



HAL
open science

Hydrodynamics of elastic micro-filaments : model comparison and applications

Clément Moreau, Laetitia Giraldi, Hermes Gadêlha

► **To cite this version:**

Clément Moreau, Laetitia Giraldi, Hermes Gadêlha. Hydrodynamics of elastic micro-filaments : model comparison and applications. CANUM, May 2018, AGDE, France. hal-01880544

HAL Id: hal-01880544

<https://inria.hal.science/hal-01880544>

Submitted on 25 Sep 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Hydrodynamics of elastic micro-filaments : model comparison and applications

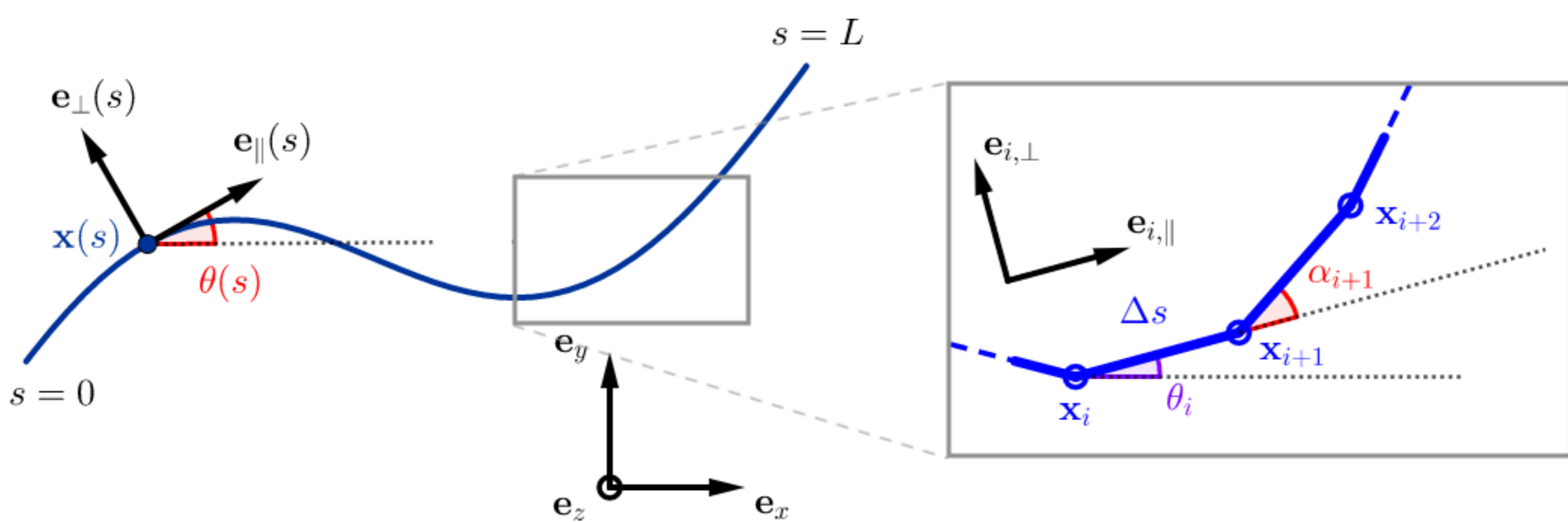
Clément Moreau, Laetitia Girdali, and Hermes Gadêlha

Motivations

- ▶ Efficiently model and simulate the dynamics of **inextensible micro-filaments** in 2D
- ▶ Deal with the coupling between elastic and hydrodynamic interactions
- ▶ Propose a model that is
 - ▷ **simple** to implement
 - ▷ numerically **robust**
 - ▷ **adaptable** for a range of applications

Modeling

- ▶ We consider an **inextensible flexible filament** immersed in a fluid at **low Reynolds number** → inertia is negligible, viscous effects dominate: **Stokes regime**



Parametrisation for the continuous and coarse-grained models

- ▶ **Hydrodynamics**: segment force density (Resistive Force Theory [2])

$$f_i(s) = \eta_{\parallel}(\mathbf{v}_i(s) \cdot \mathbf{e}_{i,\parallel})\mathbf{e}_{i,\parallel} + \eta_{\perp}(\mathbf{v}_i(s) \cdot \mathbf{e}_{i,\perp})\mathbf{e}_{i,\perp}$$

- ▶ **Elasticity**: linear model

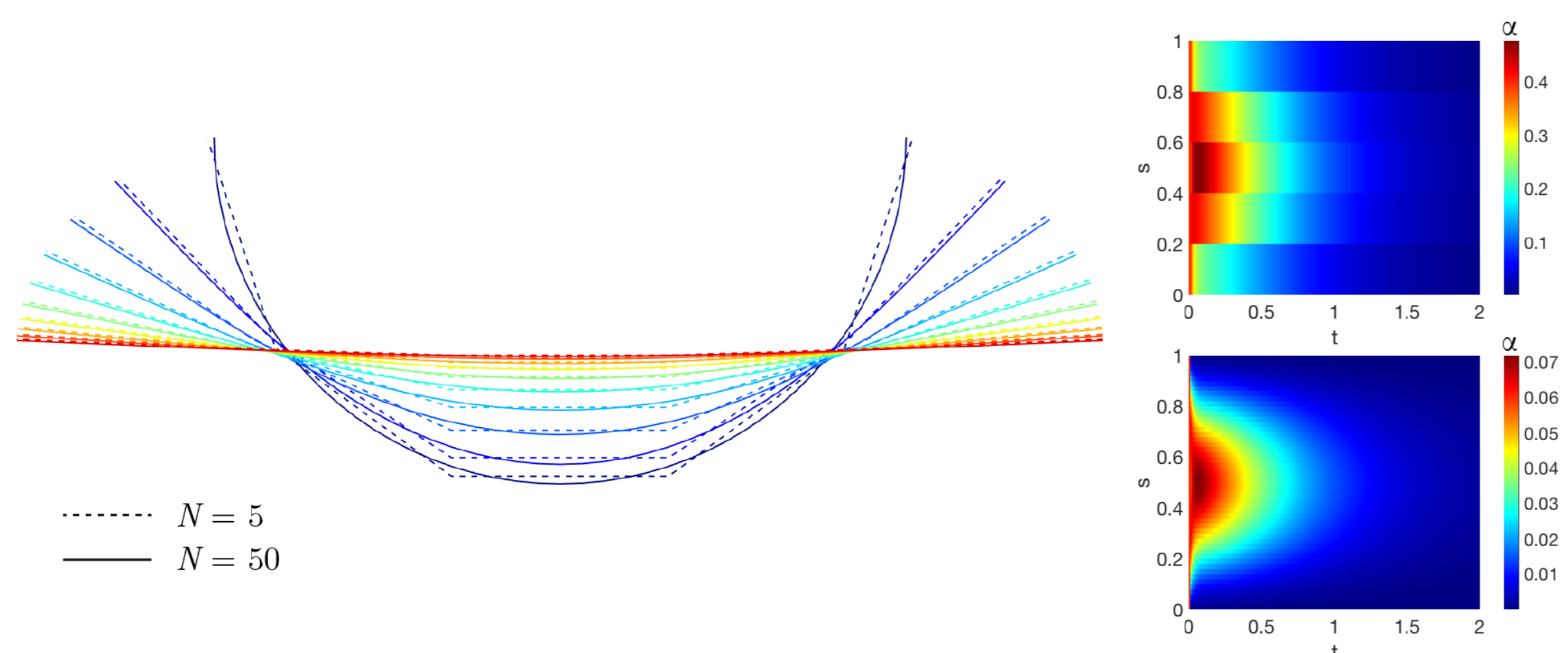
$$\mathbf{M}_i^{\text{el}} = \kappa(\theta_{i+1} - \theta_i)\mathbf{e}_z$$

- ▶ 2nd Newton law gives the balance of forces and torques :

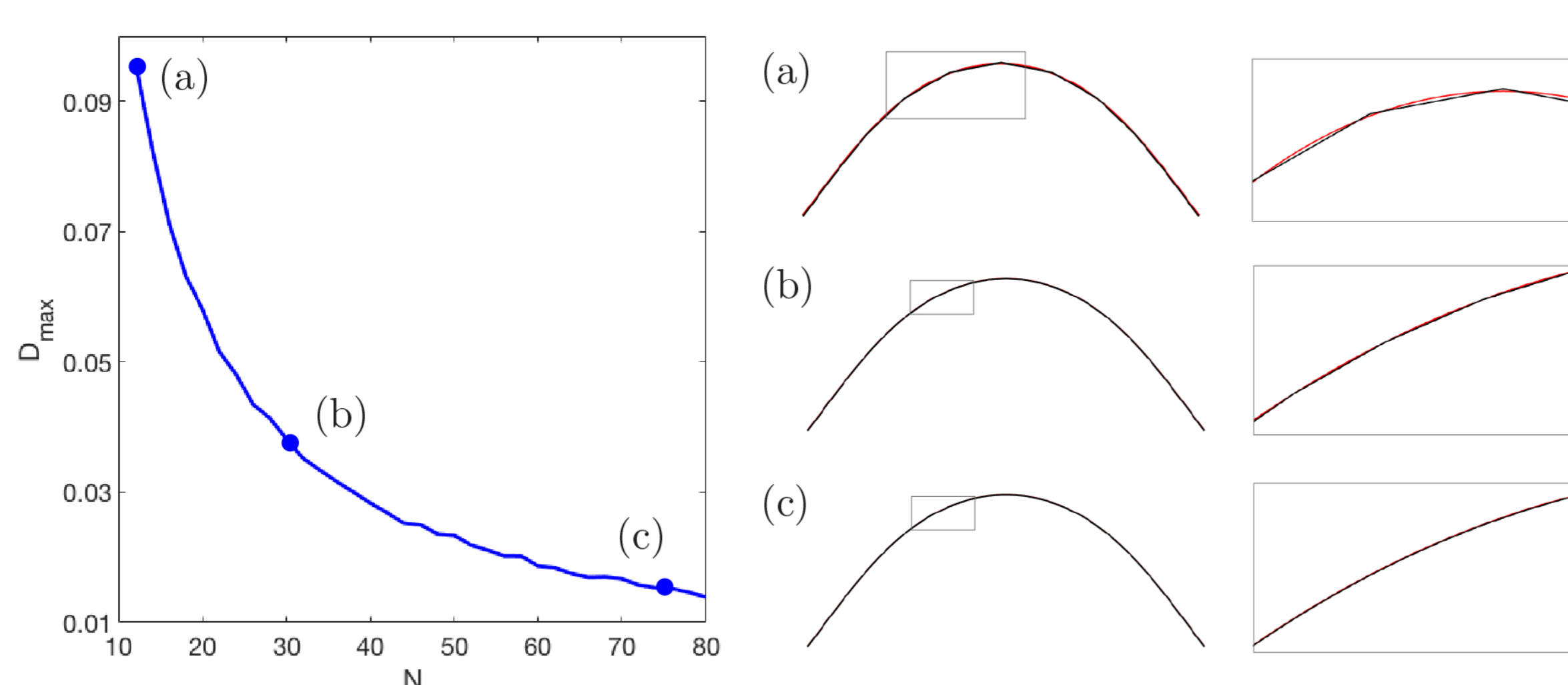
- ▷ Continuous model → **PDE system**
- ▷ Discrete model → integration on the segments → **ODE system**

Numerical Comparison

- ▶ Similar behavior on the “relaxation test”



- ▶ Fast convergence of the discrete model (error criterion $D_{\text{max}} = \max_{s,t} |\mathbf{x}_c - \mathbf{x}_{\text{cg}}|$)

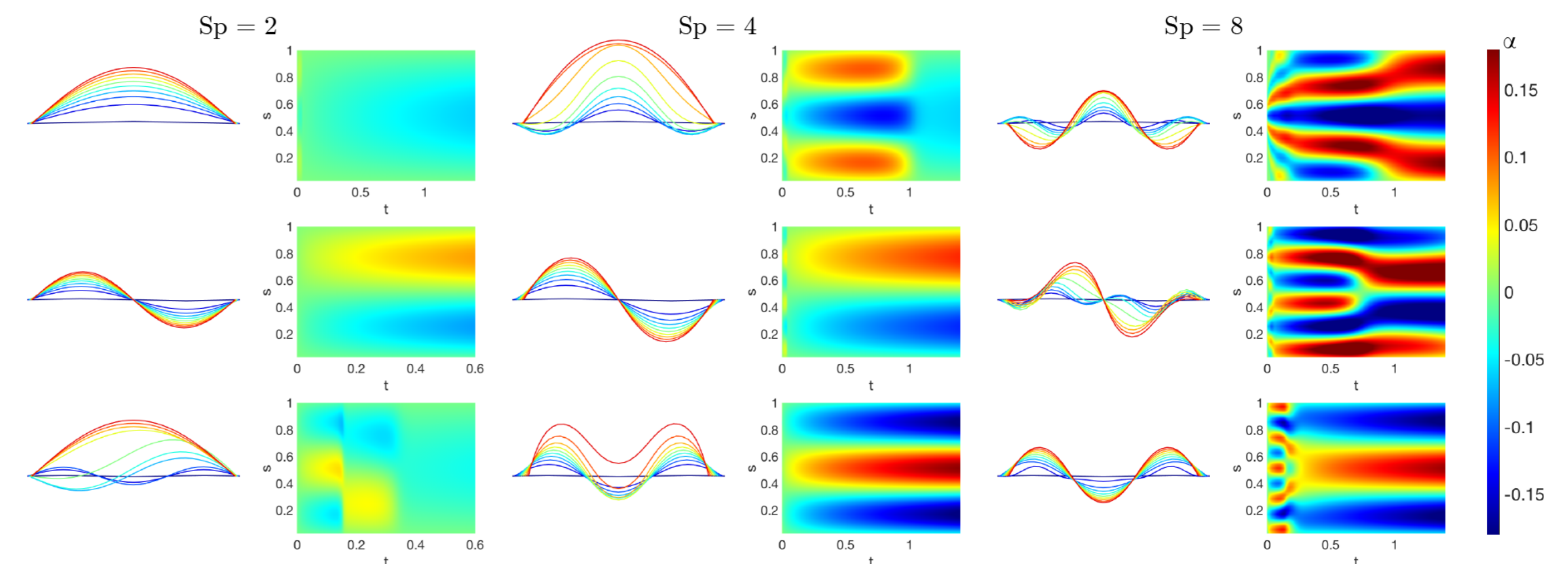


Comparison between the classical and coarse-grained systems for increasing number of segments.

- ▶ Discrete approach up to **100 times faster** than PDE approach
- ▶ **More robust** for rigid filaments and sharp curvatures

Applications : Buckling

- ▶ Filament submitted to **compression**
- ▶ Triggering different modes by changing initial condition



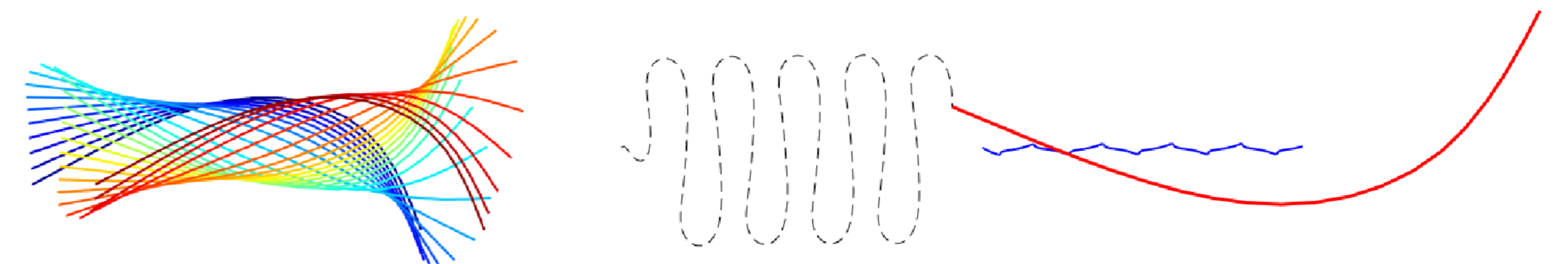
Visualization of the buckling phenomenon for three different rigidities and three different initial conditions.

- ▶ Study made easier by the discrete approach, under investigation

Applications : Magnetic Swimmers

- ▶ **Magnetised filament** – magnetisations μ_i
- ▶ External magnetic field \mathbf{H} creating a torque

$$\mathbf{T}_i^m = \mu_i(\mathbf{e}_{i,\parallel} \times \mathbf{H})$$

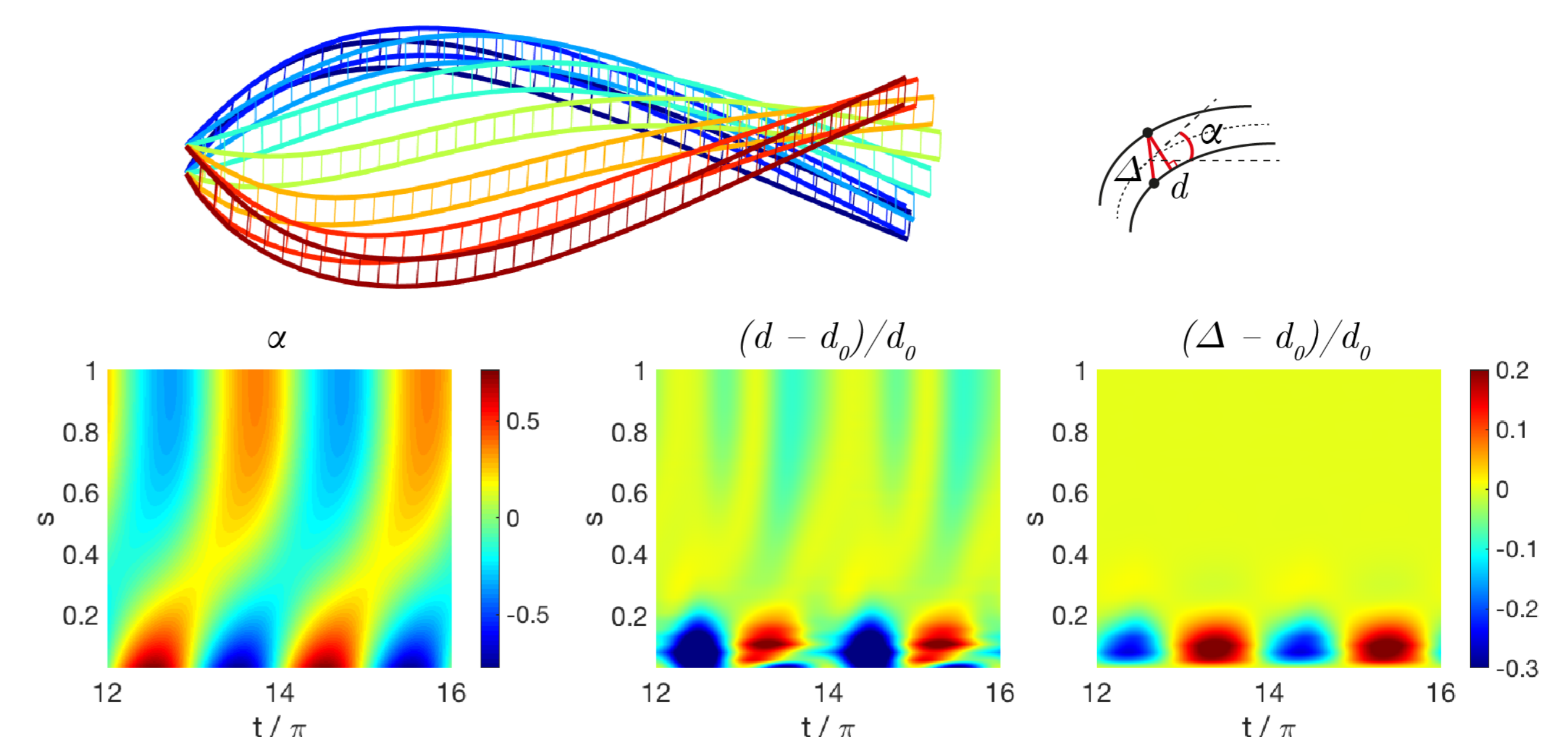


Example of magnetic drive with a sinusoidal orthogonal magnetic field.

- ▶ **Artificial self-propulsion** at micro-scale [3]
- ▶ Controllability and **optimal control**

Applications : Filament Bundle

- ▶ Model a **biological flagellum** through two filaments linked together
- ▶ Highlight non-trivial coupling phenomena



Coupling between two filaments obtained with the coarse-grained approach.

- ▶ Applications include sperm cell motility studies

References and Contact Information

- ▶ Clément Moreau, Université Côte d'Azur, Inria, clement.moreau@inria.fr
- ▶ Laetitia Girdali, Université Côte d'Azur, Inria, laetitia.girdali@inria.fr
- ▶ Hermes Gadêlha, Department of Mathematics, University of York, hermes.gadelha@york.ac.uk
- ▶ C. Moreau, L. Girdali, H. Gadêlha, *The asymptotic coarse-graining formulation of slender-rods, bio-filaments and flagella*, accepted in Journal of the Royal Society Interface, 2018.
- ▶ J. Gray, J. Hancock, *The propulsion of sea-urchin spermatozoa*, Journal of Experimental Biology, 1955.
- ▶ F. Alouges, A. DeSimone, L. Girdali, M. Zoppello, *Can magnetic multilayers propel artificial microswimmers mimicking sperm cells?*, Soft Robotics, 2015.