Handling Fluorescence in a Uni-directional Spectral Path Tracer



16 spp Monochromatic Maximum Exponential 16 spp Monochromatic Fluorescent Aware

16 spp HWSS Fluorescent Aware 65 thousand spp HWSS Fluorescent Aware

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Contributions



No volume

Fluorescent volume

Improved Tracking for Fluorescent Volume rendering

Contributions



• Hero Wavelength Spectral Sampling (HWSS) for Fluorescence

Contributions

Monochromatic 16 spp

HWSS

16 spp

4 wavelengths



• Hero Wavelength Spectral Sampling (HWSS) for Fluorescence

• **A.Glassner** - 1994 A model for Fluorescence and Phosphorescence



Fluorescence

Phosphorescence

• Wilkie et al. - EGSR 2001

Combined Rendering of Polarization and Fluorescence Effects





• Wilkie et al. - GRAPHITE 2006

A reflectance model for diffuse fluorescent surfaces



• Hullin et al. - SIGGRAPH 2010

Acquisition and Analysis of Bispectral Bidirectional Reflectance and Reradiation Distribution Functions



• M.Abdellah et al. - 2017

A Physically Plausible Model for Rendering Highly Scattering Fluorescent Participating Media



• Jung et al. - MAM 2018 A Simple Diffuse Fluorescent BBRRDF Model



- Molecule re-emits a photon from one wavelength to another
 - Absorption Spectrum





- Molecule re-emits a photon from one wavelength to another
 - Absorption Spectrum
 - Reemission Spectrum





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 - → Stokes shift





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Real World Example





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- The colour chart is not fluorescent
- The post-its is wavelength shifting
- Note how the volume turns clear!



- The colour chart is not fluorescent
- The post-its is wavelength shifting
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Re-Radiation Matrix

Represents the full
reflectance



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Re-Radiation Matrix



Re-Radiation Matrix



Re-Radiation Matrix



Non fluorescent volumes



Non fluorescent volumes



Non fluorescent volumes



Redirected paths - phase function:

$$A = \int_{\Omega} f_p(\omega_i, \omega, \lambda_i) L(\mathbf{x}_t, \omega_i, \lambda_i) \, \mathrm{d}\omega_i$$

Non fluorescent volumes





Redirected paths - phase function:

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Non fluorescent volumes



Transmittance:

$$T(\mathbf{x}, \mathbf{x}_t, \lambda_i) = e^{-\|\mathbf{x} - \mathbf{x}_t\| \cdot \sigma_t(\lambda_i)}$$
$$\sigma_t(\lambda_i) = \sigma_a(\lambda_i) + \sigma_s(\lambda_i)$$

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Spectral Radiative Transfer Equation:

$$L(\mathbf{x}, \omega, \lambda_i) = \int_{t=0}^{d} T(\mathbf{x}, \mathbf{x}_t, \lambda_i) \cdot (\sigma_s(\lambda_i) \cdot A) dt + T(\mathbf{x}, \mathbf{x}_d, \lambda_i) \cdot L(\mathbf{x}_d, \omega, \lambda_i)$$

Fluorescent volumes



Fluorescent volumes



In-shifting:

$$\sigma_{s_e}(\lambda_i) = \int_{\Lambda_a} \sigma_{s_{\Phi}}(\lambda_a, \lambda_i) \, \mathrm{d}\lambda_a$$

Spectral Radiative Transfer Equation:

$$L(\mathbf{x}, \omega, \lambda_i) = \int_{t=0}^{d} T(\mathbf{x}, \mathbf{x}_t, \lambda_i) \cdot (\sigma_s(\lambda_i) \cdot A + C) dt + T(\mathbf{x}, \mathbf{x}_d, \lambda_i) \cdot L(\mathbf{x}_d, \omega, \lambda_i)$$

Fluorescent volumes



Fluorescent volumes



In-shifting:

$$\sigma_{s_e}(\lambda_i) = \int_{\Lambda_a} \sigma_{s_{\Phi}}(\lambda_a, \lambda_i) \, \mathrm{d}\lambda$$

 \mathbf{a}

Paths redirected from other directions / wavelengths

$$B = \int_{\Omega} f_p(\omega_i, \omega, \lambda_a, \lambda_i) L(\mathbf{x}_t, \omega_i, \lambda_a) \, \mathrm{d}\omega_i$$
$$C = \int \sigma_{s_{\Phi}}(\lambda_a, \lambda_i) \cdot B \, \mathrm{d}\lambda_a$$

Spectral Radiative Transfer Equation:

$$L(\mathbf{x}, \omega, \lambda_i) = \int_{t=0}^{d} T(\mathbf{x}, \mathbf{x}_t, \lambda_i) \cdot (\sigma_s(\lambda_i) \cdot A + C) dt + T(\mathbf{x}, \mathbf{x}_d, \lambda_i) \cdot L(\mathbf{x}_d, \omega, \lambda_i)$$

Re-Radiation matrix without main diagonal

Fluorescent volumes


Fluorescent volumes



Fluorescent volumes



Fluorescent volumes



Out-shifting:

$$\sigma_{s_a}(\lambda_i) = \int_{\Lambda}$$

$$\sigma_{s_{\Phi}}(\lambda_i,\lambda_e)\,\mathrm{d}\lambda_e$$

Re-Radiation matrix without main diagonal

Transmittance:

$$\widehat{\sigma_t}(\lambda_i) = \sigma_t(\lambda_i) + \sigma_{s_a}(\lambda_i)$$

Fluorescent volumes

Contribution 1: Fluo-aware Tracking

Classic Exponential Tracking

- Problem:
 - If the medium is exclusively fluorescent, we cannot handle re-emission

Exponential tracking 128 spp

Reference

Classic Exponential Tracking

• Sampling over $\sigma_t = \overline{\sigma_a + \sigma_s}$

Classic Exponential Tracking

• Sampling over $\sigma_t = \sigma_a + \sigma_s$

In case of fluorescence

$$\hat{\sigma}_{t} = \sigma_{a} + \sigma_{s} + \sigma_{s_{a}}$$

$$\int_{\Lambda} \sigma_{s_{\phi}}(\lambda, \lambda_{e}) d\lambda_{e}$$

Classic Exponential Tracking

• Sampling over $\sigma_t = \sigma_a + \sigma_s$

In case of fluorescence

$$\hat{\sigma}_t = \sigma_a + \sigma_s + \sigma_{s_a}$$

$$\int_{\Lambda} \sigma_{s_{\phi}}(\lambda, \lambda_e) d\lambda$$

Classic Exponential Tracking

• Sampling over $\sigma_t = \sigma_a + \sigma_s$

In case of fluorescence

$$\hat{\sigma}_t = \sigma_a + \sigma_s + \sigma_{s_a}$$

$$\int_{\Lambda} \sigma_{s_{\phi}}(\lambda, \lambda_e) d\lambda_e$$

is the counterpart of σ_{s_e}

• At re-emission wavelength $\sigma_{s_e} > \sigma_{s_a} \implies \hat{\sigma}_s > \hat{\sigma}_t$

Classic Exponential Tracking

- Sampling over $\sigma_t = \sigma_a + \sigma_s$
 - In case of fluorescence

$$\sigma_{s_{e}} = \sigma_{a} + \sigma_{s} + \sigma_{s_{a}}$$

$$\int_{\Lambda} \sigma_{s_{\phi}}(\lambda, \lambda_{e}) d\lambda_{e}$$

 σ_{s_a} is the counterpart of σ_{s_e}

• At re-emission wavelength $\sigma_{s_e} > \sigma_{s_a} \implies \hat{\sigma}_s > \hat{\sigma}_t$

→We miss in-shifting

Maximum Exponential Tracking

- Quick fix:
 - Use $\max(\hat{\sigma}_s, \hat{\sigma}_t)$

Max tracking 128 spp

Fluorescent aware Tracking

- Proposed solution:
 - Fluorescence aware tracking

Max tracking 128 spp

Fluorescence aware tracking 128 spp

Reference

Fluorescence aware Tracking

$$1 \sim \int_0^d \mathrm{e}^{-t \cdot \hat{\sigma}_t} \cdot \hat{\sigma}_s \mathrm{d}t + \mathrm{e}^{-d \cdot \hat{\sigma}_t}$$

Tracking Fluorescence Aware Tracking

Fluorescence aware Tracking

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Fluorescence aware Tracking

$$p(t) \sim \begin{cases} \hat{\sigma}_s \cdot e^{-t \cdot \hat{\sigma}_t} & t < d \\ e^{-d \cdot \hat{\sigma}_t} \cdot \delta(t - d) & t \ge d \end{cases}$$

Tracking Fluorescent Aware Tracking

Fluorescence aware Tracking

$$p(t) \sim \begin{cases} \hat{\sigma}_s \cdot e^{-t \cdot \hat{\sigma}_t} & t < d \\ e^{-d \cdot \hat{\sigma}_t} \cdot \delta(t - d) & t \ge d \end{cases}$$

Contribution 2: Fluo HWSS

What does one do with the other three wavelengths?

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We've turned our render into a monochrome renderer :-(

Constant Shifting

Constant Shifting

Each wavelength is shifted according to the hero wavelength

Constant Shifting

Each wavelength is shifted according to the hero wavelength Very inefficient: a lot of wavelengths **outside reemission band**

Independent Shifting

Each wavelength is shifted independently We still have to solve the problem of wavelengths that can't be shifted

Independent Shifting

Each wavelength is shifted independently We still have to solve the problem of wavelengths that can't be shifted

Standard HWSS

Standard Rendering Equation plain vs. vectorised

$$L_{\mathrm{d}}(\mathbf{x},\boldsymbol{\omega}_{0},\boldsymbol{\lambda}) = \int_{\Omega} f(\mathbf{x},\boldsymbol{\omega}_{i} \to \boldsymbol{\omega}_{0},\boldsymbol{\lambda}) \cdot L(\mathbf{x},\boldsymbol{\omega}_{i},\boldsymbol{\lambda}) \,\mathrm{d}\boldsymbol{\omega}_{i}$$

Standard HWSS

Standard Rendering Equation plain vs. vectorised

$$L_{\mathrm{d}}(\mathbf{x},\boldsymbol{\omega}_{0},\boldsymbol{\lambda}) = \int_{\Omega} f(\mathbf{x},\boldsymbol{\omega}_{i} \to \boldsymbol{\omega}_{0},\boldsymbol{\lambda}) \cdot L(\mathbf{x},\boldsymbol{\omega}_{i},\boldsymbol{\lambda}) \,\mathrm{d}\boldsymbol{\omega}_{i}$$

$$\boldsymbol{L}_{\mathrm{d}}^{*}(\mathbf{x},\boldsymbol{\omega}_{0},\boldsymbol{\lambda}^{*}) = \int_{\Omega} \boldsymbol{f}^{*}(\mathbf{x},\boldsymbol{\omega}_{i} \to \boldsymbol{\omega}_{0},\boldsymbol{\lambda}^{*}) \diamond \boldsymbol{L}^{*}(\mathbf{x},\boldsymbol{\omega}_{i},\boldsymbol{\lambda}^{*}) \,\mathrm{d}\boldsymbol{\omega}_{i}$$

Fluo HWSS

Plain vs. vectorised

~

$$L_{\mathrm{d}}(\mathbf{x},\boldsymbol{\omega}_{0},\boldsymbol{\lambda}_{o}) = \int_{\Lambda} \int_{\Omega} f(\mathbf{x},(\boldsymbol{\omega}_{i},\boldsymbol{\lambda}_{i}) \to (\boldsymbol{\omega}_{0},\boldsymbol{\lambda}_{o})) \cdot L(\mathbf{x},\boldsymbol{\omega}_{i},\boldsymbol{\lambda}_{i}) \,\mathrm{d}\boldsymbol{\omega}_{i} \,\mathrm{d}\boldsymbol{\lambda}_{i}$$

Fluo HWSS

Plain vs. vectorised

~

$$L_{\mathrm{d}}(\mathbf{x},\boldsymbol{\omega}_{0},\boldsymbol{\lambda}_{o}) = \int_{\Lambda} \int_{\Omega} f(\mathbf{x},(\boldsymbol{\omega}_{i},\boldsymbol{\lambda}_{i}) \to (\boldsymbol{\omega}_{0},\boldsymbol{\lambda}_{o})) \cdot L(\mathbf{x},\boldsymbol{\omega}_{i},\boldsymbol{\lambda}_{i}) \,\mathrm{d}\boldsymbol{\omega}_{i} \,\mathrm{d}\boldsymbol{\lambda}_{i}$$

$$L^*_{\mathrm{d}}(\mathbf{x}, \boldsymbol{\omega}_0, \boldsymbol{\lambda}^*_{\boldsymbol{o}}) = \int_{\Lambda^n} \int_{\Omega} \boldsymbol{f}^*(\mathbf{x}, (\boldsymbol{\omega}_i, \boldsymbol{\lambda}^*_{\boldsymbol{i}}) \to (\boldsymbol{\omega}_0, \boldsymbol{\lambda}^*_{\boldsymbol{o}})) \diamond \boldsymbol{L}^*(\mathbf{x}, \boldsymbol{\omega}_i, \boldsymbol{\lambda}^*_{\boldsymbol{i}}) \,\mathrm{d}\boldsymbol{\omega}_i \Big[\mathrm{d}\boldsymbol{\lambda}_{\mathrm{i}, k} \Big]_{1..n}$$

Solution Vector

• Standard vs. fluo

$$\boldsymbol{L}_{\mathrm{d}}^{*}(\mathbf{x},\boldsymbol{\omega}_{0},\boldsymbol{\lambda}^{*}) = \begin{bmatrix} L_{\mathrm{d}}(\mathbf{x},\boldsymbol{\omega}_{0},\boldsymbol{\lambda}_{1}) \\ \vdots \\ L_{\mathrm{d}}(\mathbf{x},\boldsymbol{\omega}_{0},\boldsymbol{\lambda}_{n}) \end{bmatrix}$$

Solution Vector

• Standard vs. fluo

$$\boldsymbol{L}_{\mathrm{d}}^{*}(\mathbf{x},\boldsymbol{\omega}_{0},\boldsymbol{\lambda}^{*}) = \begin{bmatrix} L_{\mathrm{d}}(\mathbf{x},\boldsymbol{\omega}_{0},\boldsymbol{\lambda}_{1}) \\ \vdots \\ L_{\mathrm{d}}(\mathbf{x},\boldsymbol{\omega}_{0},\boldsymbol{\lambda}_{n}) \end{bmatrix}$$

$$\boldsymbol{L}_{d}^{*}(\mathbf{x},\boldsymbol{\omega}_{0},\boldsymbol{\lambda}_{\boldsymbol{o}}^{*}) = \left\|\boldsymbol{\Lambda}\right\|^{n-1} \cdot \begin{bmatrix} L_{d}(\mathbf{x},\boldsymbol{\omega}_{0},\boldsymbol{\lambda}_{0,1}) \\ \vdots \\ L_{d}(\mathbf{x},\boldsymbol{\omega}_{0},\boldsymbol{\lambda}_{0,n}) \end{bmatrix}$$

MC Integration Issue

 If each of the HWSS components does its own importance sampling, the overall sample probability becomes

$$p = p_{d1} \prod_{i=1}^{4} p_{\lambda i}$$

- MC basics: non-zero sample values must never have zero sampling probability!
- The integration domain extension offers a way to insert compensation factors to fix this, via the additional factor
 - → We zero out samples which contain a zero shifting probability

Results












D65 - No Volume











D65 - No Volume

440nm - Diff





440nm - Volume Present







D65 - No Volume

440nm - Diff





440nm - Volume Present



550nm - Volume Present





D65 - No Volume

440nm - Diff

HWSS scattering aware









2048 spp



















Handling Fluorescence in a Uni-directional Spectral Path Tracer

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