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Riemann solver for a macroscopic double-lane roundabout model

Maria Laura Delle Monache [*], Samuel Hammond[†], Benedetto Piccoli[‡]

Abstract

In this article, we introduce a Riemann solver for traffic flow on a roundabout with two lanes. The roundabout is modeled as a sequence of 2×1, 1×2 and 2×2 junctions. The Riemann solver provide a solution at junctions by taking into consideration traffic distribution, priorities, and the maximization of through flux. We prove existence and uniqueness of the solution of the Riemann problem and show some results numerically.

Keywords: Traffic modeling, Roundabouts, Macroscopic models

1 Introduction

Macroscopic traffic models were introduced during the fifties by Lighthill, Whitham [7] and independently Richards [8]. They were the first to propose a hydrodynamics model for traffic flow using a non linear scalar hyperbolic Partial Differential Equation (PDE). The PDE equipped with an initial data is commonly referred to as the LWR model. This model, was later on extended to work on networks. In fact, over the years, several authors proposed models on networks that are able to describe the dynamics at intersections, see for example [1, 5, 3] and reference therein. Each of these models consider different types of solutions for different types of junctions, according to the different number of lanes, incoming and outgoing links.

In this article, we focus on a Riemann problem for roundabouts. In particular, we analyze an urban double-lane roundabout with four approaches and exits aligned at 90 degrees. This roundabout can be seen as concatenation of 2×1 ("*merging*"), 1×2 ("*diverging*") and 2×2 ("*crossing*") junctions, but

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the approach can be generalized to a more general network. The main lanes density evolution, as well as the entries and the exits ones are described by a scalar hyperbolic conservation law. At each junction, the Riemann problem is uniquely solved using right-of-way and traffic distribution parameters.
The article is organized as follows:

- In Section 2 we introduce formally the mathematical model by describing accurately the network and the mathematical description of the traffic evolution on each link and on each junction.
- In Section 3 we introduce the Riemann Solver at junctions. We first introduce some necessary notations and we describe step by step the construction of the Riemann Solver for the different types of junctions. We also prove in this section the existence and uniqueness theorem.
- In Section 4 we describe the numerical scheme used to find the numerical solution of the problem on the roundabout.
- In Section 5 we show the results obtained using the numerical scheme introduced in Section 4.

2 Mathematical model

In this article, we describe the macroscopic model for an urban double-lane roundabout with four approaches and exits aligned at 90 degrees, see Figure 1.

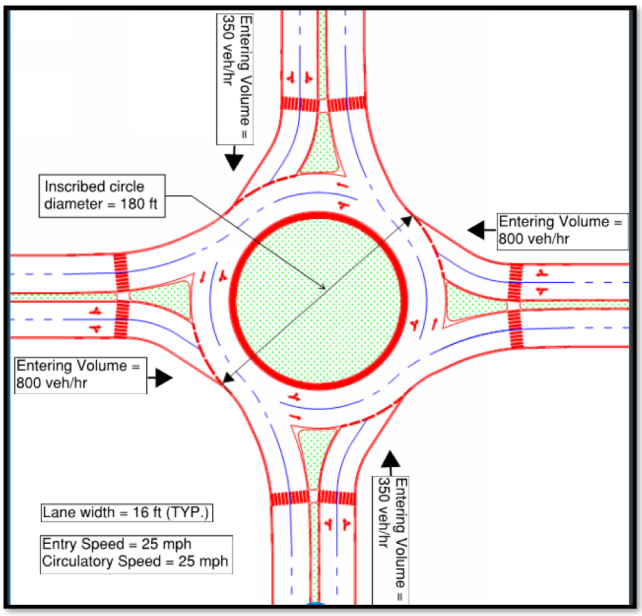

Figure 1: Roundabout modeled in the article.

A roundabout can be seen as a sequence of junctions, and it can be represented by graph in which roads are described by arcs and junctions by vertices, see Figure 2. The roundabout that we study in the following of this article is equipped with 52 links of which 16 belong to entrances, 12 belong to exits, 16 belong the the outer lane of the circle and 8 belong to the inner lane of the circle. The generalization of the study to an arbitrary number of roads being straightforward. Each link forming the roundabout is modeled by an interval $I_i = [a_i, b_i] \subset \mathbb{R}, i = 1, \ldots, 52,\ a_i < b_i$, with $b_{44} = a_{29}$ and $b_{52} = a_{45}$ to represent the roundabout circles.

The evolution of traffic flow along the circle lanes and on the entering and

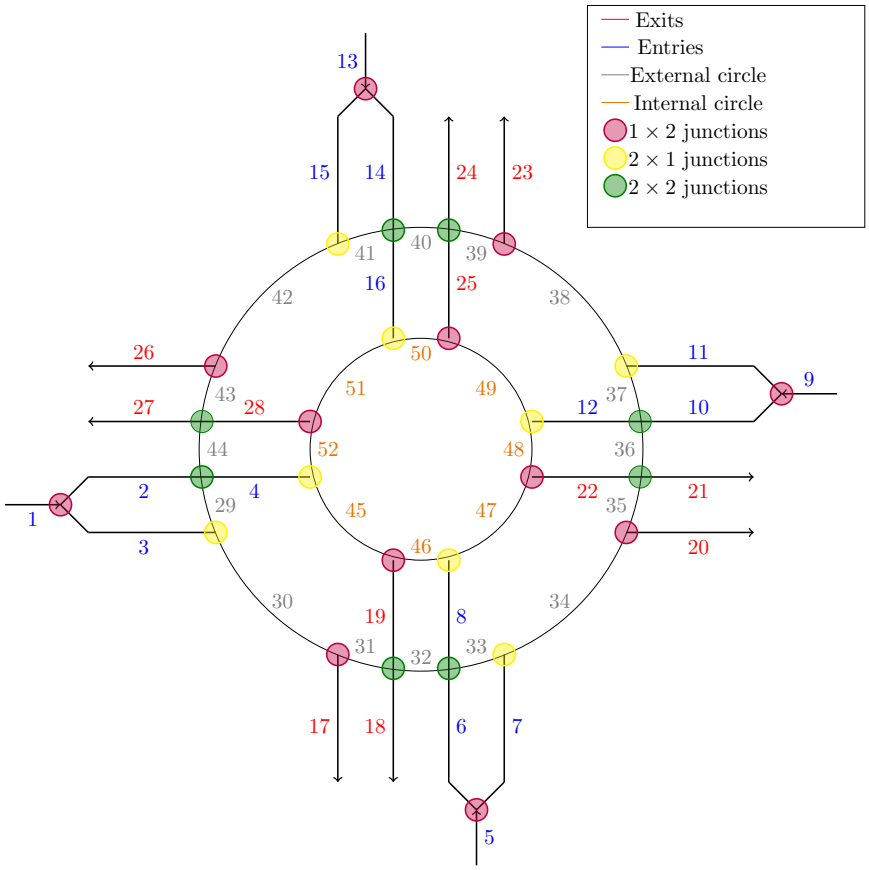

Figure 2: Schematics of the roundabout representing the different links and the different junctions types.

exiting links is described as follows

$$\partial_t \rho_i + \partial_x f(\rho_i) = 0 \qquad (x,t) \in \mathbb{R}^+ \times I_i \ i = 1, \ldots, 52, \tag{1}$$

where $\rho_i = \rho_i(t,x) \in [0, \rho_i^{\max}]$ is the mean traffic density, $\rho_i^{\max}$ is the maximal density on each single road and the flux function is given by the Greenshield

fundamental diagram described by the following equation:

$$f(\rho_i) = \rho_i v_{\max,i}\Big(1 - \frac{\rho_i}{\rho_i^{\max}}\Big) \tag{2}$$

with $v_{\max,i}$ the maximal speed on each link. In the roundabout that we are modeling, there are 3 types of junctions: merge junction (2 incoming and 1 outgoing roads), diverge junction (1 incoming and 2 outgoing links) and crossing junctions (2 incoming and 2 outgoing), see Figure 2 for the different locations of the junctions and Figure 3 for a more detailed representation of the different types of junctions used in this study.

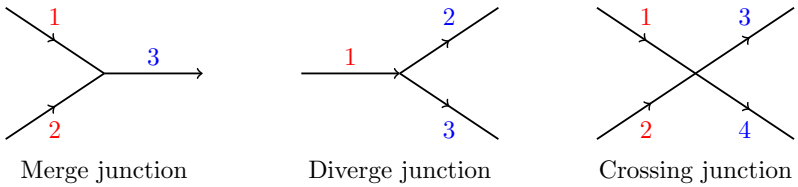

Figure 3: Different types of junctions modeled

Definition 2.1 *Consider a roundabout with 52 links $I_i = [a_i, b_i] \subset \mathbb{R}$, $a_i \leq b_i$ for $i = 1, \dots, 52$, with $b_{52} = a_{45}$ and $b_{44} = a_{29}$, 16 entrance links and 12 exit links. A collection of functions $(\rho_i)_{i=1,\dots,52} \in \prod_{i=1}^{52} \mathrm{C}^0(\mathbb{R}^+; \mathrm{L}^1 \bigcap \mathrm{BV}(I_i))$ is an admissible solution to (1) if*

1. *ρ_i is a weak solution on I_i, i.e., $\rho_i : [0, +\infty[\times I_i \to [0, \rho_i^{\max}]$, such that*

$$\int_{\mathbb{R}^+} \int_{I_i} \Big(\rho_i \partial_t \varphi_i + f(\rho_i) \partial_x \varphi_i\Big) dx dt = 0 \tag{3}$$

   for every $\varphi_i \in \mathrm{C}_c^1(\mathbb{R}^+ \times I_i), i = 1, \dots, 52$.

2. *ρ_i satisfies the Kružhkov entropy condition ([6]) on $(\mathbb{R} \times I_i)$, that is,*

$$\begin{aligned} \int_{\mathbb{R}^+} \int_{I_i} \Big(& |\rho_i - k| \partial_t \varphi_i + \mathrm{sgn}(\rho_i - k) \cdot \\ & (f(\rho_i) - f(k)) \partial_x \varphi_i \Big) dx dt \\ & + \int_{I_i} |\rho_{i,0} - k| \varphi_i(0, x) dx \geq 0 \end{aligned} \tag{4}$$

   for every $k \in [0, 1]$ and for all $\varphi_i \in \mathrm{C}_c^1(\mathbb{R} \times I_i)$, $i = 1, \dots, 52$

3. *At each junction J_i for $i = 1, \dots, 52$,*

$$\sum f(\rho_{\mathrm{inc}}(t, 0-)) = \sum f(\rho_{\mathrm{out}}(t, 0+)))$$

   where the subscripts $_{\mathrm{inc}}$, $_{\mathrm{out}}$ indicates all the links belonging to a junction. In particular, $_{\mathrm{inc}}$ indicates the incoming links and $_{\mathrm{out}}$ indicates the outgoing ones.

3 Riemann problem at the junction

In this section we describe the construction of the Riemann solver at a junction. Let us first set some notations. In the following of the paper the subscripts $_{\text{inc}}$ indicates that quantities belonging to the incoming links on a junction, while $_{\text{out}}$ indicates the outgoing ones.

Definition 3.1 *Let us define the following quantities*

1. *For every* $l \in \{\text{inc}\}$ *define*

$$\gamma_{\text{inc}}^{\max}(\rho_l) = \begin{cases} f(\rho_l) & \text{if } 0 \leq \rho_l \leq \rho_{\text{cr}} \\ f_{\max} & \text{if } \rho_{\text{cr}} \leq \rho_l \leq \rho_l^{\max}; \end{cases} \tag{5}$$

2. *for every* $j \in \{\text{out}\}$ *define*

$$\gamma_{\text{out}}^{\max}(\rho_j) = \begin{cases} f_{\max} & \text{if } 0 \leq \rho_j \leq \rho_{\text{cr}} \\ f(\rho_j) & \text{if } \rho_{\text{cr}} \leq \rho_j \leq \rho_j^{\max}. \end{cases} \tag{6}$$

Moreover, let us fix a matrix A belonging to the set of matrices:

$$\mathcal{A} := \Big\{ A = a_{1,j} j \in \{\text{out}\} : 0 \leq a_{1,j} \leq 1, \sum_{j \in \{\text{out}\}} a_{1,j} = 1 \Big\} \tag{7}$$

and a priority vectors $p = (p_1, p_2) \in \mathbb{R}^2$ with $p_l > 0$, $\sum_{l=1}^{2} p_l = 1$, indicating priorities among incoming roads.
Moreover, we define a function τ as follows. For details, see [3].

Definition 3.2 *Let* $\tau : [0,1] \to [0,1]$ *be the map such that*

- $f(\tau(\rho)) = f(\rho)$ *for every* $\rho \in [0, \rho^{\max}]$
- $\tau(\rho) \neq \rho$ *for every* $\rho \in [0,1] \setminus \{\rho_{\text{cr}}\}$

We are now ready to describe the construction of the Riemann Solver for different types of junctions.
Fix $\rho_{1,0}, \ldots, \rho_{52,0} \in [0, \rho_i^{\max}]$. Consider a Riemann problem at a junction J_i

$$\begin{cases} \partial_t \rho_i + \partial_x f(\rho_i) = 0 \\ \rho_i(0, \cdot) = \rho_{i,0} \qquad i \in 1, \ldots, 52 \end{cases} \tag{8}$$

A solution to the Riemann problem at J_i is defined as follows.

3.1 Merge junction

Let us consider first a merging junction, i.e. a junction with two incoming and one outgoing road, see Figure 3, left.
Let us fix constants $\rho_{1,0},\ \rho_{2,0},\ \rho_{3,0} \in [0, \rho_i^{\max}]$ for $i = 1, 2, 3$, and a priority parameter p. The Riemann solver $\mathcal{RS}(\rho_{1,0},\ \rho_{2,0},\ \rho_{3,0}) = (\hat{\rho}_1, \hat{\rho}_2, \hat{\rho}_3)$ at the junction is constructed in the following way.

1. Compute :

   $\gamma_1^{\max} = \gamma_{\text{inc}}^{\max}(\rho_1)$,:

   $\gamma_2^{\max} = \gamma_{\text{inc}}^{\max}(\rho_2)$, :

   $\gamma_3^{\max} = \gamma_{\text{out}}^{\max}(\rho_3)$

2. Fix:

   $\hat{\gamma}_3 = \min(\gamma_1^{\max} + \gamma_2^{\max}, \gamma_3^{\max})$,

   $\hat{\gamma}_1 = \min(\gamma_1^{\max}, \max(\hat{\gamma}_3 - \gamma_2^{\max}, p\hat{\gamma}_3))$

   $\hat{\gamma}_2 = \hat{\gamma}_3 - \hat{\gamma}_1$

3. Set $\hat{\gamma}_{\text{inc}} = (\hat{\gamma}_1, \hat{\gamma}_2)$ and $\hat{\gamma}_{\text{out}} = (\hat{\gamma}_3)$

3.2 Diverge junction

We consider a diverging junction, i.e. a junction with one incoming and two outgoing links, see Figure 3, center. Let us fix constants $\rho_{1,0},\ \rho_{2,0},\ \rho_{3,0} \in [0, \rho_i^{\max}]$ for $i = 1, 2, 3$, and a distribution matrix $A = [\alpha, 1 - \alpha]$. The Riemann solver $\mathcal{RS}(\rho_{1,0},\ \rho_{2,0},\ \rho_{3,0}) = (\hat{\rho}_1, \hat{\rho}_2, \hat{\rho}_3)$ at the junction is constructed in the following way.

1. Compute:

   $\gamma_1^{\max} = \gamma_{\text{inc}}^{\max}(\rho_1)$,

   $\gamma_2^{\max} = \gamma_{\text{out}}^{\max}(\rho_2)$,

   $\gamma_3^{\max} = \gamma_{\text{out}}^{\max}(\rho_3)$.

2. Then

   $\hat{\gamma}_1 = \min(\gamma_1^{\max}, \dfrac{\gamma_2^{\max}}{\alpha}, \dfrac{\gamma_3^{\max}}{1-\alpha})$

   $\hat{\gamma}_2 = \alpha\hat{\gamma}_1$

   $\hat{\gamma}_3 = (1 - \alpha)\hat{\gamma}_1$

3. Set $\hat{\gamma}_{\text{inc}} = (\hat{\gamma}_1)$ and $\hat{\gamma}_{\text{out}} = (\hat{\gamma}_2, \hat{\gamma}_3)$

3.3 Crossing junction

Let us consider finally a crossing junction , i.e. a junction with two incoming and two outgoing links, see Figure 3, right. In this particular setting the flux from link 1 is allowed only on link 4 and the flux from link 2 is allowed only on link 3. Let us fix constants $\rho_{1,0},\ \rho_{2,0},\ \rho_{3,0},\ \rho_{4,0} \in [0, \rho_i^{\max}]$ for $i = 1, 2, 3, 4$, the maximal capacity of the junction Γ^{max} and a priority parameter p. The Riemann solver $\mathcal{RS}(\rho_{1,0},\ \rho_{2,0},\ \rho_{3,0},\ \rho_{4,0}) = (\hat{\rho}_1, \hat{\rho}_2, \hat{\rho}_3, \hat{\rho}_4)$ at the junction is constructed in the following way.

1. Compute:

   $\gamma_1^{\max} = \gamma_{\text{inc}}^{\max}(\rho_1)$,

   $\gamma_2^{\max} = \gamma_{\text{inc}}^{\max}(\rho_2)$,

   $\gamma_3^{\max} = \gamma_{\text{out}}^{\max}(\rho_3)$

   $\gamma_4^{\max} = \gamma_{\text{out}}^{\max}(\rho_4)$

2. Set

   $\gamma_{1,4} = \min(\gamma_1^{\max}, \gamma_4^{\max})$ and

   $\gamma_{2,3} = \min(\gamma_2^{\max}, \gamma_3^{\max})$

3. Then two situations can occur

   - If $\gamma_{1,4} + \gamma_{2,3} \leq \Gamma^{\max}$ then

     $\hat{\gamma}_1 = \hat{\gamma}_4 = \gamma_{1,4}$ and

     $\gamma_2^{\max} = \gamma_3^{\max} = \gamma_{2,3}$

   - else $\hat{\gamma}_1 = \hat{\gamma}_4 = \min(\gamma_{1,4}, \max(\Gamma^{\max} - \gamma_{2,3}, p\Gamma^{\max}))$ and

     $\hat{\gamma}_2 = \hat{\gamma}_3 = \Gamma^{\max} - \hat{\gamma}_1$

4. Set $\hat{\gamma}_{\text{inc}} = (\hat{\gamma}_1, \hat{\gamma}_2)$ and $\hat{\gamma}_{\text{out}} = (\hat{\gamma}_3, \hat{\gamma}_4)$

Finally the following result holds.

Theorem 1 *Consider a junction J and fix a priority parameter $p \in]0, 1[$ in case of merge junction, a distribution matrix $A = [\alpha, 1 - \alpha]$ in case of diverge junctions, and the maximal capacity of the junction Γ^{max} and a priority parameter $p \in]0, 1[$ in case of crossing junction. Let us define with $\rho_{\text{inc}}(\rho_{\text{out}})$ all the densities belonging to the incoming (outgoing) links. For every $\rho_{\text{inc},0}, \rho_{\text{out},0}$, there exists a unique admissible solution $(\rho_{\text{inc}}(t, x), \rho_{\text{out}}(t, x))$ in the sense of Definition 2.1, compatible with the Riemann solver proposed*

in Section 3. More precisely, there exists a unique n-tuple of functions $(\hat{\rho}_{\text{inc}}, \hat{\rho}_{\text{out}})$ *such that* $\mathcal{RS}(\rho_{\text{inc},0}, \rho_{\text{out},0}) = (\hat{\rho}_{\text{inc}}, \hat{\rho}_{\text{out}})$*:*

$$\hat{\rho}_{\text{inc}} = \begin{cases} \{\rho_{\text{inc},0}\} \cup]\tau(\rho_{\text{inc},0}), 1] & \text{if } 0 \leq \rho_{\text{inc},0} \leq \rho_{\text{cr}}, \\ [\rho_{\text{cr}}, 1] & \text{if } \rho_{\text{cr}} \leq \rho_{\text{inc},0} \leq \rho^{\max}, \end{cases} \tag{9}$$
$$f(\hat{\rho}_{\text{inc}}) = \hat{\gamma}_{\text{inc}}$$

and

$$\hat{\rho}_{\text{out}} = \begin{cases} [\rho_{\text{cr}}, 1] & \text{if } 0 \leq \rho_{\text{out},0} \leq \rho_{\text{cr}}, \\ \{\rho_{\text{out},0}\} \cup [0, \tau(\rho_{\text{out},0})[& \text{if } \rho_{\text{cr}} \leq \rho_{\text{out},0} \leq \rho^{\max}, \end{cases} \tag{10}$$
$$f(\hat{\rho}_{\text{out}}) = \hat{\gamma}_{\text{inc}}$$

For the incoming road, the solution is given by the wave $(\rho_{\text{inc},0}, \hat{\rho}_{\text{inc}})$*, while for the outgoing road, the solution is given by the wave* $(\hat{\rho}_{\text{out}}, \rho_{\text{out},0})$*.*

Proof. Notice that for the case of merge and diverge junction the Riemann solver is the same as that proposed in [3] in Sections 3.2.2 and 3.2.3. In particular, the Riemann solver gives rise to a unique solution to every Riemann problem, see Section 4.2.2 of [3].
Consider now a crossing junction. Since the equality $\hat{\gamma}_1 = \hat{\gamma}_4$ and $\hat{\gamma}_2 = \hat{\gamma}_3$ must hold for every solution, the problem can be solved focusing only on incoming links 1 and 2. In this setting, after restrictions as in (2) are applied, then the solution is defined as for merge junctions. Then the Riemann solver gives rise to a unique solution. □
To guarantee that a Riemann solver provides a good solution for all times we give the following:

Definition 3.3 *A Riemann solver* $\mathcal{RS}$ *is consistent if for every* $\rho_{\text{inc},0}, \rho_{\text{out},0}$ *we have*

$$\mathcal{RS}(\rho_{\text{inc},0}, \rho_{\text{out},0}) = \mathcal{RS}(\mathcal{RS}(\rho_{\text{inc},0}, \rho_{\text{out},0})).$$

In particular consistency implies that the solution to the Riemann problem satisfies for a.e. $t > 0$:

$$(\rho_{\text{inc}}(t, 0-), \rho_{\text{out}}(t, 0+)) = \mathcal{RS}(\rho_{\text{inc}}(t, 0-), \rho_{\text{out}}(t, 0+)).$$

We have the following:

Theorem 2 *The Riemann solver proposed in Section 3 is consistent.*

Proof. For merge and diverge junctions, the results is given in Section 4.2.2 of [3]. For crossing junctions, the solution to the Riemann problem can be interpreted as a merge junction for the incoming roads 1 and 2 as for the proof of Theorem 1. Then consistency condition follows from the same property for merge junction. □

4 Numerical scheme

In this section, we describe the numerical scheme used to solve problem (8).

4.1 Network topology

The roundabout will be modeled by

- eight roads from the inner circle $(I_{45}, \ldots, I_{52})$
- sixteen roads for the outer circle $(I_{29}, \ldots, I_{44})$
- eight exit roads, which results in twelve links $(I_{17}, \ldots, I_{28})$, for details see Figure 2
- eight entrances with sixteen links indicated by $(I_1, \ldots, I_{16})$, see Figure 2

Moreover, from the topology it can deduced that the intersections on the roundabout can be represented by 2×1, 1×2 and 2×2 junctions.

4.2 Scheme

We define a numerical grid in $(0, T) \times \mathbb{R}$ using the following notation:

- Δx is the fixed grid space
- Δt is the time step given by the CFL condition
- $(t^n, x_j) = (n\Delta t, k\Delta x)$ for $n \in \mathbb{N}$ and $k \in \mathbb{Z}$ are the grid points

The scheme used for solving equation (1) is the Godunov scheme as introduced in [4] and it is based on exact solutions to the Riemann problem. Under the CFL condition, [2] it holds:

$$\Delta t \max_{k \in \mathbb{Z}} \left| \lambda_{k+\frac{1}{2}} \right| \leq \frac{1}{2} \Delta x \tag{11}$$

where $\lambda_{k+\frac{1}{2}}$ is the speed of the wave of the Riemann problem solution at the interface $x_{k+\frac{1}{2}}$ at the time t^n, the numerical scheme can be written as

$$\rho_k^{n+1} = \rho_k^n - \frac{\Delta t}{\Delta x} \Big(g(\rho_k^n, \rho_{k+1}^n) - g(\rho_{k-1}^n, g_k^n) \Big). \tag{12}$$

The numerical flux g takes in general the following expression

$$g(u, v) = \begin{cases} \min_{z \in [u,v]} f(z) & \text{if } u \leq v \\ \max_{z \in [v,y]} f(z) & \text{if } v \leq u \end{cases} \tag{13}$$

4.2.1 Boundary conditions

Each road is divided in K cells numbered from 1 to K. For the incoming roads we proceed by setting:

$$\rho_1^{n+1} = \rho_1^n - \frac{\Delta t}{\Delta x}\Big(g(\rho_1^n, \rho_1^n) - g(\rho_1^n, \rho_2^n)\Big)$$

while for the outgoing ones, we set

$$\rho_K^{n+1} = \rho_K^n - \frac{\Delta t}{\Delta x}\Big(g(\rho_{K-1}^n, \rho_K^n) - g(\rho_K^n, \rho_K^n)\Big).$$

4.2.2 Conditions at the junctions

From the incoming roads which are connected at the junction at the right endpoint, we set

$$\rho_K^{n+1} = \rho_K^n - \frac{\Delta t}{\Delta x}\Big(\hat{\gamma}_{\text{inc}} - g(\rho_{K-1}^n, \rho_K^n)\Big)$$

while for the outgoing ones, connected at the junction at the junctions at the left point, we set

$$\rho_1^{n+1} = \rho_1^n - \frac{\Delta t}{\Delta x}\Big(g(\rho_1^n, \rho_2^n) - \hat{\gamma}_{\text{out}}\Big)$$

where $\hat{\gamma}_{\text{inc}}$ and $\hat{\gamma}_{\text{out}}$ are the flux computed in Section 3.

5 Numerical Results

For illustration, we choose a concave fundamental diagram as introduced in (2) with the following values for the parameters.

$$v_{\max,i} = 1,\ \rho^{\max} = 1,\ L = 1,\ \rho_{\text{cr}} = 0.5,\ T = 2.5 \tag{14}$$

For the sake of simplicity we show simulations with normalized parameters. The extension with general values is straightforward. We choose the following initial conditions:

$$\rho_i(0,x) = 0.4 \cdot \rho_i^{\max} \qquad \text{for } i = 1,5,9,13 \tag{15}$$
$$\rho_i(0,x) = 0.2 \cdot \rho_i^{\max} \qquad \text{for } i \in [45,\dots,52] \tag{16}$$
$$\rho_i(0,x) = 0.3 \cdot \rho_i^{\max} \qquad \text{for } i \in [29,\dots,44] \tag{17}$$
$$\rho_i(0,x) = 0 \qquad \text{everywhere else} \tag{18}$$

The results obtained are shown in Figures 4, 5, 6, 7.As example we show the evolution of the density in an entrance link, an exit link, a link on the inside cirlce and a link on the outside one. In all of them we can see the evolution of the density during the simulation time.

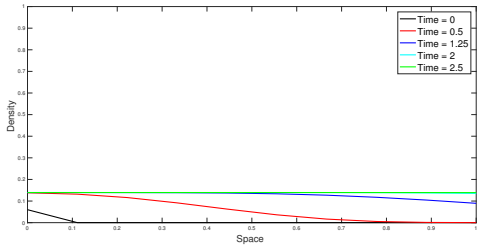

Figure 4: Evolution of the density on a roundabout entrance.

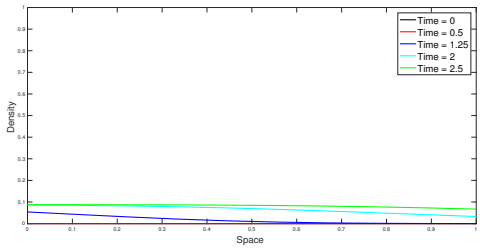

Figure 5: Evolution of the density on a roundabout exit.

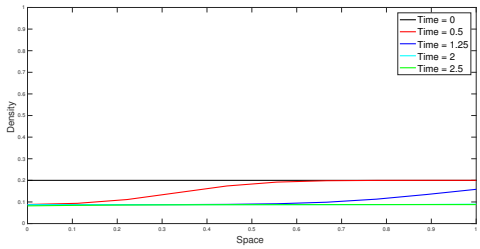

Figure 6: Evolution of the density on a roundabout link in the inside lane of the circle.

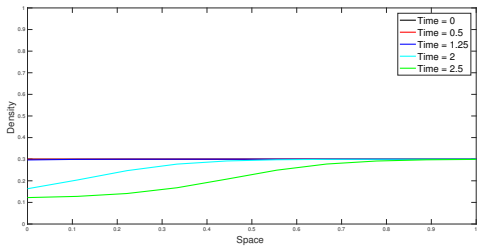

Figure 7: Evolution of the density on a roundabout link in the outside lane of the circle.

6 Conclusion

This article introduces a model for junctions on a roundabout with double lanes and 4 entrances and 4 exits. The junction flow distribution at the junction is solved by using distribution and priority parameters. We proved existence and uniqueness of solutions for the Riemann problem and we solved the problem numerically using the Godunov scheme. Some numerical tests are presented to show the accuracy of the scheme.

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References

[1] G. M. Coclite, M. Garavello, and B. Piccoli. Traffic flow on a road network. *SIAM journal on mathematical analysis*, 36(6):1862–1886, 2005.

[2] R. Courant, K. Friedrichs, and H. Lewy. On the partial difference equations of mathematical physics. *IBM journal of Research and Development*, 11(2):215–234, 1967.

[3] M. Garavello, K. Han, and B. Piccoli. *Models for vehicular traffic on networks, volume 9 of AIMS Series on Applied Mathematics*. American Institute of Mathematical Sciences (AIMS), Springfield, MO, 2016.

[4] S. Godunov. A finite difference method for the computation of discontinuous solutions of the equations of fluid dynamics. *Sbornik: Mathematics*, 47(8-9):357–393, 1959.

[5] H. Holden and N. H. Risebro. A mathematical model of traffic flow on a network of unidirectional roads. *SIAM Journal on Mathematical Analysis*, 26(4):999–1017, 1995.

[6] S. N. Kruzhkov. First order quasilinear equations in several independent variables. *Matematicheskii Sbornik*, 123(2):228–255, 1970.

[7] M. J. Lighthill and G. B. Whitham. On kinematic waves. ii. a theory of traffic flow on long crowded roads. *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, 229(1178):317–345, 1955.

[8] P. I. Richards. Shock waves on the highway. *Operations research*, 4(1):42–51, 1956.