + \frac{\kappa^2}{2|\mathbf{X}|} \left\{ \mathbf{E}^{\text{inc}}(0) + (\mathbf{e}_r \cdot \mathbf{E}^{\text{inc}}(0)) \mathbf{e}_r \right\} \end{aligned}$$

$$\begin{aligned} \hat{\mathbf{H}}_2(\mathbf{X}) = & \frac{1}{|\mathbf{X}|^5} \left\{ -\frac{1}{2} (\mathbf{e}_r \cdot \mathbf{e}_r^\top \mathbf{H}_{\mathbf{H}^{\text{inc}}}(0) \mathbf{e}_r) \mathbf{e}_r - \frac{\kappa^2}{10} (\mathbf{e}_r \cdot \mathbf{H}^{\text{inc}}(0)) \mathbf{e}_r + \frac{3}{8} \gamma_t (\mathbf{e}_r^\top \mathbf{H}_{\mathbf{H}^{\text{inc}}}(0) \mathbf{e}_r) \right. \\ & \left. - \frac{i\kappa}{4} \mathbf{e}_r \times \mathbf{J}_{\mathbf{E}^{\text{inc}}}(0) \mathbf{e}_r + \frac{3\kappa^2}{20} \gamma_t (\mathbf{H}^{\text{inc}}(0)) \right\} + \frac{i\kappa}{2|\mathbf{X}|} \mathbf{e}_r \times \mathbf{J}_{\mathbf{E}^{\text{inc}}}(0) \mathbf{e}_r \\ & + \frac{3\kappa^2}{10|\mathbf{X}|^3} \left\{ 3(\mathbf{e}_r \cdot \mathbf{H}^{\text{inc}}(0)) \mathbf{e}_r - \mathbf{H}^{\text{inc}}(0) \right\} - \frac{\kappa^2}{4|\mathbf{X}|} \left\{ \mathbf{H}^{\text{inc}}(0) + (\mathbf{e}_r \cdot \mathbf{H}^{\text{inc}}(0)) \mathbf{e}_r \right\} \end{aligned}$$

Far Field Expansion I

$$\mathbf{E}_\delta(\mathbf{x}) = \delta^3 \tilde{\mathbf{E}}_3(\mathbf{x}) + \delta^5 \tilde{\mathbf{E}}_5(\mathbf{x}) + \dots \quad \mathbf{H}_\delta(\mathbf{x}) = \delta^3 \tilde{\mathbf{H}}_3(\mathbf{x}) + \delta^5 \tilde{\mathbf{H}}_5(\mathbf{x}) + \dots$$

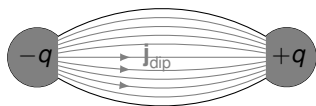
3-order (time-harmonic dipole)

$$\tilde{\mathbf{E}}_3(\mathbf{x}) = -\frac{\kappa^3}{2} h_1^{(1)}(\kappa r) \mathbf{e}_r \times \mathbf{H}^{\text{inc}}(0) - \kappa^3 \frac{\tilde{h}_1^{(1)}(\kappa r)}{i\kappa r} \gamma_t \mathbf{E}^{\text{inc}}(0) - 2\kappa^3 \frac{h_1^{(1)}(\kappa r)}{i\kappa r} (\mathbf{e}_r \cdot \mathbf{E}^{\text{inc}}(0)) \mathbf{e}_r$$

$$\tilde{\mathbf{H}}_3(\mathbf{x}) = -\kappa^3 h_1^{(1)}(\kappa r) \mathbf{e}_r \times \mathbf{E}^{\text{inc}}(0) + \frac{\kappa^3}{2} \frac{\tilde{h}_1^{(1)}(\kappa r)}{i\kappa r} \gamma_t \mathbf{H}^{\text{inc}}(0) + \kappa^3 \frac{h_1^{(1)}(\kappa r)}{i\kappa r} (\mathbf{e}_r \cdot \mathbf{H}^{\text{inc}}(0)) \mathbf{e}_r$$

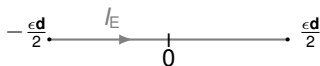
with $h_n^{(1)}$ spherical Hankel of 1st kind and $\tilde{h}_1^{(1)}(z) = h_1^{(1)}(z) + zh_1^{(1)'}(z)$

Time-Harmonic Electromagnetic Dipole



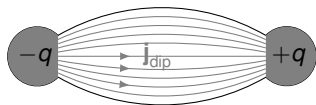
\mathbf{j}_{dip} electric current

Ideal configuration

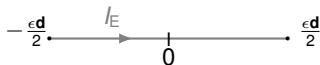


$\mathbf{d} \in \mathbb{R}^3$, ϵ small parameter

Time-Harmonic Electromagnetic Dipole

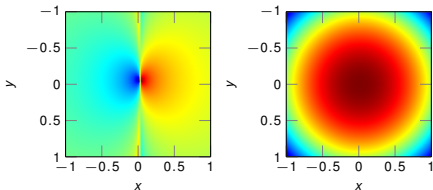


Ideal configuration



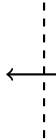
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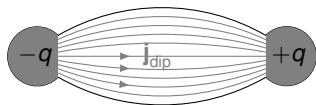
Time-harmonic electric potential V and magnetic potential \mathbf{A} (x -component) in the xy -plane

$\epsilon \rightarrow 0$

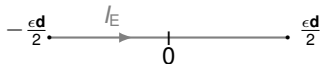


- Punctual charges with $q = \frac{1}{\epsilon}$
- Filiform current (\mathbf{d}, I_E)
- Charge conservation $I_E = -i\omega q$

Time-Harmonic Electromagnetic Dipole

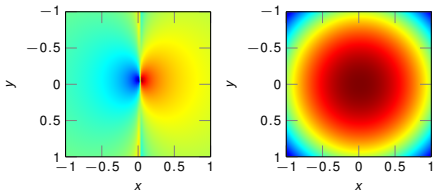


Ideal configuration



$\mathbf{d} \in \mathbb{R}^3$, ϵ small parameter

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Time-harmonic electric potential V and magnetic potential \mathbf{A} (x -component) in the xy -plane

$\epsilon \rightarrow 0$



- Punctual charges with $q = \frac{1}{\epsilon}$
- Filiform current (\mathbf{d} , I_E)
- Charge conservation $I_E = -i\omega q$

$$\mathcal{E}_{\text{dip}}^{\text{elec}}[\mathbf{d}] = -\nabla V + i\kappa \mathbf{A}$$

$$\mathcal{H}_{\text{dip}}^{\text{elec}}[\mathbf{d}] = \mathbf{curl} \mathbf{A}$$



Magnetic dipole

$$\mathcal{H}_{\text{dip}}^{\text{mag}}[\mathbf{d}] = \mathcal{E}_{\text{dip}}^{\text{elec}}[\mathbf{d}]$$

$$\mathcal{E}_{\text{dip}}^{\text{mag}}[\mathbf{d}] = -\mathcal{H}_{\text{dip}}^{\text{elec}}[\mathbf{d}]$$

Far Field Expansion II

$$\mathbf{E}_\delta(\mathbf{x}) = \delta^3 \tilde{\mathbf{E}}_3(\mathbf{x}) + \delta^5 \tilde{\mathbf{E}}_5(\mathbf{x}) + \dots$$

3-order (time-harmonic **dipole**)

$$\tilde{\mathbf{E}}_3 = \boldsymbol{\varepsilon}_{\text{dip}}^{\text{elec}} [4\pi \mathbf{E}^{\text{inc}}(0)] + \boldsymbol{\varepsilon}_{\text{dip}}^{\text{mag}} [-2\pi \mathbf{H}^{\text{inc}}(0)]$$

5-order (time-harmonic **dipole** + time-harmonic **quadrupole**)

$$\begin{aligned} \tilde{\mathbf{E}}_5(\mathbf{x}) = & \frac{3\kappa^2}{10} \boldsymbol{\varepsilon}_{\text{dip}}^{\text{elec}} [4\pi \mathbf{E}^{\text{inc}}(0)](\mathbf{x}) - \frac{3\kappa^2}{5} \boldsymbol{\varepsilon}_{\text{dip}}^{\text{mag}} [-2\pi \mathbf{H}^{\text{inc}}(0)](\mathbf{x}) \\ & - \frac{\kappa^4}{9} h_2^{(1)}(\kappa r) \mathbf{e}_r \times \mathbf{J}_{\mathbf{H}^{\text{inc}}}^{\text{sym}}(0) \mathbf{e}_r - \frac{\kappa^4}{6} \frac{\tilde{h}_2^{(1)}(\kappa r)}{i\kappa r} \gamma_t \left(\mathbf{J}_{\mathbf{E}^{\text{inc}}}^{\text{sym}}(0) \mathbf{e}_r \right) - \frac{\kappa^4}{2} \frac{h_2^{(1)}(\kappa r)}{i\kappa r} (\mathbf{e}_r \cdot \mathbf{J}_{\mathbf{E}^{\text{inc}}}(0) \mathbf{e}_r) \mathbf{e}_r \end{aligned}$$

Collected dipolar model

$$\mathbf{E}_\delta^{\text{Col}}(\mathbf{x}) = \left(\delta^3 + \frac{3\kappa^2 \delta^5}{10} \right) \boldsymbol{\varepsilon}_{\text{dip}}^{\text{elec}} [4\pi \mathbf{E}^{\text{inc}}(0)](\mathbf{x}) + \left(\delta^3 - \frac{3\kappa^2 \delta^5}{5} \right) \boldsymbol{\varepsilon}_{\text{dip}}^{\text{mag}} [-2\pi \mathbf{H}^{\text{inc}}(0)](\mathbf{x})$$

Far Field Expansion III

$$\mathbf{H}_\delta(\mathbf{x}) = \delta^3 \tilde{\mathbf{H}}_3(\mathbf{x}) + \delta^5 \tilde{\mathbf{H}}_5(\mathbf{x}) + \dots$$

3-order (time-harmonic **dipole**)

$$\tilde{\mathbf{H}}_3 = \mathcal{H}_{\text{dip}}^{\text{elec}} [4\pi \mathbf{E}^{\text{inc}}(0)] + \mathcal{H}_{\text{dip}}^{\text{mag}} [-2\pi \mathbf{H}^{\text{inc}}(0)]$$

5-order (time-harmonic **dipole** + time-harmonic **quadrupole**)

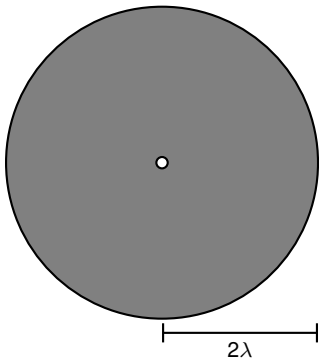
$$\begin{aligned} \tilde{\mathbf{H}}_5(\mathbf{x}) = & \frac{3\kappa^2}{10} \mathcal{H}_{\text{dip}}^{\text{elec}} [4\pi \mathbf{E}^{\text{inc}}(0)](\mathbf{x}) - \frac{3\kappa^2}{5} \mathcal{H}_{\text{dip}}^{\text{mag}} [-2\pi \mathbf{H}^{\text{inc}}(0)](\mathbf{x}) \\ & - \frac{\kappa^4}{6} h_2^{(1)}(\kappa r) \mathbf{e}_r \times \mathbf{J}_{\text{Einc}}^{\text{sym}}(0) \mathbf{e}_r + \frac{\kappa^4}{9} \frac{\tilde{h}_2^{(1)}(\kappa r)}{i\kappa r} \gamma_t \left(\mathbf{J}_{\text{Hinc}}^{\text{sym}}(0) \mathbf{e}_r \right) + \frac{\kappa^4}{3} \frac{h_2^{(1)}(\kappa r)}{i\kappa r} (\mathbf{e}_r \cdot \mathbf{J}_{\text{Hinc}}(0) \mathbf{e}_r) \mathbf{e}_r \end{aligned}$$

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Simulation Description

3D Whole domain



Small parameter

$$\delta \in \left\{ \frac{1}{10^p}, p = 0.5 : 0.1 : 4 \right\}$$

Physical parameters

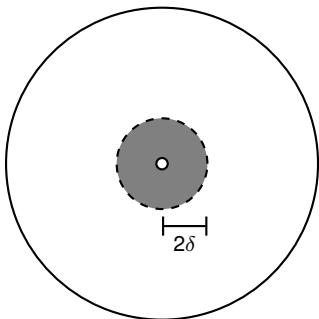
- $\varepsilon = \mu = 1.0, \quad \sigma = 0$
- $\lambda = 5.0,$
 $\omega = \kappa = \frac{2\pi}{5.0} \approx 1.26$

Incident plane wave

- $\mathbf{E}^{\text{inc}} = \vec{p} \exp(-i\vec{k} \cdot \mathbf{x})$
- $\mathbf{H}^{\text{inc}} = \frac{1}{i\kappa} \text{curl } \mathbf{E}^{\text{inc}}$
- $\vec{p} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \vec{k} = \begin{pmatrix} 0 \\ 0 \\ \kappa \end{pmatrix}$

Simulation Description

Near field domain



Small parameter

$$\delta \in \left\{ \frac{1}{10^p}, p = 0.5 : 0.1 : 4 \right\}$$

Physical parameters

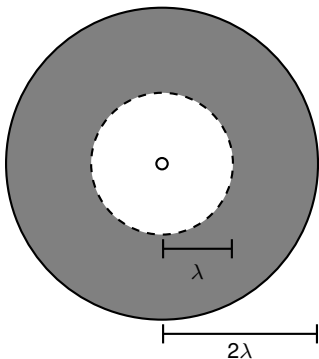
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- $\mathbf{H}^{\text{inc}} = \frac{1}{i\kappa} \text{curl } \mathbf{E}^{\text{inc}}$
- $\vec{p} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \vec{k} = \begin{pmatrix} 0 \\ 0 \\ \kappa \end{pmatrix}$

Simulation Description

Far field domain



Small parameter

$$\delta \in \left\{ \frac{1}{10^p}, p = 0.5 : 0.1 : 4 \right\}$$

Physical parameters

- $\varepsilon = \mu = 1.0, \sigma = 0$
- $\lambda = 5.0,$
 $\omega = \kappa = \frac{2\pi}{5.0} \approx 1.26$

Incident plane wave

- $\mathbf{E}^{\text{inc}} = \vec{p} \exp(-i\vec{k} \cdot \mathbf{x})$
- $\mathbf{H}^{\text{inc}} = \frac{1}{i\kappa} \text{curl } \mathbf{E}^{\text{inc}}$
- $\vec{p} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \vec{k} = \begin{pmatrix} 0 \\ 0 \\ \kappa \end{pmatrix}$

Near field approximations

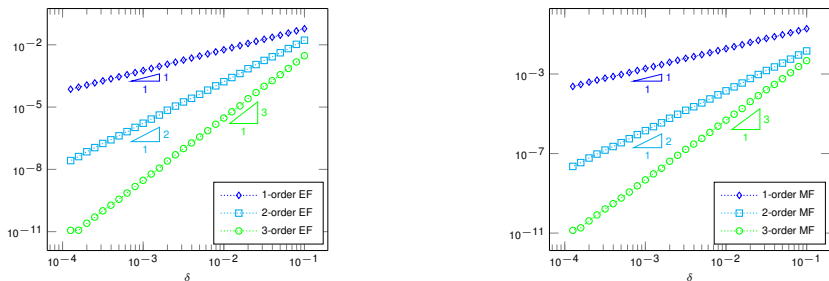
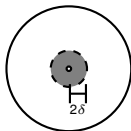


Figure: Relative $L^2(\Omega_\delta^{2\delta})$ -errors for electric near field (left) magnetic near field (right)



$$\text{1-order EF : } \|\mathbf{E}_\delta - \widehat{\mathbf{E}}_0(\frac{\cdot}{\delta})\|_{L^2(\Omega_\delta^{2\delta})} / \|\mathbf{E}_\delta\|_{L^2(\Omega_\delta^{2\delta})}$$

$$\text{2-order EF : } \|\mathbf{E}_\delta - \widehat{\mathbf{E}}_0(\frac{\cdot}{\delta}) - \delta \widehat{\mathbf{E}}_1(\frac{\cdot}{\delta})\|_{L^2(\Omega_\delta^{2\delta})} / \|\mathbf{E}_\delta\|_{L^2(\Omega_\delta^{2\delta})}$$

$$\text{3-order EF : } \|\mathbf{E}_\delta - \widehat{\mathbf{E}}_0(\frac{\cdot}{\delta}) - \delta \widehat{\mathbf{E}}_1(\frac{\cdot}{\delta}) - \delta^2 \widehat{\mathbf{E}}_2(\frac{\cdot}{\delta})\|_{L^2(\Omega_\delta^{2\delta})} / \|\mathbf{E}_\delta\|_{L^2(\Omega_\delta^{2\delta})}$$

Far field approximations

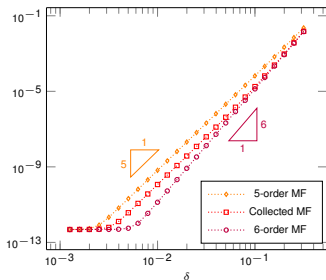
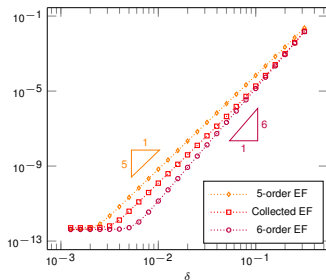
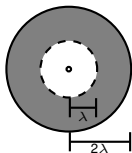


Figure: Absolute $L^2(\Omega_\lambda^{2\lambda})$ -errors for electric far field (left) magnetic far field (right)



$$\text{5-order EF : } \|\mathbf{E}_\delta - \delta^3 \tilde{\mathbf{E}}_3\|_{L^2(\Omega_\lambda^{2\lambda})}$$

$$\text{6-order EF : } \|\mathbf{E}_\delta - \delta^3 \tilde{\mathbf{E}}_3 - \delta^5 \tilde{\mathbf{E}}_5\|_{L^2(\Omega_\lambda^{2\lambda})}$$

Outline

1. Asymptotic expansions for the single electromagnetic scattering

- First terms of the asymptotics
- Numerical results

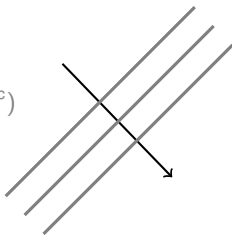
2. Multiple electromagnetic scattering by small spheres

- Born approximation
- Foldy-Lax approximation
- Preliminary numerical results

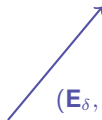
3. Conclusions and perspectives

3D-Multiple Scattering Problem by Small Spheres

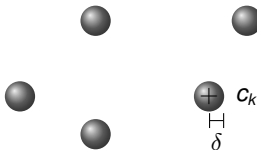
Incident field $(\mathbf{E}^{\text{inc}}, \mathbf{H}^{\text{inc}})$
with wavelength λ



$(\mathbf{E}_\delta, \mathbf{H}_\delta)$ Scattered field



Exterior domain $\Omega_\delta \subset \mathbb{R}^3$



$$\Omega_\delta = \mathbb{R}^3 \setminus \bigcup_{k=1}^{N_{\text{obs}}} \overline{\mathcal{B}(c_k, \delta)}$$

Born Approximation

$$\mathbf{E}_{\delta}^{\text{Born}}(\mathbf{x}) = \sum_{k=1}^{N_{\text{obs}}} \mathbf{E}_{\delta,k}(\mathbf{x})$$

$$\mathbf{H}_{\delta}^{\text{Born}}(\mathbf{x}) = \sum_{k=1}^{N_{\text{obs}}} \mathbf{H}_{\delta,k}(\mathbf{x})$$

- Each obstacle is modeled as a **dipolar source** around c_k

$$\mathbf{E}_{\delta,k}(\mathbf{x}) = \mathcal{E}_{\text{dip}}^{\text{elec}}[\mathbf{d}_{\delta,k}^{\text{E}}](\mathbf{x} - c_k) + \mathcal{E}_{\text{dip}}^{\text{mag}}[\mathbf{d}_{\delta,k}^{\text{H}}](\mathbf{x} - c_k)$$

$$\mathbf{H}_{\delta,k}(\mathbf{x}) = \mathcal{H}_{\text{dip}}^{\text{elec}}[\mathbf{d}_{\delta,k}^{\text{E}}](\mathbf{x} - c_k) + \mathcal{H}_{\text{dip}}^{\text{mag}}[\mathbf{d}_{\delta,k}^{\text{H}}](\mathbf{x} - c_k)$$

- The directions $\mathbf{d}_{\delta,k}^{\text{E}}$ and $\mathbf{d}_{\delta,k}^{\text{H}}$ depend on the **nature** of the obstacles

Born Approximation

$$\mathbf{E}_\delta^{\text{Born}}(\mathbf{x}) = \sum_{k=1}^{N_{\text{obs}}} \mathbf{E}_{\delta,k}(\mathbf{x})$$

$$\mathbf{H}_\delta^{\text{Born}}(\mathbf{x}) = \sum_{k=1}^{N_{\text{obs}}} \mathbf{H}_{\delta,k}(\mathbf{x})$$

- Each obstacle is modeled as a **dipolar source** around c_k

$$\mathbf{E}_{\delta,k}(\mathbf{x}) = \mathcal{E}_{\text{dip}}^{\text{elec}}[\mathbf{d}_{\delta,k}^{\text{E}}](\mathbf{x} - c_k) + \mathcal{E}_{\text{dip}}^{\text{mag}}[\mathbf{d}_{\delta,k}^{\text{H}}](\mathbf{x} - c_k)$$

$$\mathbf{H}_{\delta,k}(\mathbf{x}) = \mathcal{H}_{\text{dip}}^{\text{elec}}[\mathbf{d}_{\delta,k}^{\text{E}}](\mathbf{x} - c_k) + \mathcal{H}_{\text{dip}}^{\text{mag}}[\mathbf{d}_{\delta,k}^{\text{H}}](\mathbf{x} - c_k)$$

- The directions $\mathbf{d}_{\delta,k}^{\text{E}}$ and $\mathbf{d}_{\delta,k}^{\text{H}}$ depend on the **nature** of the obstacles

Case of perfectly conducting spheres

- 3-order approximation

$$\mathbf{d}_{\delta,k}^{\text{E}} = 4\pi\delta^3 \mathbf{E}^{\text{inc}}(c_k) \quad \mathbf{d}_{\delta,k}^{\text{H}} = -2\pi\delta^3 \mathbf{H}^{\text{inc}}(c_k)$$

- Corrected dipolar approximation

$$\mathbf{d}_{\delta,k}^{\text{E}} = 4\pi\delta^3 \left(1 + \frac{3(\kappa\delta)^2}{10}\right) \mathbf{E}^{\text{inc}}(c_k)$$

$$\mathbf{d}_{\delta,k}^{\text{H}} = -2\pi\delta^3 \left(1 - \frac{3(\kappa\delta)^2}{5}\right) \mathbf{H}^{\text{inc}}(c_k)$$

Important restrictions

- Very small obstacles
- Small number of obstacles
- No interaction taken into account

Foldy-Lax Approximation

$$\mathbf{E}_\delta^{\text{Foldy}}(\mathbf{x}) = \sum_{k=1}^{N_{\text{obs}}} \mathbf{E}_{\delta,k}(\mathbf{x})$$

$$\mathbf{H}_\delta^{\text{Foldy}}(\mathbf{x}) = \sum_{k=1}^{N_{\text{obs}}} \mathbf{H}_{\delta,k}(\mathbf{x})$$

- Each obstacle is modeled as a **dipolar source** around c_k

$$\mathbf{E}_{\delta,k}(\mathbf{x}) = \boldsymbol{\varepsilon}_{\text{dip}}^{\text{elec}} [\mathbf{d}_{\delta,k}^{\text{E}}](\mathbf{x} - c_k) + \boldsymbol{\varepsilon}_{\text{dip}}^{\text{mag}} [\mathbf{d}_{\delta,k}^{\text{H}}](\mathbf{x} - c_k)$$

$$\mathbf{H}_{\delta,k}(\mathbf{x}) = \boldsymbol{\mathcal{H}}_{\text{dip}}^{\text{elec}} [\mathbf{d}_{\delta,k}^{\text{E}}](\mathbf{x} - c_k) + \boldsymbol{\mathcal{H}}_{\text{dip}}^{\text{mag}} [\mathbf{d}_{\delta,k}^{\text{H}}](\mathbf{x} - c_k)$$

Case of perfectly conducting spheres

- 3-order approximation

$$\mathbf{d}_{\delta,k}^{\text{E}} = 4\pi\delta^3 \left(\mathbf{E}^{\text{inc}}(c_k) + \sum_{\substack{\ell=1 \\ \ell \neq k}}^{N_{\text{obs}}} \mathbf{E}_{\delta,\ell}(c_k) \right) \quad \mathbf{d}_{\delta,k}^{\text{H}} = -2\pi\delta^3 \left(\mathbf{H}^{\text{inc}}(c_k) + \sum_{\substack{\ell=1 \\ \ell \neq k}}^{N_{\text{obs}}} \mathbf{H}_{\delta,\ell}(c_k) \right)$$

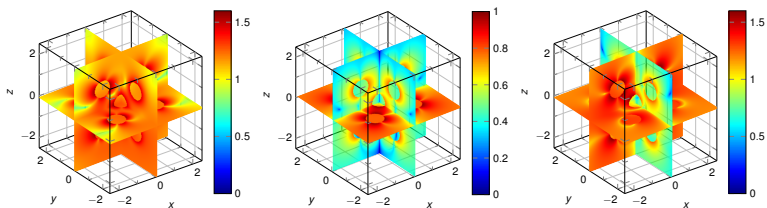
- Corrected dipolar approximation

$$\mathbf{d}_{\delta,k}^{\text{E}} = 4\pi\delta^3 \left(1 + \frac{3(\kappa\delta)^2}{10} \right) \left(\mathbf{E}^{\text{inc}}(c_k) + \sum_{\substack{\ell=1 \\ \ell \neq k}}^{N_{\text{obs}}} \mathbf{E}_{\delta,\ell}(c_k) \right) \quad \mathbf{d}_{\delta,k}^{\text{H}} = -2\pi\delta^3 \left(1 - \frac{3(\kappa\delta)^2}{5} \right) \left(\mathbf{H}^{\text{inc}}(c_k) + \sum_{\substack{\ell=1 \\ \ell \neq k}}^{N_{\text{obs}}} \mathbf{H}_{\delta,\ell}(c_k) \right)$$

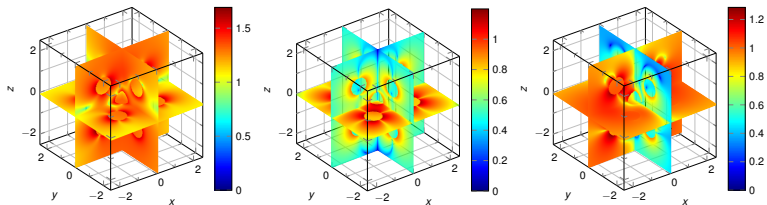
First Numerical Results

13 obstacles ; $d_{jk} \approx 1.0$; $\delta = 0.4$

Born approximation



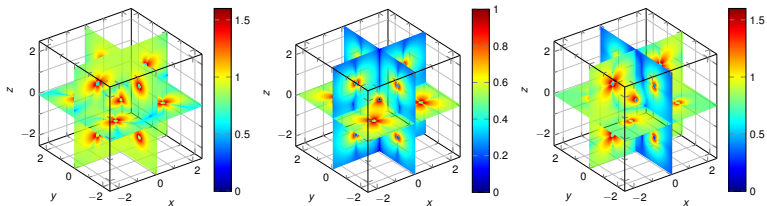
Foldy-Lax approximation



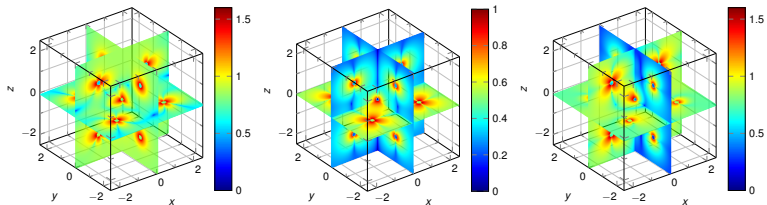
First Numerical Results

13 obstacles ; $d_{jk} \approx 1.25$; $\delta = 0.1$

Born approximation



Foldy-Lax approximation



Outline

1. Asymptotic expansions for the single electromagnetic scattering

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- Preliminary numerical results

3. Conclusions and perspectives

Conclusions and Perspectives

Conclusions and on-going works

- Single scattering
 - ✓ Derivation of first terms of asymptotics for one sphere
 - ✓ Numerical validation



J. Labat, V. Péron, S. Tordeux (Inria Research Report n°9169, 2018)

Asymptotic Modeling of the Electromagnetic Scattering by Small Spheres Perfectly Conducting.

- Multiple scattering
 - ✓ Born approximation
 - ✓ Foldy-Lax approximation
 - ~ High-order spectral method
 - ~ Numerical validation of Born and Foldy-Lax models

Perspectives

- Single scattering
 - ✗ Justification of matched asymptotic expansions
 - ✗ Extension to obstacles of arbitrary shape
- Multiple scattering
 - ✗ Justification of Foldy-Lax approximation
 - ✗ Justification of the high-order spectral method