

# Asymptotic modeling of the multiple electromagnetic wave scattering by small spheres

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PhD student in Applied Mathematics

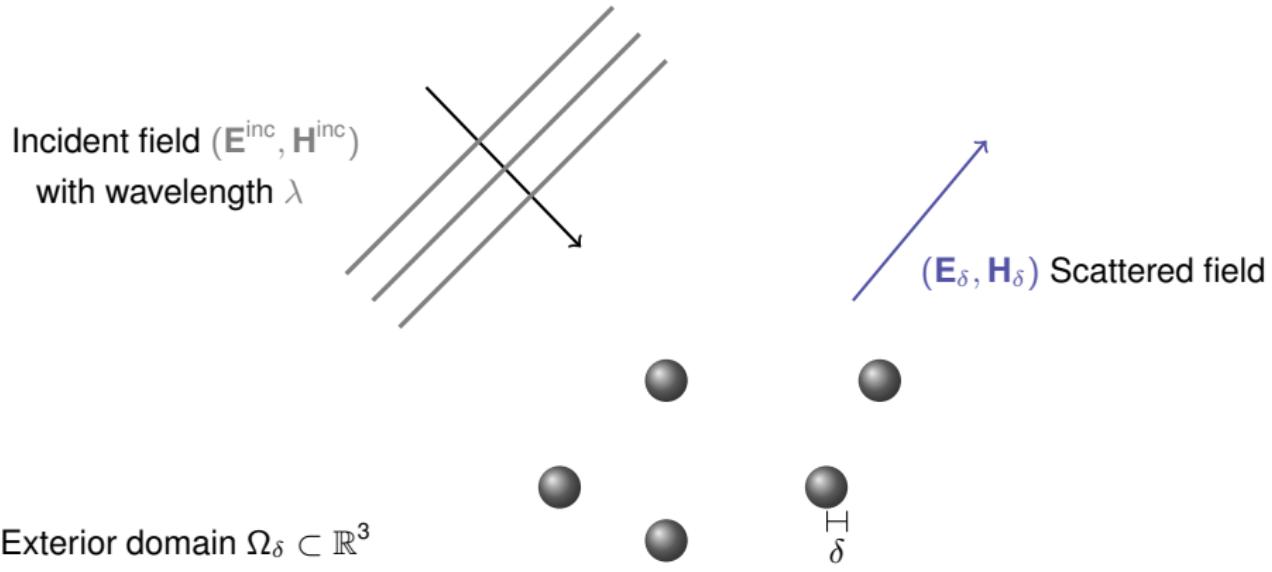
EPC Magique 3D — UPPA-E2S, INRIA Bordeaux Sud-Ouest, LMAP UMR CNRS 5142

6th European Conference on Computational Mechanics (ECCM 6)  
7th European Conference on Computational Fluid Dynamics (ECFD 7)

Glasgow, June, 14th 2018

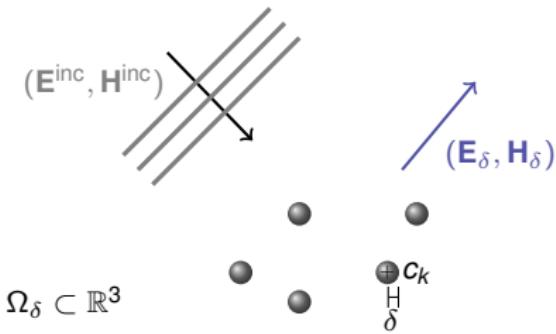


# 3D-Multiple Scattering Problem by Small Spheres



Asymptotic assumption :  $\delta \ll \lambda$

# Model Problem



$$\Omega_\delta \subset \mathbb{R}^3$$

- Time-harmonic domain
- Homogeneous & isotropic medium  $\Omega_\delta$
- Perfect conductors  $\mathcal{B}(c_k, \delta)$ ,  $k = 1 \dots N_{\text{obs}}$

Time-harmonic Maxwell equations

$$\begin{cases} \operatorname{curl} \mathbf{E}_\delta - i\kappa \mathbf{H}_\delta = 0 & \text{in } \Omega_\delta \\ \operatorname{curl} \mathbf{H}_\delta + i\kappa \mathbf{E}_\delta = 0 & \text{in } \Omega_\delta \end{cases}$$

$$\text{with } \kappa^2 = \omega^2 \mu(\epsilon + \frac{i\sigma}{\omega}), \Im(\kappa) \geq 0$$

Boundary condition

$$\mathbf{n} \times \mathbf{E}_\delta = -\mathbf{n} \times \mathbf{E}^{\text{inc}} \quad \text{on } \partial\Omega_\delta$$

Silver-Müller radiation condition

$$r(\mathbf{H}_\delta \times \hat{x} - \mathbf{E}_\delta) \xrightarrow[r \rightarrow \infty]{} 0 \quad \text{unif. in } \hat{x} = \frac{\mathbf{x}}{r}$$

## Mathematical well-posedness

For any  $\mathbf{E}^{\text{inc}} \in \mathbf{H}_{\text{loc}}(\operatorname{curl}, \Omega_\delta)$  there exist a unique solution  $(\mathbf{E}_\delta, \mathbf{H}_\delta) \in \mathbf{H}_{\text{loc}}(\operatorname{curl}, \Omega_\delta)^2$  to the exterior Maxwell problem.

# Approaches and Objectives

- Asymptotic models



[H. Ammari, M.S. Vogelius, D. Volkov \(2001\)](#)

Asymptotic formulas for perturbations in the electromagnetic fields due to the presence of inhomogeneities of small diameter II. The full Maxwell equations



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Multiple scattering of electromagnetic waves by finitely many point-like obstacles



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Asymptotics of solutions for stationary and nonstationary Maxwell systems in a domain with small holes

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## ● High-order numerical solution



M. Ganesh, S.C. Hawkins (2009)

A high-order algorithm for multiple electromagnetic scattering in three dimensions



H. Barucq, J. Chabassier, H. Pham, S. Tordeux (2017)

Numerical robustness of single-layer method with Fourier basis for multiple obstacle acoustic scattering in homogeneous media

## Asymptotic models

- ✗ Restricted to small obstacles
- ✓ Low computational cost
- ✓ Meshless method

# Motivations

## Asymptotic models

- ✗ Restricted to small obstacles
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Multiple scattering



## Foldy-Lax model

- ✓ Interactions taken into account
- ✓ Low computational cost
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Superposition principle



## Born approximation

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## High-order numerical solution

- ✗ High computational cost
  - ✗ volumical methods FEM, DG, ...
  - ~ boundary element methods

- ✗ Mesh refinement

- ✓ High-order spectral method

# Outline

## 1. Asymptotic expansions for the single electromagnetic scattering

- First terms of the asymptotics
- Numerical results

## 2. Multiple electromagnetic scattering by small spheres

- Born approximation
- Foldy-Lax approximation
- Preliminary numerical results

## 3. Conclusions and perspectives

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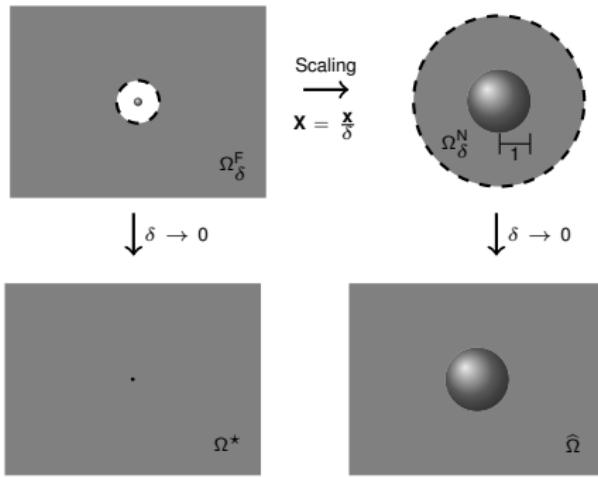
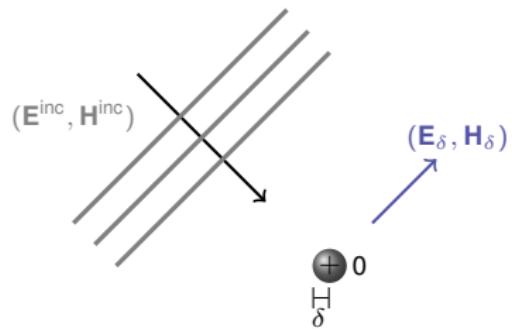
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# Approximation for Single Scattering

## Method of matched asymptotic expansions

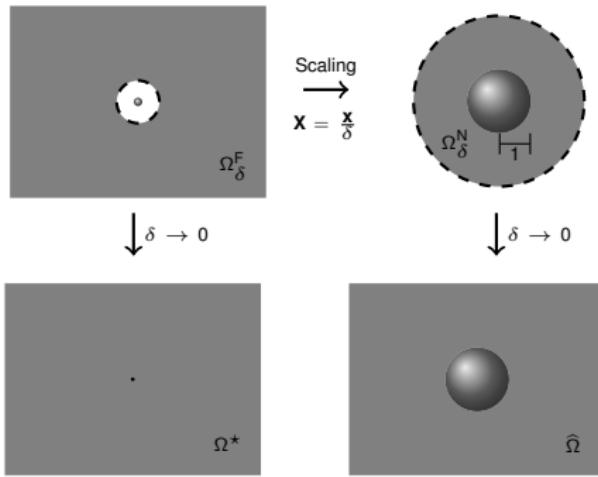
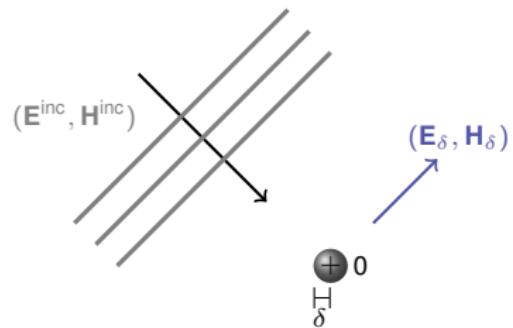
- Domain decomposition
- Local approximations
- Matching procedure



# Approximation for Single Scattering

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## Asymptotic expansions

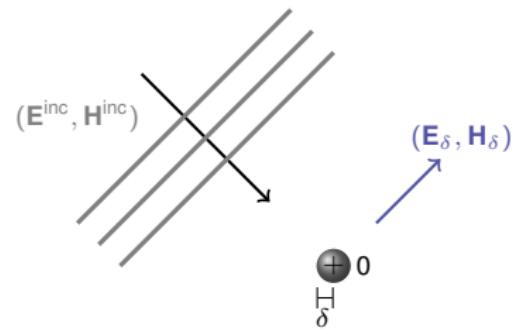
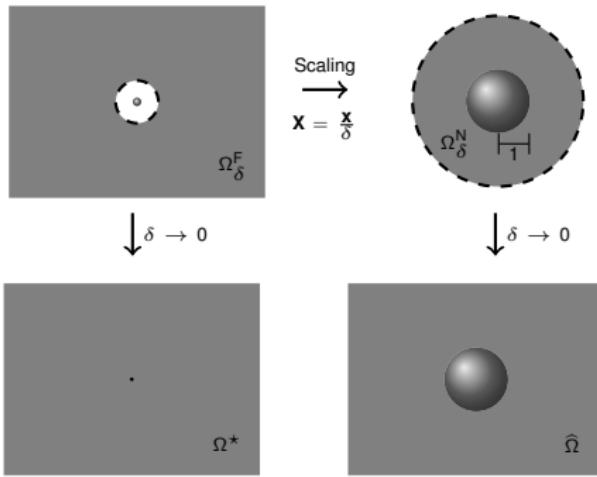
**Far field** expansion in  $\Omega^* = \mathbb{R}^3 \setminus \{0\}$

**Near field** expansion in  $\widehat{\Omega} = \mathbb{R}^3 \setminus \overline{\mathcal{B}(0, 1)}$

# Approximation for Single Scattering

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## Asymptotic expansions

**Far field** expansion in  $\Omega^* = \mathbb{R}^3 \setminus \{0\}$

**Near field** expansion in  $\hat{\Omega} = \mathbb{R}^3 \setminus \overline{\mathcal{B}(0, 1)}$

Near field  $\implies$  Far field

# Near Field Expansion

$$\mathbf{E}_\delta(\delta \mathbf{X}) = \widehat{\mathbf{E}}_0(\mathbf{X}) + \delta \widehat{\mathbf{E}}_1(\mathbf{X}) + \delta^2 \widehat{\mathbf{E}}_2(\mathbf{X}) + \dots$$

## 0-order (static dipole)

$$\widehat{\mathbf{E}}_0(\mathbf{X}) = \frac{1}{|\mathbf{X}|^3} \left( 3(\mathbf{e}_r \cdot \mathbf{E}^{\text{inc}}(0))\mathbf{e}_r - \mathbf{E}^{\text{inc}}(0) \right)$$

## 1-order (static quadrupole + static dipole)

$$\widehat{\mathbf{E}}_1(\mathbf{X}) = -\frac{1}{|\mathbf{X}|^4} \gamma_l \left( \mathbb{J}_{\mathbf{E}^{\text{inc}}}^{\text{sym}}(0) \mathbf{e}_r \right) + \frac{3}{2|\mathbf{X}|^4} (\mathbf{e}_r \cdot \mathbb{J}_{\mathbf{E}^{\text{inc}}}(0) \mathbf{e}_r) \mathbf{e}_r - \frac{i\kappa}{2|\mathbf{X}|^2} \mathbf{H}^{\text{inc}}(0) \times \mathbf{e}_r$$

with  $\gamma_l \mathbf{u} = \mathbf{u} - (\mathbf{e}_r \cdot \mathbf{u}) \mathbf{e}_r$  and  $\mathbb{J}^{\text{sym}} = \frac{1}{2} (\mathbb{J} + \mathbb{J}^\top)$  symmetric Jacobian

## 2-order

$\widehat{\mathbf{E}}_2(\mathbf{X}) = \text{octupole} + \text{quadrupole} + \text{dipole}$

# Near Field Expansion

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# Far Field Expansion I

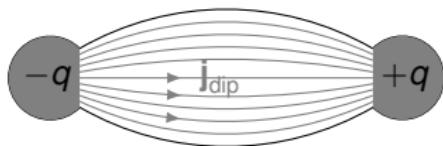
$$\mathbf{E}_\delta(\mathbf{x}) = \delta^3 \tilde{\mathbf{E}}_3(\mathbf{x}) + \delta^5 \tilde{\mathbf{E}}_5(\mathbf{x}) + \dots$$

**3-order** ( time-harmonic **dipole**)

$$\tilde{\mathbf{E}}_3(\mathbf{x}) = -\frac{\kappa^3}{2} h_1^{(1)}(\kappa r) \mathbf{e}_r \times \mathbf{H}^{\text{inc}}(0) - \kappa^3 \tilde{h}_1^{(1)}(\kappa r) \gamma_t \mathbf{E}^{\text{inc}}(0) - 2\kappa^3 \frac{h_1^{(1)}(\kappa r)}{i\kappa r} (\mathbf{e}_r \cdot \mathbf{E}^{\text{inc}}(0)) \mathbf{e}_r$$

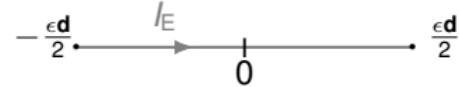
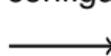
with  $h_n^{(1)}$  spherical Hankel of 1st kind and  $\tilde{h}_n^{(1)}(z) = \frac{h_n^{(1)}(z) + zh_n^{(1)'}(z)}{iz}$

# Time-Harmonic Electromagnetic Dipole



$\mathbf{j}_{\text{dip}}$  electric current

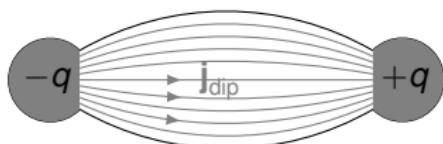
Ideal configuration



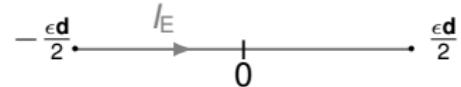
$\mathbf{d} \in \mathbb{R}^3$ ,  $\epsilon$  small parameter

- Ponctual charges with  $q = \frac{1}{\epsilon}$
- Filiform current ( $\mathbf{d}, l_E$ )
- Charge conservation  $l_E = -i\omega q$

# Time-Harmonic Electromagnetic Dipole

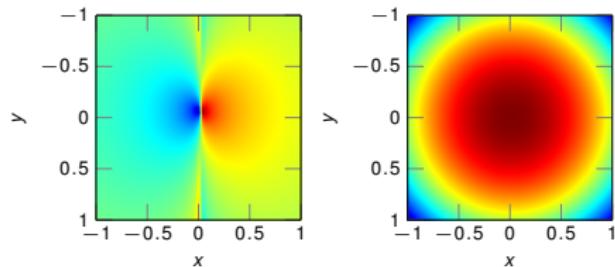


Ideal configuration



$\mathbf{d} \in \mathbb{R}^3$ ,  $\epsilon$  small parameter

$j_{\text{dip}}$  electric current



Time-harmonic electric potential  $V$  and magnetic potential  $\mathbf{A}$  ( $x$ -component) in the  $xy$ -plane

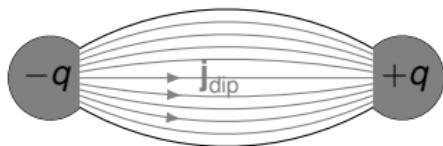
$\epsilon \rightarrow 0$

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- Filiform current ( $\mathbf{d}, I_E$ )
- Charge conservation  $I_E = -i\omega q$

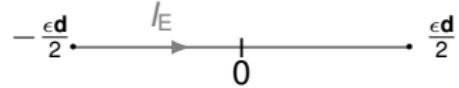
$$\mathcal{E}_{\text{dip}}^{\text{elec}}[\mathbf{d}] = -\nabla V + i\kappa \mathbf{A}$$

$$\mathcal{H}_{\text{dip}}^{\text{elec}}[\mathbf{d}] = \text{curl } \mathbf{A}$$

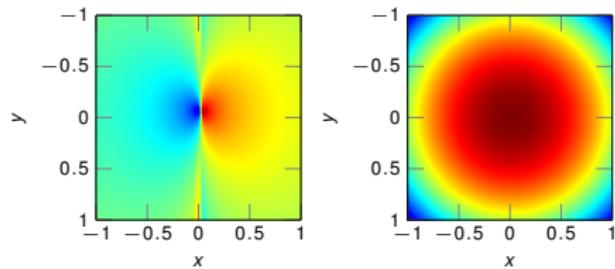
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Magnetic dipole

$$\mathcal{H}_{\text{dip}}^{\text{mag}}[\mathbf{d}] = \mathcal{E}_{\text{dip}}^{\text{elec}}[\mathbf{d}]$$

$$\mathcal{E}_{\text{dip}}^{\text{mag}}[\mathbf{d}] = -\mathcal{H}_{\text{dip}}^{\text{elec}}[\mathbf{d}]$$

## Far Field Expansion II

$$\mathbf{E}_\delta(\mathbf{x}) = \delta^3 \tilde{\mathbf{E}}_3(\mathbf{x}) + \delta^5 \tilde{\mathbf{E}}_5(\mathbf{x}) + \dots$$

**3-order**

$$\tilde{\mathbf{E}}_3 = \mathcal{E}_{\text{dip}}^{\text{elec}} [4\pi \mathbf{E}^{\text{inc}}(0)] + \mathcal{E}_{\text{dip}}^{\text{mag}} [-2\pi \mathbf{H}^{\text{inc}}(0)]$$

**5-order**

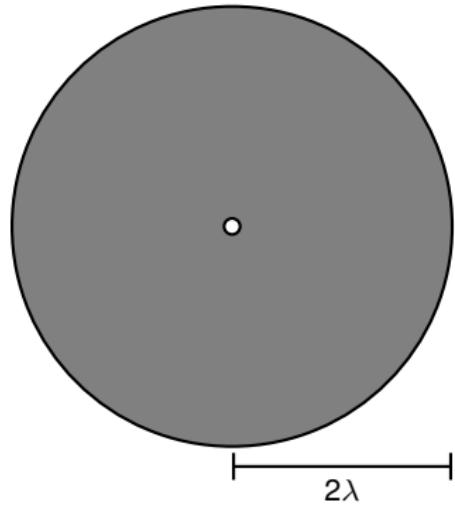
$$\tilde{\mathbf{E}}_5(\mathbf{x}) = \frac{3\kappa^2}{10} \mathcal{E}_{\text{dip}}^{\text{elec}} [4\pi \mathbf{E}^{\text{inc}}(0)](\mathbf{x}) - \frac{3\kappa^2}{5} \mathcal{E}_{\text{dip}}^{\text{mag}} [-2\pi \mathbf{H}^{\text{inc}}(0)](\mathbf{x}) + \text{time-harmonic quadrupole}$$

**Collected dipolar model**

$$\mathbf{E}_\delta^{\text{Col}}(\mathbf{x}) = \left( \delta^3 + \frac{3\kappa^2 \delta^5}{10} \right) \mathcal{E}_{\text{dip}}^{\text{elec}} [4\pi \mathbf{E}^{\text{inc}}(0)](\mathbf{x}) + \left( \delta^3 - \frac{3\kappa^2 \delta^5}{5} \right) \mathcal{E}_{\text{dip}}^{\text{mag}} [-2\pi \mathbf{H}^{\text{inc}}(0)](\mathbf{x})$$

# Simulation Description

3D Whole domain



Small parameter

$$\delta \in \left\{ \frac{1}{10^p}, p = 0.5 : 0.1 : 4 \right\}$$

Physical parameters

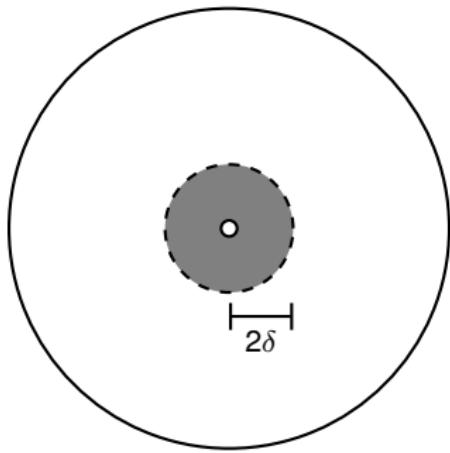
- $\varepsilon = \mu = 1.0, \sigma = 0$
- $\lambda = 5.0,$   
 $\omega = \kappa = \frac{2\pi}{5.0} \approx 1.26$

Incident plane wave

- $\mathbf{E}^{\text{inc}} = \vec{p} \exp(-i\vec{k} \cdot \mathbf{x})$
- $\mathbf{H}^{\text{inc}} = \frac{1}{i\kappa} \mathbf{curl} \mathbf{E}^{\text{inc}}$
- $\vec{p} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \vec{k} = \begin{pmatrix} 0 \\ 0 \\ \kappa \end{pmatrix}$

# Simulation Description

## Near field domain



Small parameter

$$\delta \in \left\{ \frac{1}{10^p}, p = 0.5 : 0.1 : 4 \right\}$$

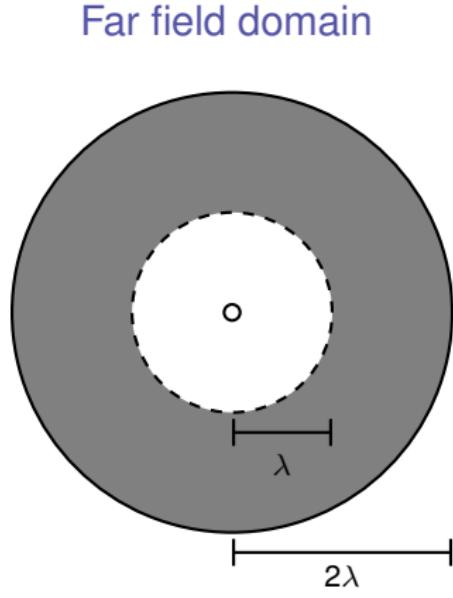
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# Numerical Convergence I

## Near field approximations

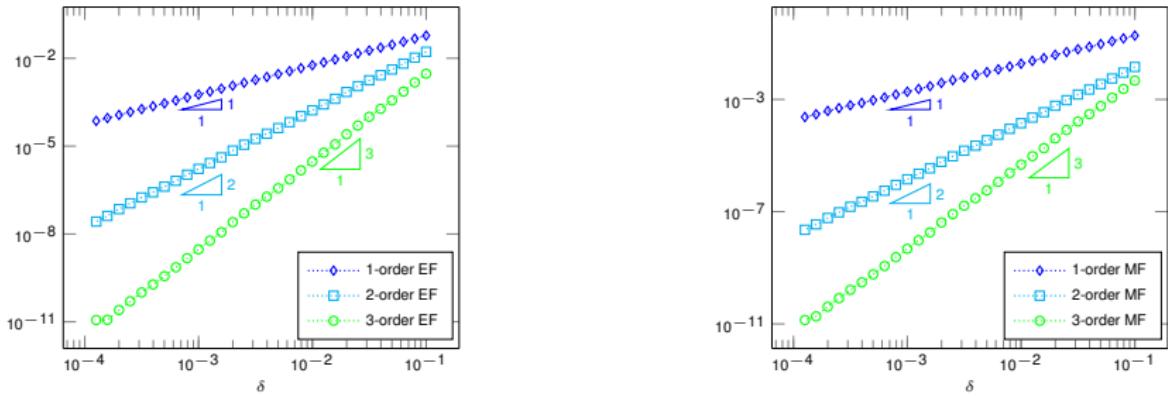
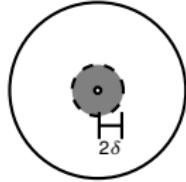


Figure: Relative  $L^2(\Omega_\delta^{2\delta})$ -errors for electric near field (left) magnetic near field (right)



$$\text{1-order EF} : \|\mathbf{E}_\delta - \widehat{\mathbf{E}}_0\left(\frac{\cdot}{\delta}\right)\|_{L^2(\Omega_\delta^{2\delta})} / \|\mathbf{E}_\delta\|_{L^2(\Omega_\delta^{2\delta})}$$

$$\text{2-order EF} : \|\mathbf{E}_\delta - \widehat{\mathbf{E}}_0\left(\frac{\cdot}{\delta}\right) - \delta \widehat{\mathbf{E}}_1\left(\frac{\cdot}{\delta}\right)\|_{L^2(\Omega_\delta^{2\delta})} / \|\mathbf{E}_\delta\|_{L^2(\Omega_\delta^{2\delta})}$$

$$\text{3-order EF} : \|\mathbf{E}_\delta - \widehat{\mathbf{E}}_0\left(\frac{\cdot}{\delta}\right) - \delta \widehat{\mathbf{E}}_1\left(\frac{\cdot}{\delta}\right) - \delta^2 \widehat{\mathbf{E}}_2\left(\frac{\cdot}{\delta}\right)\|_{L^2(\Omega_\delta^{2\delta})} / \|\mathbf{E}_\delta\|_{L^2(\Omega_\delta^{2\delta})}$$

# Numerical Convergence II

## Far field approximations

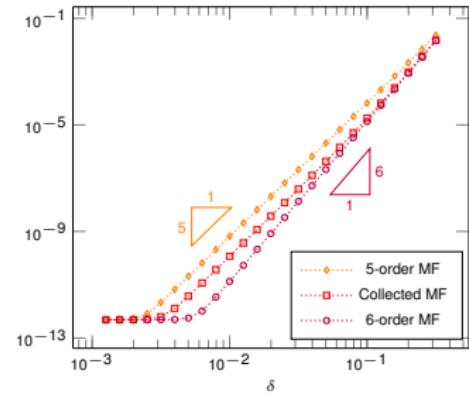
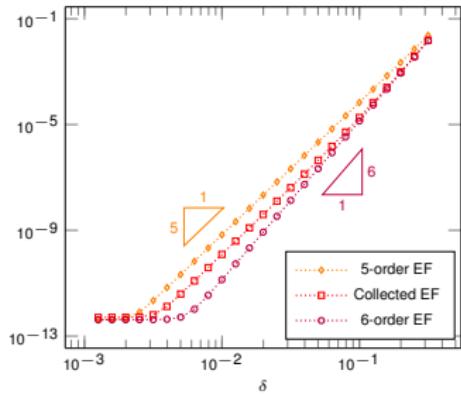
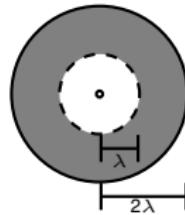


Figure: Absolute  $L^2(\Omega_2^{2\lambda})$ -errors for electric far field (left) magnetic far field (right)



$$5\text{-order EF} : \|\mathbf{E}_\delta - \delta^3 \tilde{\mathbf{E}}_3\|_{L^2(\Omega_2^{2\lambda})}$$

$$6\text{-order EF} : \|\mathbf{E}_\delta - \delta^3 \tilde{\mathbf{E}}_3 - \delta^5 \tilde{\mathbf{E}}_5\|_{L^2(\Omega_2^{2\lambda})}$$

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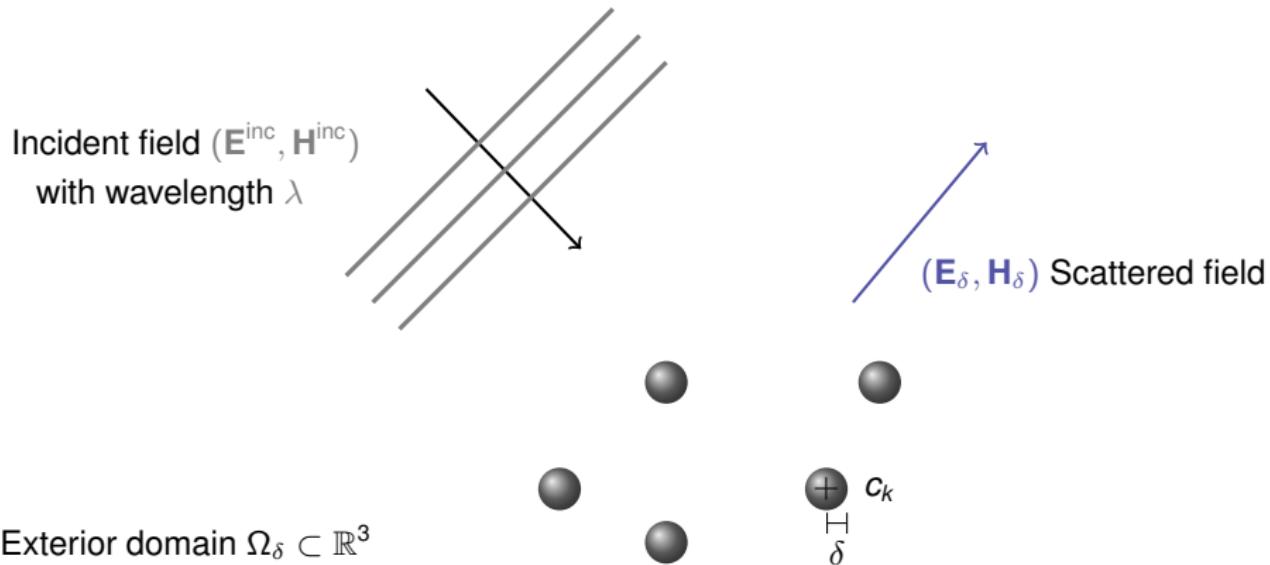
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# 3D-Multiple Scattering Problem by Small Spheres



$$\Omega_\delta = \mathbb{R}^3 \setminus \bigcup_{k=1}^{N_{\text{obs}}} \overline{\mathcal{B}(c_k, \delta)}$$

## Born Approximation

$$\mathbf{E}_\delta^{\text{Born}}(\mathbf{x}) = \sum_{k=1}^{N_{\text{obs}}} \mathbf{E}_{\delta,k}(\mathbf{x}) \quad \mathbf{H}_\delta^{\text{Born}}(\mathbf{x}) = \sum_{k=1}^{N_{\text{obs}}} \mathbf{H}_{\delta,k}(\mathbf{x})$$

- Each obstacle is modeled as a **dipolar source** around  $c_k$

$$\mathbf{E}_{\delta,k}(\mathbf{x}) = \mathcal{E}_{\text{dip}}^{\text{elec}}[\mathbf{d}_{\delta,k}^E](\mathbf{x} - \mathbf{c}_k) + \mathcal{E}_{\text{dip}}^{\text{mag}}[\mathbf{d}_{\delta,k}^H](\mathbf{x} - \mathbf{c}_k)$$

$$\mathbf{H}_{\delta,k}(\mathbf{x}) = \mathcal{H}_{\text{dip}}^{\text{elec}}[\mathbf{d}_{\delta,k}^E](\mathbf{x} - \mathbf{c}_k) + \mathcal{H}_{\text{dip}}^{\text{mag}}[\mathbf{d}_{\delta,k}^H](\mathbf{x} - \mathbf{c}_k)$$

- The directions  $\mathbf{d}_{\delta,k}^E$  and  $\mathbf{d}_{\delta,k}^H$  depend on the **nature** of the obstacles

# Born Approximation

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$$\mathbf{H}_\delta^{\text{Born}}(\mathbf{x}) = \sum_{k=1}^{N_{\text{obs}}} \mathbf{H}_{\delta,k}(\mathbf{x})$$

- Each obstacle is modeled as a **dipolar source** around  $c_k$

$$\mathbf{E}_{\delta,k}(\mathbf{x}) = \mathcal{E}_{\text{dip}}^{\text{elec}}[\mathbf{d}_{\delta,k}^E](\mathbf{x} - c_k) + \mathcal{E}_{\text{dip}}^{\text{mag}}[\mathbf{d}_{\delta,k}^H](\mathbf{x} - c_k)$$

$$\mathbf{H}_{\delta,k}(\mathbf{x}) = \mathcal{H}_{\text{dip}}^{\text{elec}}[\mathbf{d}_{\delta,k}^E](\mathbf{x} - c_k) + \mathcal{H}_{\text{dip}}^{\text{mag}}[\mathbf{d}_{\delta,k}^H](\mathbf{x} - c_k)$$

- The directions  $\mathbf{d}_{\delta,k}^E$  and  $\mathbf{d}_{\delta,k}^H$  depend on the **nature** of the obstacles

## Case of perfectly conducting spheres

- 3-order approximation

$$\mathbf{d}_{\delta,k}^E = 4\pi\delta^3 \mathbf{E}^{\text{inc}}(c_k) \quad \mathbf{d}_{\delta,k}^H = -2\pi\delta^3 \mathbf{H}^{\text{inc}}(c_k)$$

- Corrected dipolar approximation

$$\mathbf{d}_{\delta,k}^E = 4\pi\delta^3 \left(1 + \frac{3(\kappa\delta)^2}{10}\right) \mathbf{E}^{\text{inc}}(c_k)$$

$$\mathbf{d}_{\delta,k}^H = -2\pi\delta^3 \left(1 - \frac{3(\kappa\delta)^2}{5}\right) \mathbf{H}^{\text{inc}}(c_k)$$

## Important restrictions

- Very small obstacles
- Small number of obstacles
- No interaction taken into account

# Foldy-Lax Approximation

$$\mathbf{E}_\delta^{\text{Foldy}}(\mathbf{x}) = \sum_{k=1}^{N_{\text{obs}}} \mathbf{E}_{\delta,k}(\mathbf{x})$$

$$\mathbf{H}_\delta^{\text{Foldy}}(\mathbf{x}) = \sum_{k=1}^{N_{\text{obs}}} \mathbf{H}_{\delta,k}(\mathbf{x})$$

- Each obstacle is modeled as a **dipolar source** around  $c_k$

$$\mathbf{E}_{\delta,k}(\mathbf{x}) = \mathcal{E}_{\text{dip}}^{\text{elec}}[\mathbf{d}_{\delta,k}^{\text{E}}](\mathbf{x} - c_k) + \mathcal{E}_{\text{dip}}^{\text{mag}}[\mathbf{d}_{\delta,k}^{\text{H}}](\mathbf{x} - c_k)$$

$$\mathbf{H}_{\delta,k}(\mathbf{x}) = \mathcal{H}_{\text{dip}}^{\text{elec}}[\mathbf{d}_{\delta,k}^{\text{E}}](\mathbf{x} - c_k) + \mathcal{H}_{\text{dip}}^{\text{mag}}[\mathbf{d}_{\delta,k}^{\text{H}}](\mathbf{x} - c_k)$$

## Case of perfectly conducting spheres

- 3-order approximation

$$\mathbf{d}_{\delta,k}^{\text{E}} = 4\pi\delta^3 \left( \mathbf{E}^{\text{inc}}(c_k) + \sum_{\substack{\ell=1 \\ \ell \neq k}}^{N_{\text{obs}}} \mathbf{E}_{\delta,\ell}(c_k) \right) \quad \mathbf{d}_{\delta,k}^{\text{H}} = -2\pi\delta^3 \left( \mathbf{H}^{\text{inc}}(c_k) + \sum_{\substack{\ell=1 \\ \ell \neq k}}^{N_{\text{obs}}} \mathbf{H}_{\delta,\ell}(c_k) \right)$$

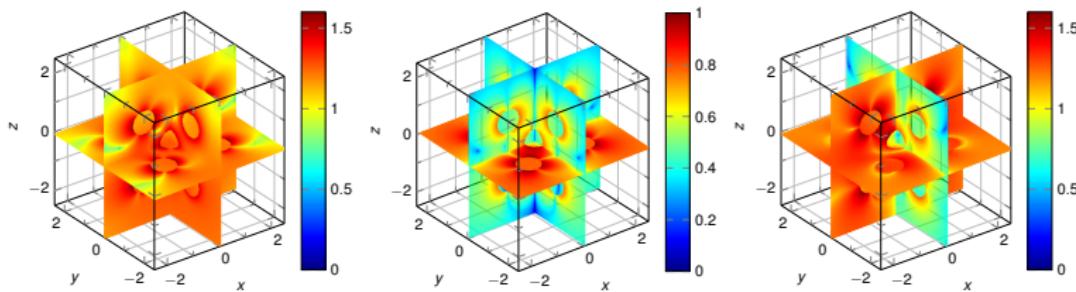
- Corrected dipolar approximation

$$\mathbf{d}_{\delta,k}^{\text{E}} = 4\pi\delta^3 \left( 1 + \frac{3(\kappa\delta)^2}{10} \right) \left( \mathbf{E}^{\text{inc}}(c_k) + \sum_{\substack{\ell=1 \\ \ell \neq k}}^{N_{\text{obs}}} \mathbf{E}_{\delta,\ell}(c_k) \right) \quad \mathbf{d}_{\delta,k}^{\text{H}} = -2\pi\delta^3 \left( 1 - \frac{3(\kappa\delta)^2}{5} \right) \left( \mathbf{H}^{\text{inc}}(c_k) + \sum_{\substack{\ell=1 \\ \ell \neq k}}^{N_{\text{obs}}} \mathbf{H}_{\delta,\ell}(c_k) \right)$$

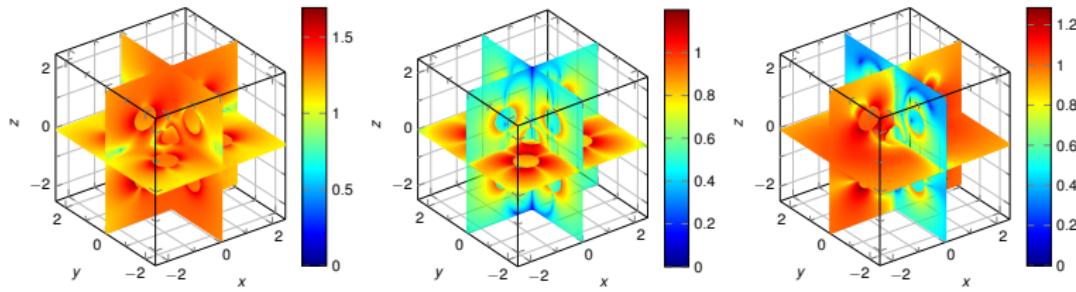
# First Numerical Results

13 obstacles ;  $d_{jk} \approx 1.0$  ;  $\delta = 0.4$

**Born** approximation



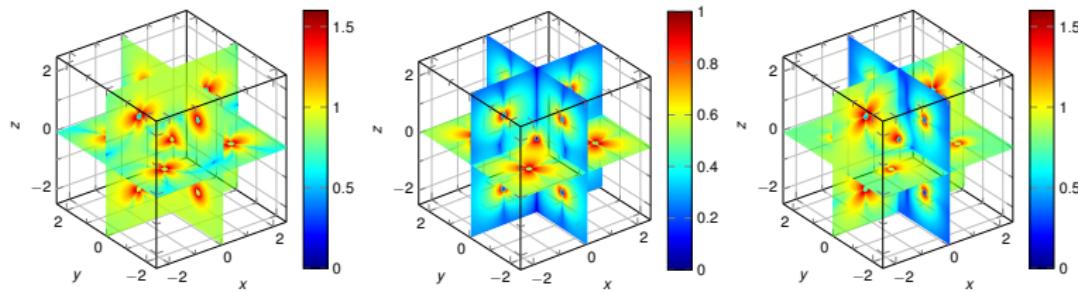
**Foldy-Lax** approximation



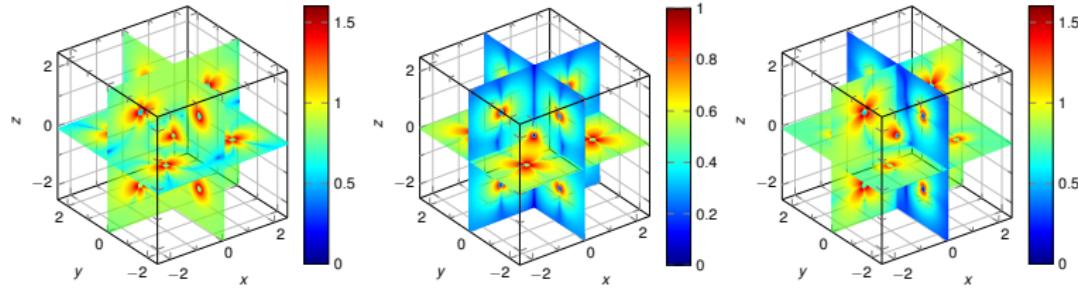
# First Numerical Results

13 obstacles ;  $d_{jk} \approx 1.25$  ;  $\delta = 0.1$

Born approximation



Foldy-Lax approximation



# Outline

## 1. Asymptotic expansions for the single electromagnetic scattering

- First terms of the asymptotics
- Numerical results

## 2. Multiple electromagnetic scattering by small spheres

- Born approximation
- Foldy-Lax approximation
- Preliminary numerical results

## 3. Conclusions and perspectives

# Conclusions and Perspectives

## Conclusions and on-going works

- Single scattering
  - ✓ Derivation of first terms of asymptotics for one sphere
  - ✓ Numerical validation
- J. Labat, V. Péron, S. Tordeux (Inria Research Report n°9169, 2018)  
Asymptotic Modeling of the Electromagnetic Scattering by Small Spheres Perfectly Conducting.
- Multiple scattering
  - ✓ Born approximation
  - ✓ Foldy-Lax approximation
  - ~ Numerical validation of Born and Foldy-Lax models with **finite element solutions**
  - ~ High-order spectral method

## Perspectives

- Single scattering
  - ✗ Justification of matched asymptotic expansions
  - ✗ Extension to obstacles of arbitrary shape
- Multiple scattering
  - ✗ Justification of Foldy-Lax approximation
  - ✗ Justification of the high-order spectral method