

blockcluster, simerge and C++ with R

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▶ To cite this version:

Serge Iovleff, Seydou Nourou Sylla. blockcluster, simerge and C++ with R. Mixture Models: Theory and Applications, Jun 2018, Paris, France. hal-01884822

HAL Id: hal-01884822 https://inria.hal.science/hal-01884822

Submitted on 1 Oct 2018 $\,$

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blockcluster, simerge and C++ with R

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Summary

blockcluster package

simerge: Block clustering of binary data with Gaussian co-variables

C++ Programming with R: The simerge package

Preliminary Results

References



Co-Clustering

"Aims to organize data-set into a set of homogeneous blocks by simultaneous clustering of individuals and variables."

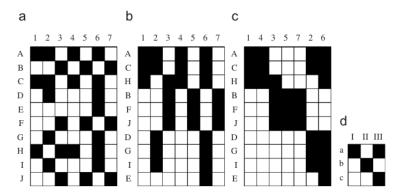


Figure: Binary data set (a), data reorganized by a partition on I (b), by partitions on I and J simultaneously (c) and summary matrix (d).



Model Based Approach

x is data set doubly indexed by a set I with n elements (individuals) and a set J with m elements (variables).

 $\mathbf{z} = (z_{11}, \ldots, z_{ng})$ with $z_{ik} = 1$ if *i* belongs to cluster *k* and $z_{ik} = 0$ otherwise,

 $\mathbf{w} = (w_{11}, \dots, w_{md})$ with $w_{j\ell} = 1$ if j belongs to cluster ℓ and $w_{j\ell} = 0$ otherwise,

$$f(\mathbf{x}; \boldsymbol{\theta}) = \sum_{(\mathbf{z}, \mathbf{w}) \in \mathcal{Z} \times \mathcal{W}} p(\mathbf{z}; \boldsymbol{\theta}) p(\mathbf{w}; \boldsymbol{\theta}) f(\mathbf{x} | \mathbf{z}, \mathbf{w}; \boldsymbol{\theta})$$
(1)

where \mathcal{Z} and \mathcal{W} denote the sets of all possible labelling \mathbf{z} of I and \mathbf{w} of J. There is $g^n \times d^m$ labelling possible.



blockcluster: R Package For coclustering

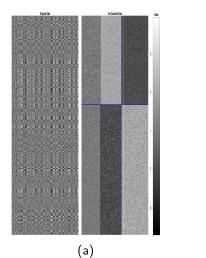
- ▶ R interface to C++ library coclust (using STK++ in background),
- Simple and Robust API,
- Extend four basic functions "Plot", "Summary", "Show", "Print",
- Implements "intelligent" estimation strategy.

Example

```
data(gaussiandata)
out<-coclusterGaussian(gaussiandata,model="
    pi_rho_sigma2kl",nbcocluster=c(2,3))
plot(out)
plot(out, type="distribution")</pre>
```



Example : Gaussian distribution



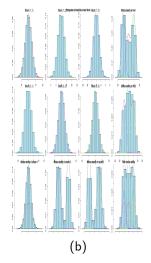


Figure: Simulated and co-clustered data (a), Data block-distributions (b)



Example : Binary distribution

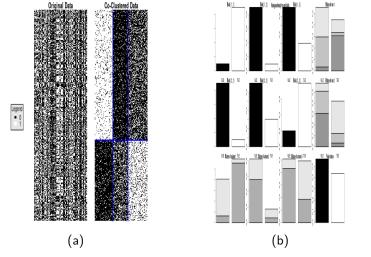
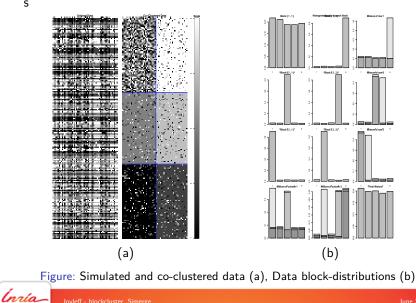


Figure: Simulated and co-clustered data (a), Data block-distributions (b)



Example : Categorical distribution

s



Example : Poisson distribution

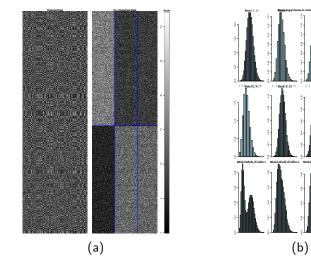


Figure: Simulated and co-clustered data (a), Data block-distributions (b)



Development history

- First versions developed during ADT coclust (October 2011-October 2013). Implement binary, Poisson, Gaussian models; BEM and BCEM algorithms.
- Release 3.0 in 2014 add:
 - 1. Support for categorical data,
 - 2. Add Bayesian inference estimation algorithms,
 - 3. But stay unstable in certain situations (crashes..).
- Release 4.0 in November/December 2015 :
 - 1. Use STK++ as background library (code became cleaner and more compact).
 - 2. Fix (a lot of) crashes issues,
- Enhancement in release 4.2 in November/December 2016 (ADT Massicc)
 - 1. Adding selection criteria,
 - 2. Adding Gibbs estimation algorithms.



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lovleff - blockcluster, Simerge

Simerge : Statistical Inference for the Management of Extreme Risks, Genetics and Global Epidemiology http://mistis.inrialpes.fr/simerge/index.html

SIMERGE is a LIRIMA project-team started in January 2015. It includes

- Mistis (Inria Grenoble Rhône-Alpes, France)
- LERSTAD (Laboratoire d'Etudes et de Recherches en Statistiques et Développement, Université Gaston Berger, Sénégal)
- IRD (Institut de Recherche pour le Développement, équipe G4BBM, Dakar, Sénégal)
- LEM (Lille Economie et Management, Université Lille 2)
- Modal (Inria Lille Nord-Europe)

The Associate team is built on two research themes:

- 1. Spatial extremes, application to management of extreme risks
- 2. Classification, application to genetics and global epidemiology



Challenge

Build statistical models in order to test association between diseases and human host genetics in a context of genome-wide screening.

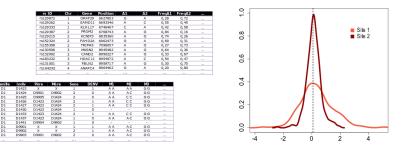


Figure: Genotypes on 719,656 SNPs (Single Nucleotide Polymorphism) typed on 481 individuals in Senegal, in rural area where malaria and arboviral diseases are endemic. 1 malaria quantitative phenotype on two sites: the individual effect on the risk of having malaria attack (iPFA).



Statistical Model

"Pour que blockcluster mette en évidence une cause génétique à l'iPFA, il faudrait que les populations aient été exposées à la maladie pendant plusieurs millénaires"(Cheick Loucoubar, head of G4BBM)

x is a binary data-set. **y** is a data-set (co-variables) of \mathbb{R}^p indexed by *I*.

Classical block model formulation for binary data is extended

$$f(\mathbf{x}, \mathbf{y}; \boldsymbol{\theta}) = \sum_{(\mathbf{z}, \mathbf{w}) \in \mathcal{Z} \times \mathcal{W}} p(\mathbf{z}; \boldsymbol{\theta}) p(\mathbf{w}; \boldsymbol{\theta}) f(\mathbf{x} | \mathbf{y}, \mathbf{z}, \mathbf{w}; \boldsymbol{\theta}) f(\mathbf{y} | \mathbf{z}; \boldsymbol{\theta}).$$
(2)

Dependency between x_{ij} and \mathbf{y}_i modeled by canonical link for binary response data

$$f(x_{ij}|\mathbf{y}_i, \boldsymbol{\beta}_{z_i w_j}) = \mathsf{logis}(\boldsymbol{\beta}_{z_i w_j}^T \mathbf{y}_i)^{x_{ij}} \left(1 - \mathsf{logis}(\boldsymbol{\beta}_{z_i w_j}^T \mathbf{y}_i)\right)^{1 - x_{ij}}$$
(3)

 $f(\mathbf{y}|\mathbf{z}; \boldsymbol{\theta}) = \prod_{i} \phi(\mathbf{y}_{i}; \boldsymbol{\mu}_{z_{i}}, \boldsymbol{\Sigma}_{z_{i}})$ with ϕ multivariate Gaussian density.

Estimation

EM algorithm not feasible as quantity $e_{ikj\ell} = P(z_{ik}w_{j\ell} = 1 | \mathbf{x}, \mathbf{y}, \theta)$ is not computable.

Take $q(\mathbf{z}, \mathbf{w}) = t(\mathbf{z})r(\mathbf{w}) = \mathbf{t} \times \mathbf{r}$ with \mathbf{t} and \mathbf{r} matrices of sizes (n, g) and (m, d), then

$$I(\boldsymbol{\theta}) = \tilde{F}_{C}(\mathbf{t}, \mathbf{r}, \boldsymbol{\theta}) + KL(q(\mathbf{z}, \mathbf{w}) \parallel p(\mathbf{z}, \mathbf{w} | \mathbf{x}, \mathbf{y}, \boldsymbol{\theta}))$$
(4)

with $KL(q \parallel p)$ denoting the Kullback-Liebler divergence and \tilde{F}_C denoting the Free Energy or Fuzzy Criterion

$$\tilde{\mathcal{F}}_{C}(\mathbf{t}, \mathbf{r}, \boldsymbol{\theta}) = \sum_{k} t_{.k} \log \pi_{k} + \sum_{\ell} r_{.\ell} \log \rho_{l} + \sum_{i, j, k, \ell} t_{ik} r_{j\ell} \log f(x_{ij}, \mathbf{y}_{i}; \boldsymbol{\theta}_{k\ell}) + H(\mathbf{t}) + H(\mathbf{r})$$
(5)

and $H(\mathbf{t})$, $H(\mathbf{r})$ denoting the entropy of \mathbf{t} and \mathbf{r} . Maximization of likelihood $l(\boldsymbol{\theta})$ is replaced by the following maximization

$$\operatorname*{argmax}_{\mathbf{t},\mathbf{r},\boldsymbol{\theta}} \tilde{F}_{C}(\mathbf{t},\mathbf{r},\boldsymbol{\theta}).$$

BEM algorithm

Initialization Set $\mathbf{t}^{(0)}, \mathbf{r}^{(0)}$ and $\boldsymbol{\theta}^{(0)} = (\pi^{(0)}, \boldsymbol{\rho}^{(0)}, \boldsymbol{\beta}^{(0)}, \boldsymbol{\mu}^{(0)}, \boldsymbol{\Sigma}^{(0)}).$ (a) Row-EStep Compute $\mathbf{t}^{(c+1)}$ using formula

$$t_{ik}^{(c+1)} = \frac{\pi_k^{(c)} \prod_{jl} \left(f(x_{ij} | \mathbf{y}_i; \boldsymbol{\beta}_{kl}^{(c)}) \phi(\mathbf{y}_i; \boldsymbol{\mu}_k^{(c)}, \boldsymbol{\Sigma}_k^{(c)}) \right)^{r_{jl}^{(c)}}}{\sum_k \pi_k^{(c)} \prod_{jl} \left(f(x_{ij} | \mathbf{y}_i; \boldsymbol{\beta}_{kl}^{(c)}) \phi(\mathbf{y}_i; \boldsymbol{\mu}_k^{(c)}, \boldsymbol{\Sigma}_k^{(c)}) \right)^{r_{jl}^{(c)}}}.$$

(b) Row-MStep Compute π^(c+1), μ^(c+1), Σ^(c+1) and estimate β^(c+1/2).
(c) Col-EStep Compute r^(c+1) using formula

$$r_{jl}^{(c+1)} = \frac{\rho_l^{(c)} \prod_{ik} f(x_{ij} | \mathbf{y}_i; \boldsymbol{\beta}_{kl}^{(c+1/2)})^{t_{ik}^{(c+1)}}}{\sum_l \rho_l^{(c)} \prod_{ik} f(x_{ij} | \mathbf{y}_i; \boldsymbol{\beta}_{kl}^{(c+1/2)})^{t_{ik}^{(c+1)}}}.$$

(d) Col-MStep Compute $\rho^{(c+1)}$ and estimate $\beta^{(c+1)}$. Iterate Iterate (a)-(b)-(c)-(d) until convergence.



Measuring contribution of a variable

 m_l denotes the number of columns with label l, i.e $m_l = \#\{w_{jl} = 1, j = 1, ..., m\}$ and for a row i fixed let m_{il} denotes the number of elements such that $w_{jl} = 1$ and $x_{ij} = 1$, i.e. $m_{il} = \#\{w_{jl}x_{ij} = 1, j = 1, ..., m\}$. The posterior probability of the co-variable **y** is

$$f(\mathbf{y}|\mathbf{x}, \mathbf{z}, \mathbf{w}, \boldsymbol{\theta}) \propto \prod_{i=1}^{n} \pi_{z_i} \phi(\mathbf{y}_i; \boldsymbol{\mu}_{z_i}, \boldsymbol{\Sigma}_{z_i}) \prod_{l=1}^{d} \rho_l^{m_l} \frac{e^{m_l \mathbf{y}_l^{\mathsf{T}} \boldsymbol{\beta}_{z_i l}}}{\left(1 + e^{\mathbf{y}_l^{\mathsf{T}} \boldsymbol{\beta}_{z_i l}}\right)^{m_l}} \quad (6)$$

Taking log, contribution of the *j*th variable is computed as

$$I(j) = \log \rho_{w_j} + \sum_{i=1}^{n} \left(x_{ij} \mathbf{y}_i^T \boldsymbol{\beta}_{z_i w_j} - \log(1 + \exp(\mathbf{y}_i^T \cdot \boldsymbol{\beta}_{z_i w_j})) \right).$$
(7)

using MAP estimator for z and w.



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Extreme Programming $(XP)^1$

Extreme Programming is a discipline of software development based on values of simplicity, communication, feedback, courage, and respect.

- Simple Design: XP teams build software to a simple but always adequate design. They start simple, and through programmer testing and design improvement, they keep it that way.
- Pair Programming: All production software in XP is built by two programmers, sitting side by side, at the same machine.
- Test-Driven Development: XP is obsessed with feedback, and in software development, good feedback requires good testing.
- Design Improvement (Refactoring): XP focuses on delivering business value in every iteration. To accomplish this over the course of the whole project, the software must be well-designed.
- Coding Standard: XP teams follow a common coding standard, so that all the code in the system looks as if it was written by a single – very competent – individual.

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Design and Coding Standard

Use S4 class for R side and a mirror C++ class

```
setClass(
 Class = "CoClusterBinary",
 representation = representation(
   # y part
   yid = "matrix", # covariables
   mukd = "matrix", # means of yid
   sigmakd = "matrix", # standard deviations
   isCoMixture = "logical", # yid is a mixture ?
   # x part
              = "matrix",
   xij
# . . . .
# Constructor of the S4 class
setMethod(
 f="initialize",
 signature=c("CoClusterBinary"),
 definition=function(.Object, x, y, nbcocluster,
     isCoMixture)
```



Design and Coding Standard

Use S4 class for R side and a mirror C++ class

```
class CoClusterBinaryModel: public STK::IRunnerBase
{
    public:
    // constructor of the C++ class
    CoClusterBinaryModel(Rcpp::S4 s4Model);
    //...
    STK::RMatrix<double> yid_;
    STK::RMatrix<double> mukd_;
    STK::RMatrix<double> sdkd_;
    bool isCoMixture_;
    STK::RMatrix<double> xij_;
```

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Design and Coding Standard

Use S4 class for R side and a mirror C++ class C++ constructor get R structure and wrap them as STK++ arrays

```
# Constructor of the S4 class
setMethod(
  f="initialize",
   signature=c("CoClusterBinary"),
   definition=function(.Object, x, y, nbcocluster,
        isCoMixture)
```

```
CoClusterBinaryModel::CoClusterBinaryModel( Rcpp::S4
    s4Model):
    //....
, yid_((SEXP)s4Model.slot("yid"))
, mukd_((SEXP)s4Model.slot("mukd"))
, sdkd_((SEXP)s4Model.slot("sigmak d"))
, isCoMixture_(s4Model.slot("isCoMixture"))
, xij_((SEXP)s4Model.slot("xij"))
    //....
```

lovleff - blockcluster, Simerge

Exemple: Computation of the Fuzzy Criterion \tilde{F}_C

R side

```
setMethod(
  f="logLikelihood",
  signature = "CoClusterBinary",
  definition = function(object)
  {
    .Call("logLikelihood",object,package="simerge")
  }
)
```

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```
C side
```

```
extern "C" SEXP logLikelihood( SEXP model)
{
    Rcpp::S4 s4model(model);
    CoClusterBinaryModel coclust(model);
    coclust.computeLogLikelihood();
    coclust.getValues(model);
    return model;
}
```



$$\begin{split} \tilde{F}_{C}(\mathbf{t},\mathbf{r};\boldsymbol{\theta}) &= \sum_{k} t_{.k} \log \pi_{k} + \sum_{\ell} r_{.\ell} \log \rho_{l} + H(\mathbf{t}) + H(\mathbf{r}) \\ &+ \sum_{i,j,k,\ell} t_{ik} r_{j\ell} (\log(1 + \exp(\mathbf{y}_{i}^{T} \boldsymbol{\beta}_{kl})) + x_{ij} \mathbf{y}_{i}^{T} \boldsymbol{\beta}_{kl}) + \log(\phi(\mathbf{y}_{i};\boldsymbol{\mu}_{k},\boldsymbol{\Sigma}_{k})) \end{split}$$

```
setMethod(
  f="entropy",
  signature = "CoClusterBinary",
  definition = function(object)
  {
    epsilon <- 1e-15
    tik <- object@tik
    rjl <- object@tik
    rjl <- object@rjl
    object@rowEntropy <- -sum(tik*log(epsilon+tik))
    object@colEntropy <- -sum(rjl*log(epsilon+rjl))
    return(object)
  }
</pre>
```

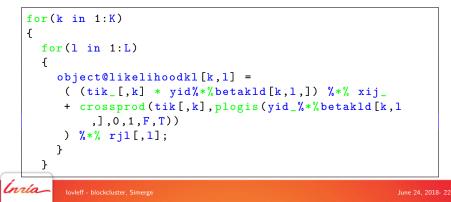
$$\begin{split} \tilde{\mathcal{F}}_{\mathcal{C}}(\mathbf{t},\mathbf{r};\boldsymbol{\theta}) &= \sum_{k} t_{.k} \log \pi_{k} + \sum_{\ell} r_{.\ell} \log \rho_{l} + H(\mathbf{t}) + H(\mathbf{r}) \\ &+ \sum_{i,j,k,\ell} t_{ik} r_{j\ell} (\log(1 + \exp(\mathbf{y}_{i}^{\mathsf{T}} \boldsymbol{\beta}_{kl})) + x_{ij} \mathbf{y}_{i}^{\mathsf{T}} \boldsymbol{\beta}_{kl}) + \log(\phi(\mathbf{y}_{i};\boldsymbol{\mu}_{k},\boldsymbol{\Sigma}_{k})) \end{split}$$

C++ side

rowEntropy_= -tik_.prod((tik_+RealMin).log()).sum(); colEntropy_= -rjl_.prod((rjl_+RealMin).log()).sum();

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$$\begin{split} \tilde{F}_{C}(\mathbf{t},\mathbf{r};\boldsymbol{\theta}) &= \sum_{k} t_{.k} \log \pi_{k} + \sum_{\ell} r_{.\ell} \log \rho_{l} + H(\mathbf{t}) + H(\mathbf{r}) \\ &+ \sum_{i,j,k,\ell} t_{ik} r_{j\ell} (\log(1 + \exp(\mathbf{y}_{i}^{T} \boldsymbol{\beta}_{kl})) + x_{ij} \mathbf{y}_{i}^{T} \boldsymbol{\beta}_{kl}) + \log(\phi(\mathbf{y}_{i};\boldsymbol{\mu}_{k},\boldsymbol{\Sigma}_{k})) \end{split}$$



$$\begin{split} \tilde{\mathcal{F}}_{\mathcal{C}}(\mathbf{t},\mathbf{r};\boldsymbol{\theta}) &= \sum_{k} t_{.k} \log \pi_{k} + \sum_{\ell} r_{.\ell} \log \rho_{l} + H(\mathbf{t}) + H(\mathbf{r}) \\ &+ \sum_{i,j,k,\ell} t_{ik} r_{j\ell} (\log(1 + \exp(\mathbf{y}_{i}^{\mathsf{T}} \boldsymbol{\beta}_{kl})) + x_{ij} \mathbf{y}_{i}^{\mathsf{T}} \boldsymbol{\beta}_{kl}) + \log(\phi(\mathbf{y}_{i};\boldsymbol{\mu}_{k},\boldsymbol{\Sigma}_{k})) \end{split}$$

```
for(int k=0; k<K_; ++k)</pre>
{
  for(int l=0; l<L_; ++1)</pre>
  Ł
    likelihoodkl_(k,l)
     = ( tik_.col(k).prod( yid_*betakld_(k,1)).
         transpose() * xij_
       + tik_.col(k).dot( (yid_*betakld_(k,l)).lcdfc(
           logis_) )
       ) * rjl_.col(1);
    }
```

$$\begin{split} \tilde{F}_{C}(\mathbf{t},\mathbf{r};\boldsymbol{\theta}) &= \sum_{k} t_{.k} \log \pi_{k} + \sum_{\ell} r_{.\ell} \log \rho_{l} + H(\mathbf{t}) + H(\mathbf{r}) \\ &+ \sum_{i,j,k,\ell} t_{ik} r_{j\ell} (\log(1 + \exp(\mathbf{y}_{i}^{\mathsf{T}} \boldsymbol{\beta}_{kl})) + x_{ij} \mathbf{y}_{i}^{\mathsf{T}} \boldsymbol{\beta}_{kl}) + \log(\phi(\mathbf{y}_{i};\boldsymbol{\mu}_{k},\boldsymbol{\Sigma}_{k})) \end{split}$$



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Data set

n = 444 individuals and m = 515721 SNPs conserved.

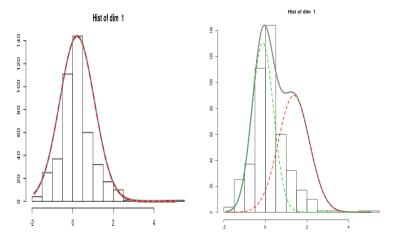


Figure: Histogram of the iPFA variable and fitted Gaussian mixture models obtained with MixAll package



Preliminary Results

Model selection

ICL BIC-like approximations leads to the following BIC(g, d)

 $-2\max_{\theta} \log f(\mathbf{x}, \mathbf{y}; \theta) + (g-1) \log n + \lambda \log n + (d-1) \log m + gd(p+1) \log(mn)$

with λ the number of parameters of the ${\bf y}$ distribution.

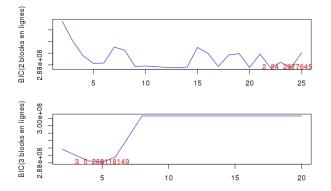


Figure: Choosing the number of blocks (Note: implemented criteria was wrong)

Preliminary Results

Results with (g, d) = (2, 22) and **y** Gaussian mixture

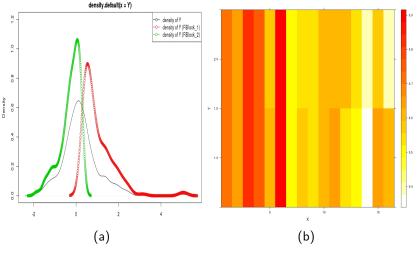


Figure: iPFA density (a), Proportion of mutation (b), BIC = 290551317



Preliminary Results

Results with (g, d) = (2, 22) and **y** Gaussian rv

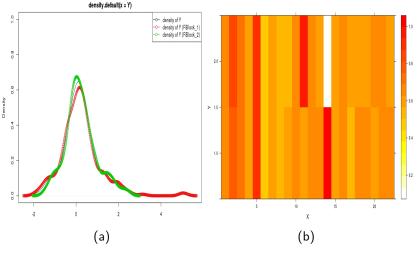


Figure: iPFA density (a), Proportion of mutation (b), BIC = 287770996



Influence Measure

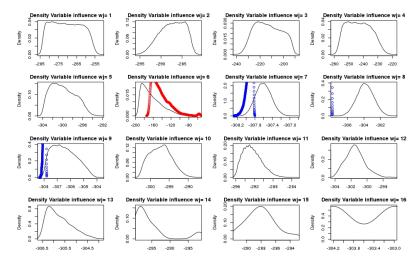


Figure: Repartition of the influence in clusters (by columns)



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Innía

Merci à la G4BBM team

Cheikh LOUCOUBAR - Biomathematician



PhD in Statistical Genetics Head of the Group Dakar University / Paris 5

Marvam DIARRA - Biomathematician



PhD in Applied Mathematics Saint Louis University (UGB)

Mamadou DIOP - Computer Scientist



Bioinformatician Master in Computer Science Saint Louis University (UGB)

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DTS in Computer Science

Sevdou Nourou SYLLA - Biomathematician



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Amadou DIALLO – Biomathematician



Bachelor in Mathematics Minot State University, USA



Mareme S. THIAM – Master Fellow in Mathematics

M2 Mathematics - Big Data AIMS



Aboubacry GAYE – Master Fellow in Mathematics

M2 Mathematics Saint Louis University (UGB)



§ Research on human host genetic diversity and implication in malaria phenotypes § New grant application

Other Activities

- § Support IPD units in data management and analysis
- § Teaching in collaborations with
 - universities



Mame Malick DIENG - Computer Scientist

Master in Computer Science Saint Louis University (UGB)



Links

- http://www.pasteur.sn/recherche/ biostatistique-bio-informatique-et-modelisation/
- https://cran.r-project.org/package=blockcluster
- https://cran.r-project.org/package=rtkore
- https://cran.r-project.org/package=MixAll
- http://www.stkpp.org
- https://modal.lille.inria.fr/wikimodal/doku.php

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