



HAL
open science

Inverse design of a suspended Kirchhoff rod: From theory to practice

Victor Romero, Florence Bertails-Descoubes, Alexandre Derouet-Jourdan,
Arnaud Lazarus

► To cite this version:

Victor Romero, Florence Bertails-Descoubes, Alexandre Derouet-Jourdan, Arnaud Lazarus. Inverse design of a suspended Kirchhoff rod: From theory to practice. ESMC10 2018 - 10th European Solid Mechanics Conference, Jul 2018, Bologne, Italy. pp.184302 - 184302. hal-01778474v2

HAL Id: hal-01778474

<https://inria.hal.science/hal-01778474v2>

Submitted on 23 May 2018 (v2), last revised 23 May 2018 (v3)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Inverse design of a suspended Kirchhoff rod: From theory to practice

Victor Romero¹, Florence Bertails-Descoubes¹, Alexandre Derouet-Jourdan², Arnaud Lazarus³

¹ Univ. Grenoble Alpes, Inria, CNRS, Grenoble INP, LJK, 38000 Grenoble, France

E-mail: victor.romero-gramegna@inria.fr

² OLM Digital Inc., Mikami Bldg. 2F, 1-18-10, Wakabayashi, Setagaya, Tokyo, 154-0023, Japan

³ Sorbonne Universités, UPMC Univ Paris 06, CNRS, UMR 7190, Institut Jean Le Rond d'Alembert, F-75005, Paris, France

Keywords: inverse design, Kirchhoff rod, experimental and theoretical curve framing

Estimating the natural shape of a given hanging deformed rod, made of a known material, is a relevant problem in many industrial, graphic design, or even biological applications [1, 2, 3]. To tackle this problem we examine the static Kirchhoff equations for thin elastic rods [4] in the reverse direction, that is, when the shape at equilibrium is known and the natural shape is unknown. Our study is focussed on the case of an isotropic rod clamped at one end and free at the other, subjected to gravity. The input shape is described as a mere geometric curve that we subsequently frame to compute a material curvature field and feed our inverse problem. One interesting property we prove is that the natural shape of the rod satisfying equilibrium exists and is unique, regardless of the infinity of framings compatible with the input curve. The natural shape is computed efficiently by solving in sequence three linear initial value problems, starting from any framing of the input curve.

We illustrate our theoretical results through numerical examples of well-known curves to which we apply our inverse procedure. We validate, by direct simulation [2], that indeed those natural shapes fall, under the effect of gravity, onto the expected equilibrium. We stress on the fact that the obtained rest shapes are complex and far from intuitive, making the simplicity of this procedure very useful. For example, we show that the intricate rest shape depicted in Figure 1 (left) will fall into a perfect helix of radius 15 mm and pitch 20 mm (see fig. 1 right) if the rod is made of a material with elastic modulus of 5 Mpa and a 3mm cross section diameter.

Finally, we complement this study with experimental corroborations. By means of a standard array of cameras, we spatially reconstruct real elastic hanging rods with well-defined geometrical features. We find some good agreements with model prediction despite the experimental limitations on the estimation of the curvature fields of the rod's center lines.

Aknowlegment: This work was supported by the European Research Council grant GEM (StG-2014-639139).

References

- [1] Goyal, S. et al., "Nonlinear dynamics and loop formation in kirchhoff rods with implications to the mechanics of dna and cables", *J. Comput. Phys.*, **209**, 371389, (2006)
- [2] Bertails et al., "Super-helices for predicting the dynamics of natural hair", *ACM Trans. Graph (Proc. ACM SIGGRAPH06)*, **25** 11801187, 2006
- [3] Goriely, A. and Neukirch, S., "Mechanics of climbing and attachment in twining plants", *Phys. Rev. Lett.*, **97**, 184302, (2006)
- [4] Audoly, B. and Pomeau, Y., *Elasticity and Geometry: from hair curls to the nonlinear response of shells*, Oxford University Press (2010)

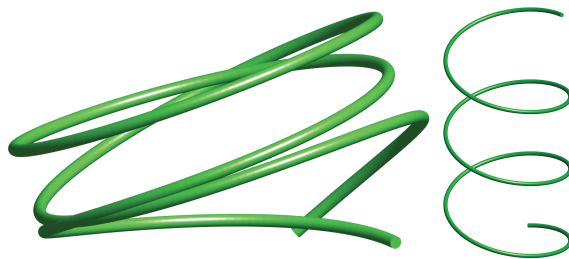


Figure 1: **Left.-** Inverse natural shape for a perfect helix under gravity. **Right.-** Equilibrium shape.