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## 4D Variational Data Assimilation with Incompact3d using Large Eddy Models

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# 4D Variational Data Assimilation with Incompact3d using Large Eddy Models

Pranav CHANDRAMOULI

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INRIA - Rennes (Fluminance)  
Incompact3d Meeting - London, 2018

28/03/2018

# Plan

- 1 Large Eddy Simulation
  - SGS Models
  - LES - Results
- 2 Data Assimilation
  - Introduction to Data Assimilation
  - Code Formulation
  - VDA with LES - Results

# Objective

To Identify an apt Sub-Grid Scale (SGS) model for coarse LES in terms of statistical accuracy and numerical stability.

## LES - SGS Models

Smagorinsky Variants :

$$\text{Classic} : \nu_t = (C_s * \Delta)^2 \bar{S}, \quad (1)$$

$$\text{Dynamic} : C_s^2 = -\frac{1}{2} \frac{\mathcal{L}_{ij} \bar{M}_{ij}}{M_{kl} \bar{M}_{kl}}. \quad (2)$$

where,

$$\mathcal{L}_{ij} = T_{ij} - \tilde{\tau}_{ij} \quad (3)$$

$$M_{ij} = \tilde{\Delta}^2 |\tilde{S}| \tilde{S}_{ij} - \bar{\Delta}^2 (\widetilde{|\bar{S}| \bar{S}_{ij}}), \quad (4)$$

WALE :

$$\nu_t = (C_w \Delta)^2 \frac{(\varsigma_{ij}^d \varsigma_{ij}^d)^{3/2}}{(\bar{S}_{ij} \bar{S}_{ij})^{5/2} + (\varsigma_{ij}^d \varsigma_{ij}^d)^{5/4}} \quad (5)$$

# LES - Models under Location Uncertainty (MULU)

$$\frac{d\mathbf{X}_t}{dt} = \mathbf{w}(\mathbf{X}_t, t) + \sigma(\mathbf{X}_t, t)\dot{\mathbf{B}} \quad (6)$$

- $\mathbf{w}(\mathbf{X}, t)$  is the large scale drift
- $\sigma(\mathbf{X}, t)d\dot{\mathbf{B}}_t$  stands for small scales represented by a noise

Mémin, Etienne. "Fluid flow dynamics under location uncertainty." Geophysical & Astrophysical Fluid Dynamics 108.2 (2014) : 119-146.

## MULU

NS formulation as derived in Memin [2014] :

**Mass conservation :**

$$d_t \rho_t + \nabla \cdot (\rho \tilde{\mathbf{w}}) dt + \nabla \rho \cdot \boldsymbol{\sigma} d\mathbf{B}_t = \frac{1}{2} \nabla \cdot (\mathbf{a} \nabla q) dt, \quad (7)$$

$$\tilde{\mathbf{w}} = \mathbf{w} - \frac{1}{2} \nabla \cdot \mathbf{a} \quad (8)$$

For an incompressible fluid :

$$\nabla \cdot (\boldsymbol{\sigma} d\mathbf{B}_t) = 0, \quad \nabla \cdot \tilde{\mathbf{w}} = 0, \quad (9)$$

**Momentum conservation :**

$$\left( \partial_t \mathbf{w} + \mathbf{w} \nabla^T \left( \mathbf{w} - \frac{1}{2} \nabla \cdot \mathbf{a} \right) - \frac{1}{2} \sum_{ij} \partial_{x_i} (a_{ij} \partial_{x_j} \mathbf{w}) \right) \rho = \rho \mathbf{g} - \nabla p + \mu \Delta \mathbf{w}. \quad (10)$$

## MULU

Modelling of  $\mathbf{a}$  :

Stochastic Smagorinsky model (StSm) :

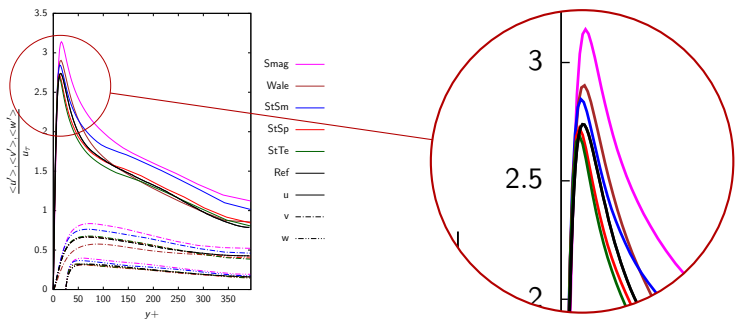
$$\mathbf{a}(\mathbf{x}, t) = C \|\mathbf{S}\| \mathbb{I}_3, \quad (11)$$

Local variance based models (StSp / StTe) :

$$\mathbf{a}(\mathbf{x}, n\delta t) = \frac{1}{|\Gamma| - 1} \sum_{x_i \in \eta(\mathbf{x})} (\mathbf{w}(x_i, n\delta t) - \bar{\mathbf{w}}(\mathbf{x}, n\delta t)) (\mathbf{w}(x_i, n\delta t) - \bar{\mathbf{w}}(\mathbf{x}, n\delta t))^T C_{st}, \quad (12)$$

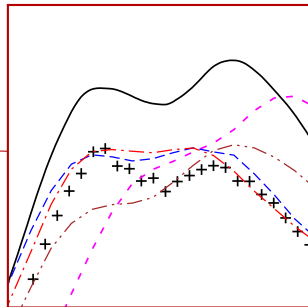
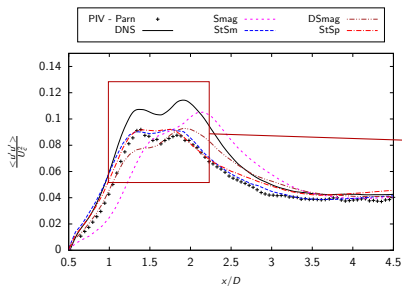


## Channel Flow

Velocity fluctuation profiles for turbulent channel flow at  $Re_\tau = 395$ 

	$n_x \times n_y \times n_z$	$l_x \times l_y \times l_z$	$\Delta x$	$\Delta y$	$\Delta z$	$\Delta t$
cLES	$48 \times 81 \times 48$	$6.28 \times 2 \times 3.14$	0.13	0.005-0.12	0.065	0.002
Ref	$256 \times 257 \times 256$	$6.28 \times 2 \times 3.14$	0.024	0.0077	0.012	-

## Wake Flow



$\langle u'u' \rangle$  profile in the streamwise direction  $u$  on the centre-line behind the cylinder.

	Re	$n_x \times n_y \times n_z$	$l_x/D \times l_y/D \times l_z/D$	$\Delta x/D$	$\Delta y/D$	$\Delta z/D$	$U\Delta t/D$
cLES	3900	241×241×48	20×20×3.14	0.083	0.024-0.289	0.065	0.003
PIV - Parn	3900	160×128×1	3.6×2.9×-	0.023	0.023	-	0.01
DNS	3900	1537×1025×96	20×20×3.14	0.013	0.0056-0.068	0.033	0.00075

# Plan

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  - Code Formulation
  - VDA with LES - Results

# Objective

An efficient method to incorporate Experimental Fluid Dynamics (EFD) as a guiding mechanism within Computational Fluid Dynamics (CFD) (Data Assimilation) - within the realm of Large Eddy Simulations (LES).

# Data Assimilation - Types

- Sequential approach : Cuzol and Mémin (2009), Colburn et al. (2011), Kato and Obayashi (2013), Combes et al. (2015)
- **Variational approach (VDA)** : Papadakis and Mémin (2009), Suzuki et al. (2009), Heitz et al. (2010), Lemke and Sesterhenne (2013), Gronskis et al. (2013), Dovetta et al. (2014)
- Hybrid approach : Yang (2014)

# VDA - General Formulation

**Objective** : Estimate the unknown true state of interest  $\mathbf{x}^t(t, \mathbf{x})$

**Formulation** :

$$\partial_t \mathbf{x}(t, \mathbf{x}) + \mathbb{M}(\mathbf{x}(t, \mathbf{x})) = \mathbf{q}(t, \mathbf{x}), \quad (13)$$

$$\mathbf{x}(t_0, \mathbf{x}) = \mathbf{x}_0^b + \boldsymbol{\eta}(\mathbf{x}) \quad (14)$$

$$\mathcal{Y}(t, \mathbf{x}) = \mathbb{H}(\mathbf{x}(t, \mathbf{x})) + \boldsymbol{\epsilon}(t, \mathbf{x}) \quad (15)$$

$\mathbf{q}(t, \mathbf{x})$  - model error (Covariance matrix  $\mathbf{Q}$ )

$\boldsymbol{\eta}(\mathbf{x})$  - background error (Covariance matrix  $\mathbf{B}$ )

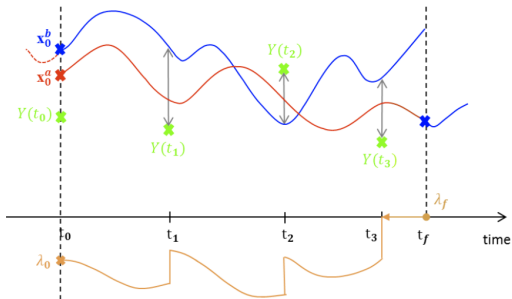
$\boldsymbol{\epsilon}(t, \mathbf{x})$  - observation error (Covariance matrix  $\mathbf{R}$ )

# VDA - General Formulation

Cost Function :

$$J(\mathbf{x}_0) = \frac{1}{2}(\mathbf{x}_0 - \mathbf{x}_0^b)^T \mathbf{B}^{-1}(\mathbf{x}_0 - \mathbf{x}_0^b) + \frac{1}{2} \int_{t_0}^{t_f} (\mathbb{H}(\mathbf{x}_t) - \mathcal{Y}(t))^T \mathbf{R}^{-1}(\mathbb{H}(\mathbf{x}_t) - \mathcal{Y}(t)). \quad (16)$$

## 4DVar - Adjoint Method



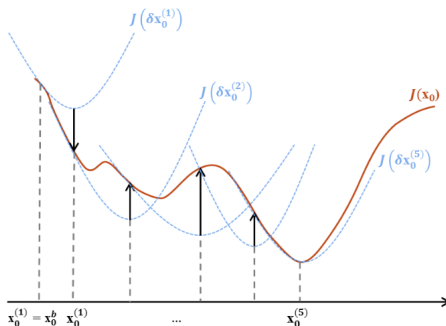
Evolution of the state of interest ( $x_0$ ) as a function of time

$$\frac{\partial J}{\partial \eta} = -\lambda(\mathbf{t}_0) + \mathbf{B}^{-1}(\partial \mathbf{x}(\mathbf{t}_0) - \partial \mathbf{x}_0). \quad (17)$$

Le Dimet, Francois-Xavier, and Olivier Talagrand. "Variational algorithms for analysis and assimilation of meteorological observations : theoretical aspects." *Tellus A : Dynamic Meteorology and Oceanography* 38.2 (1986) : 97-110.



# 4DVar - Incremental Optimisation



Evolution of the cost function ( $J(x_0)$ ) as a function of time

Courtier, P., J. N. Thépaut, and Anthony Hollingsworth. "A strategy for operational implementation of 4D-Var, using an incremental approach." Quarterly Journal of the Royal Meteorological Society 120.519 (1994) : 1367-1387.

# Additional Control

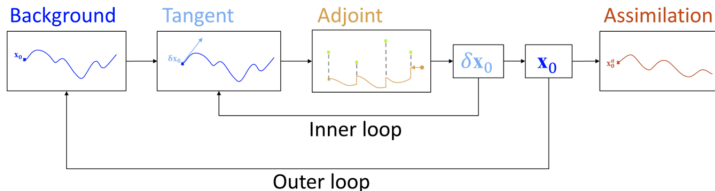
Minimisation of  $J$  with respect to additional incremental control parameter  $\delta u$  :  $\delta\gamma^{(i)} = \{\delta\mathbf{x}_0^{(i)}, \delta\mathbf{u}^{(i)}\}$

$$\begin{aligned} J(\delta\gamma^{(i)}) &= \frac{1}{2} \|\delta\mathbf{x}_0^{(i)} + \mathbf{x}_0^{(i)} - \mathbf{x}_0^{(b)}\|_{\mathbf{B}_{-1}}^2 \\ &+ \frac{1}{2} \int_{t_0}^{t_f} \|\delta\mathbf{u}^{(i)} + \mathbf{u}^{(i)} - \mathbf{u}^{(b)}\|_{\mathbf{B}_c}^2 + \dots \end{aligned} \quad (18)$$

constrained by :

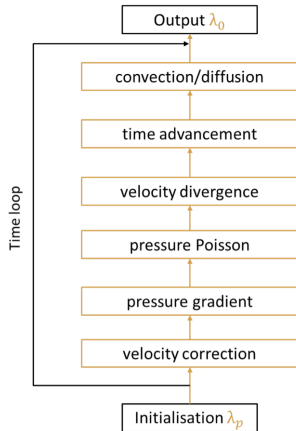
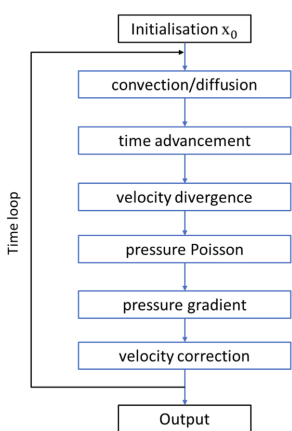
$$\partial_t \delta\mathbf{x}^{(i)} + \partial_{\mathbf{x}} \mathbb{M}(\mathbf{x}^{(i)}) \cdot \partial\mathbf{x}^{(i)} + \partial_{\mathbf{u}} \mathbb{M}(\mathbf{u}^{(i)}) \cdot \partial\mathbf{u}^{(i)} = \mathbf{0} \quad (19)$$

# 4DVar - Flow Chart

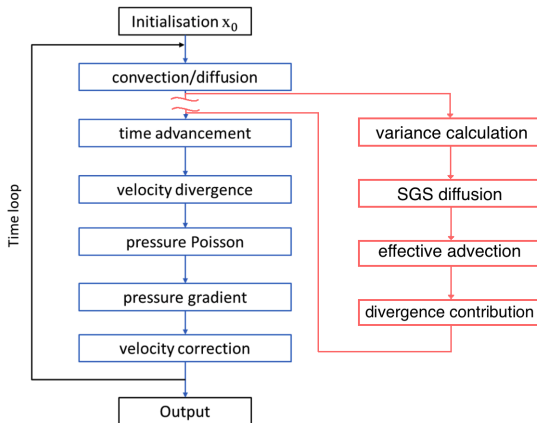


4DVar incremental Data Assimilation using Adjoint methodology.

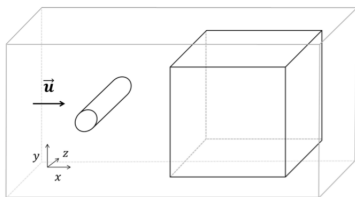
# Incompact3d (Laizet et al. 2010) - DNS



# Incompact3d (Laizet et al. 2010) - LES



## Synthetic Assimilation at Re 3900

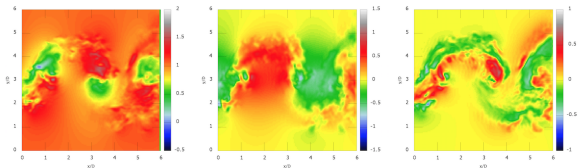


Optimisation Parameter -  $\mathbf{U}(\mathbf{x}, \mathbf{y}, \mathbf{z}, t_0)$   
 Control Parameters -  $\mathbf{U}_{in}(\mathbf{1}, \mathbf{y}, \mathbf{z}, t)$   
 $C_{st}(\mathbf{x}, \mathbf{y}, \mathbf{z})$

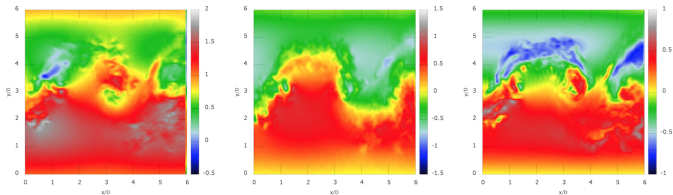
	Re	$n_x \times n_y \times n_z$	$l_x/D \times l_y/D \times l_z/D$	$U\Delta t/D$	Duration
Full Domain	3900	$361 \times 361^s \times 48$	$20 \times 20 \times 3.14$	0.003	$40100\Delta t$
Observation	3900	$145^i \times 145^i \times 48$	$6 \times 6 \times 3.14$	0.003	$100\Delta t$
4DVAR	3900	$145 \times 145 \times 48$	$6 \times 6 \times 3.14$	0.003	$100\Delta t$

s - Stretched ; i - Interpolated

# Background Velocity - Biased

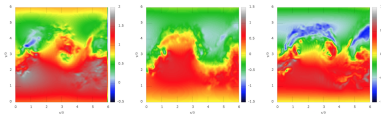


+  $\sin y^3$  noise

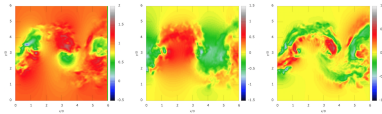


# Assimilated Velocity

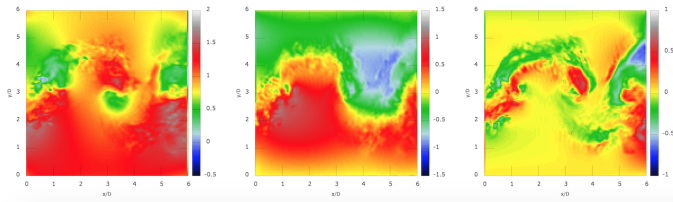
Background



Reference



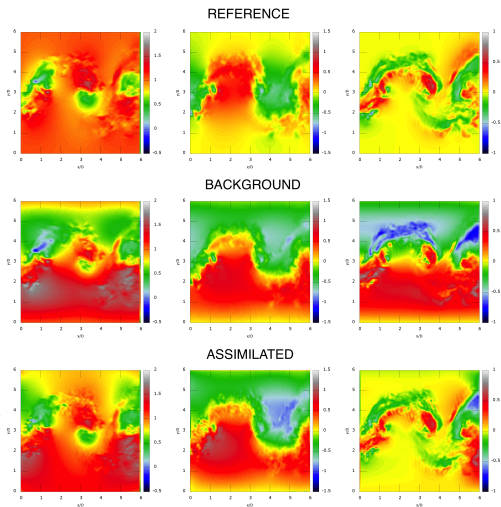
4DVAR Algorithm



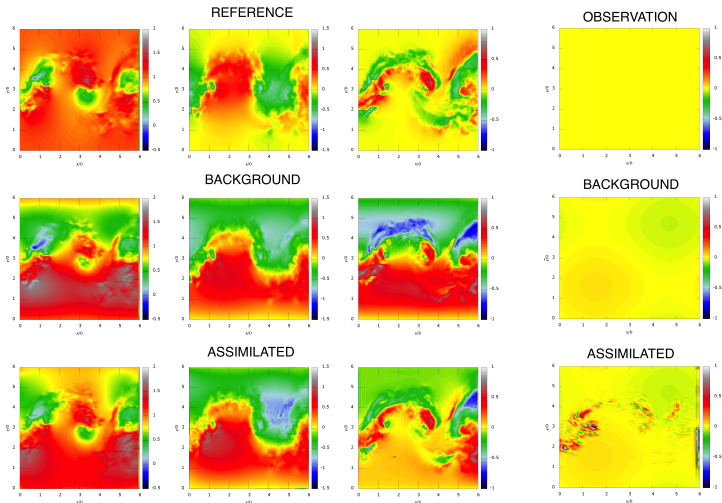
Assimilated



# Collated Results - Initial Condition Correction



# LES Coefficient Estimation - Preliminary Results



# Conclusions

- Efficient cost reduction is possible using LES with VDA
- An improvement on the background is obtained using LES based VDA
- An estimation of the LES coefficient can be obtained using VDA

## Future Work

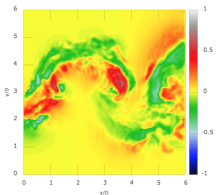
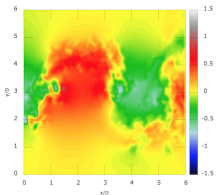
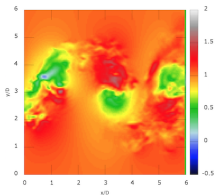
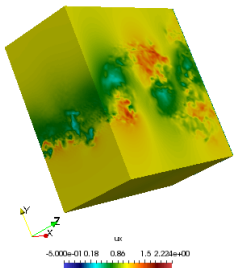
- Perform LES DA with experimental data
- Use sliding windows in VDA
- Include outlet condition in optimisation algorithm
- Assimilate over a larger temporal domain

# References

- P. Chandramouli, E. Memin, D. Heitz, and S. Laizet. Coarse large-eddy simulations in a transitional wake flow with flow models under location uncertainty. *Comput. Fluids*, 168 :170-189, 2018.
- P. Chandramouli, E. Memin, D. Heitz, and S. Laizet. A Comparative Study of LES Models Under Location Uncertainty. In *Proceedings of the Congr es Franais de M canique.*, 2017b.
- P. Chandramouli, E. Memin, and D. Heitz. 4d Turbulent Wake Reconstruction using Large Eddy Simulation based Variational Data Assimilation. Delft, Netherlands, 2017a.
- E. Memin. Fluid flow dynamics under location uncertainty. *Geophys. Astro. Fluid*, 108(2) : 119-146, March 2014.
- A. Gronskis, D. Heitz, and E. Memin. Inflow and initial conditions for direct numerical simulation based on adjoint data assimilation. *J. Comput. Phys.*, 242 :480-497, June 2013.

Thanks for your attention

# Reference - Synthetic Velocity Fields



# Models under Location Uncertainty

Modelling of  $\mathbf{a}$  :

Stochastic Smagorinsky model (StSm) :

$$\mathbf{a}(\mathbf{x}, t) = C \|\mathbf{S}\| \mathbb{I}_3, \quad (20)$$

Local variance based models (StSp / StTe) :

$$\mathbf{a}(\mathbf{x}, n\delta t) = \frac{1}{|\Gamma| - 1} \sum_{x_i \in \eta(\mathbf{x})} (\mathbf{w}(x_i, n\delta t) - \bar{\mathbf{w}}(\mathbf{x}, n\delta t)) (\mathbf{w}(x_i, n\delta t) - \bar{\mathbf{w}}(\mathbf{x}, n\delta t))^T C_{sp}, \quad (21)$$

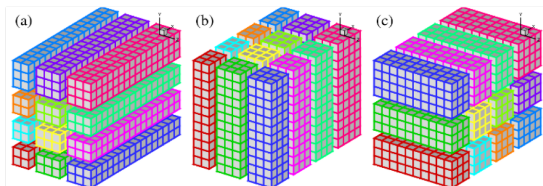


# Plan

3 Channel Flow

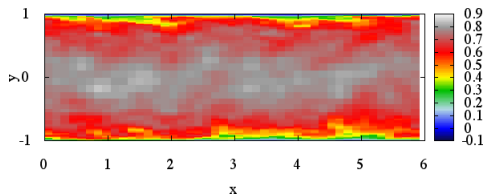
4 Wake Flow

# Flow Solver - Incompact3d



- Fortran based parallelised flow solver - 2Decomp & FFT (Laizet S. & Lamballais E. (2009))
- Simulates the incompressible Navier Stokes equations
- Spectral pressure treatment with collocated or staggered grid formulation

## Flow Specifications

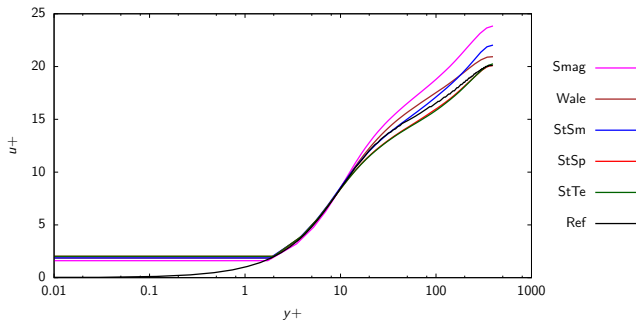


Instantaneous streamwise velocity contour

	$n_x \times n_y \times n_z$	$l_x \times l_y \times l_z$	$\Delta x$	$\Delta y$	$\Delta z$	$\Delta t$
cLES	$48 \times 81 \times 48$	$6.28 \times 2 \times 3.14$	0.13	0.005-0.12	0.065	0.002
Ref	$256 \times 257 \times 256$	$6.28 \times 2 \times 3.14$	0.024	0.0077	0.012	-

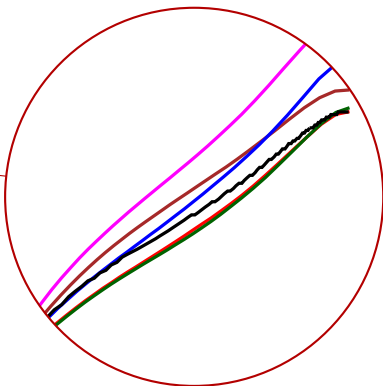
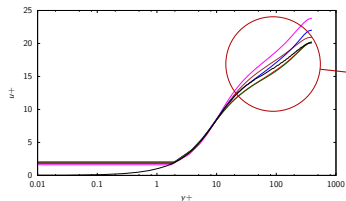
# Channel Flow Statistics

Mean Velocity Profile :

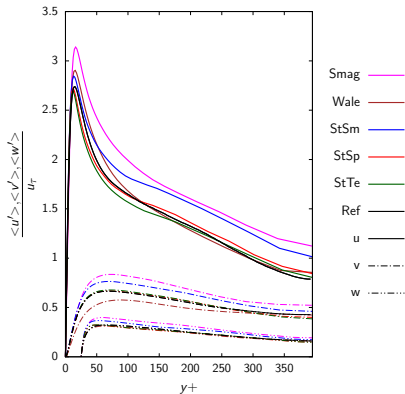


Mean velocity profile for turbulent channel flow at  $Re_\tau = 395$

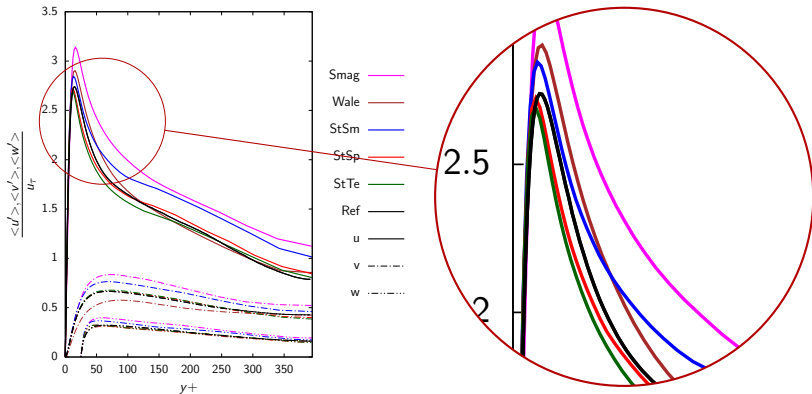
# Channel Flow



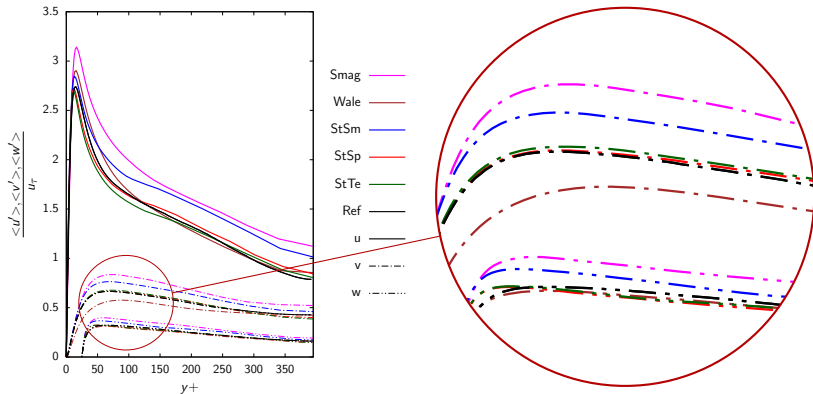
## Channel Flow

Velocity fluctuation profiles for turbulent channel flow at  $Re_\tau = 395$

# Channel Flow



# Channel Flow



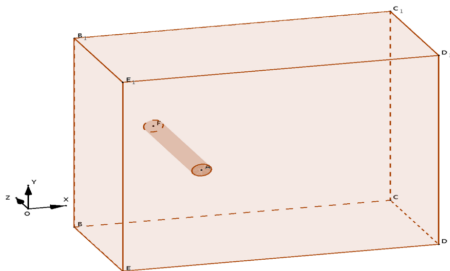


# Plan

3 Channel Flow

4 Wake Flow

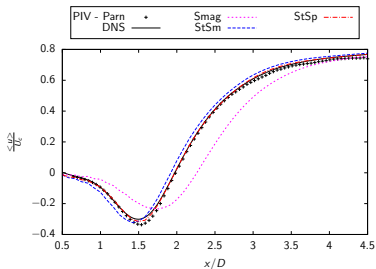
## Flow Parameters



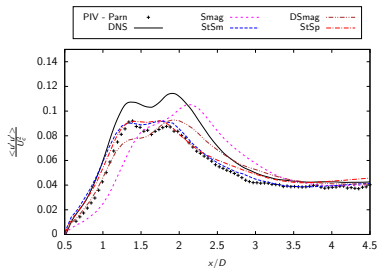
	Re	$n_x \times n_y \times n_z$	$l_x/D \times l_y/D \times l_z/D$	$\Delta x/D$	$\Delta y/D$	$\Delta z/D$	$U\Delta t/D$
cLES	3900	241×241×48	20×20×3.14	0.083	0.024-0.289	0.065	0.003
PIV - Parn	3900	160×128×1	3.6×2.9×-	0.023	0.023	-	0.01
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DNS	3900	1537×1025×96	20×20×3.14	0.013	0.0056-0.068	0.033	0.00075

## Flow Statistics

Centerline statistical comparison :

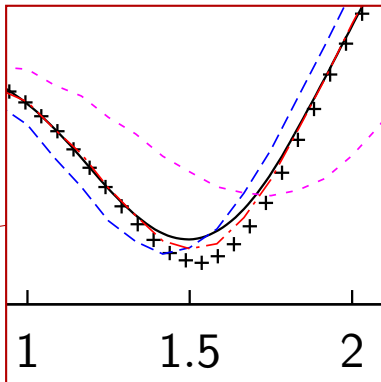
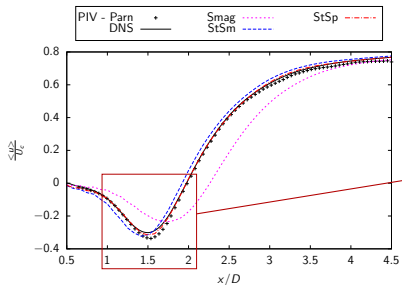


Mean streamwise velocity in the streamwise direction along the centerline behind the cylinder.

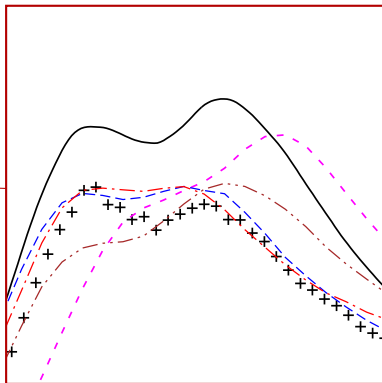
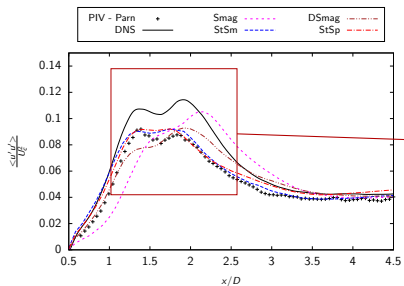


Fluctuating streamwise velocity in the streamwise direction along the centerline behind the cylinder.

## Flow Statistics



## Flow Statistics



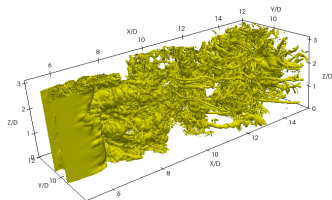
# Vorticity Structures

Smagorinsky

Stochastic Smagorinsky

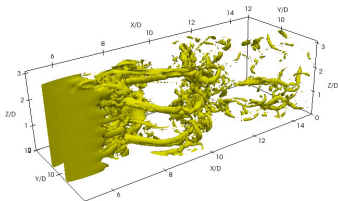
Stochastic Spatial

DNS

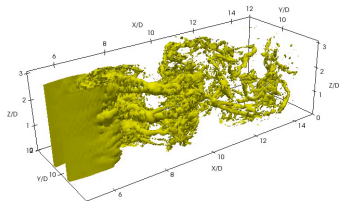


# Vorticity Structures

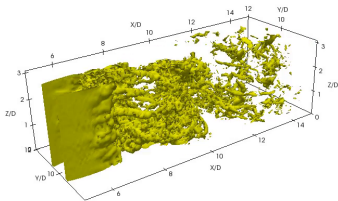
Smagorinsky



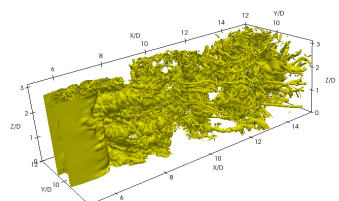
Stochastic Smagorinsky



Stochastic Spatial



DNS



# Simulation Cost

Model	w/o model	Smag	StSm	StSp
Computational Cost [s/iteration]	1.34	2.877	3.654	5.711

TABLE – Computational cost [s] per iteration for each model.

Cost of StSp under coarse resolution = **0.46%** cost of DNS



## 4D-Var with Incompact3d - DNS

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**Algorithm 27** Incompact3d\_b ( $\mathbf{u}^0, \bar{\mathbf{u}}^0$ )
 

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**Initialization:**  $k = N, \mathbf{u}^0, \bar{\mathbf{u}}^N = 0, \bar{F}^{N-1} = 0, \bar{F}^{N-2} = 0$ 
**while**  $k > 0$  **do**

 corgp\_b( $\mathbf{u}^k, \bar{\mathbf{u}}^k, \nabla p, \nabla \bar{p}$ )

 gradp\_b( $\nabla p, \nabla \bar{p}, p, \bar{p}$ )

 poisson\_b( $p, \bar{p}, \mathbf{u}^{**}, \bar{\mathbf{u}}^{**}$ )

 pre\_correc\_b( $p, \bar{p}$ )

 intt\_b( $\mathbf{u}^*, \bar{\mathbf{u}}^*, F^{k-1}, \bar{F}^{k-1}, F^{k-2}, \bar{F}^{k-2}$ )

 convdiff\_b( $\mathbf{u}^{k-1}, \bar{\mathbf{u}}^{k-1}, F^{k-1}, \bar{F}^{k-1}, F^{k-2}, \bar{F}^{k-2}$ )

 $k = k - 1$ 
**end while**


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