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CASCADE: CHANNEL-AWARE STRUCTURED COSPARSE AUDIO DECLIPPER

Clément Gaultier*, Nancy Bertin*, Rémi Gribonval*

* Univ Rennes, Inria, CNRS, IRISA, France

ABSTRACT
This work features a new algorithm, CASCADE, which leverages a structured cosparse prior across channels to address the multichannel audio declipping problem. CASCADE technique outperforms the state-of-the-art method A-SPADE applied on each channel separately in all tested settings, while retaining similar runtime.

Index Terms — declipping, multichannel, structured sparsity, cosparse

1. INTRODUCTION
Clipping, also known as saturation, is a common phenomenon that can arise from hardware or software limitations in any audio acquisition pipeline. It results in severely distorted audio recordings. Declipping consists in performing the inverse process, in order to restore saturated audio signals and improve their quality.

1.1. State-of-the-art in single-channel declipping
While we can trace back some attempts to address this issue, e.g. with autoregressive models [1], to several decades, significant progress towards efficient desaturation was recently made in several directions. First, the declipping problem was recast as an undetermined, linear inverse problem, akin to inpainting, which could be addressed by means of a sparse regularization [2]. On this basis, algorithmic frameworks evolved from usual greedy algorithms to thresholding [3] then to non-convex approaches [4]. In parallel, a switch from a (now) traditional sparse synthesis approach, to a sparse analysis (also known as cosparse model [5]) was proposed, as well as some model refinements exploiting notions of structured sparsity, especially that of social sparsity in the time-frequency domain [6]. These layers, in line of which the current paper is written, led to significant improvements in reconstruction accuracy and computational efficiency.

1.2. Multichannel declipping
However, it must be noted that all these methods were developed and tested for mono signals, while multichannel data now represent a large part of available audio content, from stereo to more and more channels. Intuitively, we expect that a joint processing of all channels could be more efficient than declipping independently each channel with the previous single-channel algorithms. To date, the multichannel joint declipping problem has only been addressed by [7] through a modeling of the signals as mixtures of sound sources, in order to encompass inter-channel correlations. This approach requires prior knowledge or estimation of the mixing process.

In this work, we propose a blinder approach to joint declipping of multichannel audio from compact antennas, which operates purely at the signal level and does not require any kind of spatial information (including the microphone positions). Our method is based on the aforementioned ideas, namely a cosparse model of data, with the original addition of a structured sparsity prior across channels which allows to take implicitly into account the spatial correlation.

1.3. Structured sparsity
In the field of sparse representations and techniques, the notion of structure which is basically the idea that the nonzero coefficients of expectedly sparse quantities may not be “indifferently” distributed, is manifold. It has given rise to various definitions and developments, all of which were initially defined in the context of sparse synthesis, but can all be straightforwardly extended to the sparse analysis point of view.

• Joint or simultaneous sparsity. Several vectors are gathered and assumed to admit a sparse decomposition on the same dictionary, and the sparse decomposition can be jointly performed [8] [9].

• Group sparsity. The index set of the considered sparse vector is partitioned into non-overlapping groups, and the signal is assumed to be sparse at the group level but not within an active group. This prior is typically enforced by mixed-norms such as the $\ell_{1,2}$-norm [10].

• Social sparsity. The previous structure is extended to the case of possibly overlapping groups or neighborhoods enforced by the use of a Persistent Empirical Wiener shrinkage [11].

1.4. Contributions and outline
In this paper, we propose an original multichannel declipping method based on a twofold prior: simultaneous cosparse of all channels, and group sparsity across channels. Section 2 introduces our notations and models. The resulting algorithm, which outperforms the naive channel-by-channel declipping strategy while keeping computation time in the same order of magnitude, is presented in Section 3 and experimentally validated on real eight-channel audio data in Section 4. We conclude in Section 5.

2. NOTATIONS AND MODEL

2.1. Notations
We observe a time-domain multichannel clipped audio signal composed of $K$ channels, $Y_n \in \mathbb{R}^{n \times K}$ the $n^{th}$ windowed frame that signal and its clean version $X_n$. We define $Z_n \simeq AX_n$ a frequency representation of $X_n$ such that $Z_n \in \mathbb{C}^{p \times K}$ and $A \in \mathbb{C}^{p \times n}$, where $A$ performs an analysis frequency transform (possibly redundant when

1State-of-the-art results can be appreciated for instance from the SPADE software webpage: [https://spade.inria.fr/](https://spade.inria.fr/)
We use the forward hard-clipping degradation model for the involved quantities. Let $Z \in \mathbb{C}^{P \times K}$ be a local multichannel frequency representation to sparsify. Let $p_k$ be coordinates of such a point and $z_{pk} \in \mathbb{C}^{1 \times K}$ the $p$-th row from matrix $Z$ (corresponding to a group, as illustrated on Fig. 1). GEW is defined as:

$$S_p(Z)_{jk} = Z_{pk} \cdot \left(1 - \mu^2 \left\|Z_{jk}\right\|_2^2\right)_+,$$

with $(\cdot)_+ = \max(\cdot, 0)$ the positive part and $\mu$ the parameter controlling the amount of shrinkage to apply. This shrinkage explicitly promotes group sparsity of $Z$ along the channel dimension.

### 3.2. Projection on the declipping constraint

For the data fidelity constraint, we define $\Omega_r$ the set of reliable samples indices $jk$ in $Y_n$. We also define respectively $\Omega_+$ and $\Omega_-$ the sets of clipped positive and clipped negative samples indices. We note that in the case of hard-clipping, the sets are easily retrieved comparing each sample to the clipping level. The notation $V_{\Omega}$ denotes the matrix formed by considering only those indices of $V$ indexed by $\Omega$ while $\prec, \succ, \preceq, \succeq$ are used for entry-wise comparisons between matrices. The data fidelity projection is the solution of the following optimization problem:

$$\min_{\hat{X} \in \Theta} \left\|AX - Z\right\|_F^2,$$

with $\Theta$ the magnitude constraint convex set expressed as:

$$\Theta = \left\{X \mid X_{\Omega_+} = Y_{\Omega_+}; X_{\Omega_-} \succ Y_{\Omega_-}; X_{\Omega_-} \prec Y_{\Omega_-}\right\}.$$

From (4), we can rewrite the solution of (3) as the following component-wise magnitude constraints:

$$\hat{X}_{jk} = \begin{cases} Y_{jk} & \text{ if } jk \in \Omega_+; \\ (A^H Z)_{jk} & \text{ if } jk \in \Omega_+; (A^H Z)_{jk} \geq \tau; \\ \text{sgn}(Y_{jk})\tau_k & \text{ if } jk \in \Omega_-; (A^H Z)_{jk} \leq -\tau_c; \\ \text{sgn}(Y_{jk})\tau_k & \text{ otherwise.} \end{cases}$$

### 3.3. Overall functioning for the algorithm

As for A-SPADE [4], the algorithm is built on the Alternating Direction Method of Multipliers (ADMM) numerical scheme [13]. Pseudo code in Algorithm 1 presents the functioning of the CASCADE algorithm for a given frame $Y \in \mathbb{R}^{J \times K}$ with $U \in \mathbb{C}^{J \times K}$ the ADMM dual variable. In Algorithm 1, the parameter $\alpha$ is of great importance as it tells the procedure how aggressively to perform the sparsification step. This value is updated along with the iterations following a geometric progression of common ratio $\alpha (0 < \alpha < 1)$. This way the algorithm relaxes the sparsity constraint while it progresses. Typical values for $\mu(0)$ and $\alpha$ are given in section 4.1. The procedure is applied in a frame-based manner and outputs the $X_n$ declipped estimates, which are used to rebuild the full length estimated signal by means of overlap-and-add synthesis.

### 4. EXPERIMENTS

We perform experiments on 8 channels recordings excerpts from the VoiceHome2 Corpus [14]. We use the 359 clean speech available at [http://voice-home.gforge.inria.fr/voiceHome2_corpus.html](http://voice-home.gforge.inria.fr/voiceHome2_corpus.html)

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### 2.2. Clipping model

We use the forward hard-clipping degradation model for $Y_n$:

$$y_{jk} = \begin{cases} x_{jk} & \text{for } |x_{jk}| \leq \tau_n; \\ \text{sgn}(x_{jk})\tau_k & \text{otherwise;} \end{cases}$$

with $y_{jk}$ (resp. $x_{jk}$) the $j$th sample recorded on the $k$th channel from $Y_n$ (resp. $X_n$) and $\tau_n$ the hard-clipping level in the $k$th channel.

### 2.3. Channel-aware structured cosparse modeling

The main model characteristics for this work derive from the relation between $Z_n$ and $X_n$ as well as properties of $Z_n$ which are: i) $A \in \mathbb{C}^{P \times X}; P \geq J$; ii) $Z_n \simeq AX_n, Z_n \in \mathbb{C}^{P \times K};$ iii) $\left\|Z_n\right\|_0 \ll P \times K$; iv) $Z_n$ is "structured across channels". The underlying hypothesis behind the structure (group sparsity) in the frequency representation $Z_n$ is that non zero coefficients are roughly distributed equivalently from one channel to another. This way we encompass a channel-aware structured sparse prior on $Z_n$ and a cosparse prior on $X_n$, so the name of the proposed algorithm: "Channel-Aware Structured Cosparse Audio DEclipper (CASCADE)".

### 3. MULTICHANNEL DECLIPPING ALGORITHM

The goal of the algorithm is to simultaneously declip each channel in the observation $Y_n$ to output an estimate $X_n$ which satisfies: i) the channel-aware structured cosparse modeling constraint, ii) the data fidelity constraint regarding the clipped $Y_n$. For that we use an iterative algorithm which alternatively projects the solution on the modeling and the declipping constraints. Projection on the modeling constraint is achieved using the Group Empirical Wiener (GEW) operator presented below.

### 3.1. Sparserisying operator

For clarity, in the following we remove the time-frame index $n$ subscript and consider $X, Y, Z$, matrices of size $(J \times K)$ or $(P \times K)$ corresponding to the $n$th time-frame. To handle structured cosparse constraints, we use the GEW operator as in [12] as a sparsifying step in the declipping procedure. Let $Z \in \mathbb{C}^{P \times K}$ be a local multichannel frequency representation to sparsify. Let $p_k$ be coordinates of such a point and $z_{pk} \in \mathbb{C}^{1 \times K}$ the $p$-th row from matrix $Z$ (corresponding to a group, as illustrated on Fig. 1). GEW is defined as:

$$S_p(Z)_{jk} = Z_{pk} \cdot \left(1 - \mu^2 \left\|Z_{jk}\right\|_2^2\right)_+,$$

with $(\cdot)_+ = \max(\cdot, 0)$ the positive part and $\mu$ the parameter controlling the amount of shrinkage to apply. This shrinkage explicitly promotes group sparsity of $Z$ along the channel dimension.

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Fig. 1. Schematic representation of the data. In a given channel $k$, the extracted layer is a time-frequency representation of this channel. At a given time frame $n$, the channel-frequency layer $Z_n$ is group sparse in the channel dimension.

P > J, $P$ is the number of frequency bins used for the frequency representation, $J$ is the number of time-domain samples in a frame. We consider $P = RJ$ with $R$ the redundancy factor of $A$. Lastly, $K$ is the number of channels. Fig. 1 displays a schematic representation of the involved quantities.

**Pseudo code in Algorithm 1 presents the functioning of the CASCADE algorithm for a given frame $Y \in \mathbb{R}^{J \times K}$ with $U \in \mathbb{C}^{J \times K}$ the ADMM dual variable. In Algorithm 1, the parameter $\alpha$ is of great importance as it tells the procedure how aggressively to perform the sparsification step. This value is updated along with the iterations following a geometric progression of common ratio $\alpha (0 < \alpha < 1)$. This way the algorithm relaxes the sparsity constraint while it progresses. Typical values for $\mu(0)$ and $\alpha$ are given in section 4.1. The procedure is applied in a frame-based manner and outputs the $X_n$ declipped estimates, which are used to rebuild the full length estimated signal by means of overlap-and-add synthesis.**
Table 1. CASCADE parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Window size [samples]</th>
<th>Overlap [%]</th>
<th>Window Type</th>
<th>Channel number</th>
<th>Maximum iterations</th>
<th>Accuracy</th>
<th>Analysis operator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>J = 1024</td>
<td>75</td>
<td>Hamming</td>
<td>K = 8</td>
<td>$i_{\text{max}} = 10^4$</td>
<td>$\beta = 10^{-3}$</td>
<td>A = DFT</td>
</tr>
</tbody>
</table>

![Fig. 2. Speech declipping numerical results: SDR improvement [dB]](image1)

![Fig. 3. Speech & Music declipping numerical results: SDR improvement [dB]](image2)
EXAMPLE 1: CASCADE algorithm

**Require:** $A$, $Y$, $\Gamma$, $\alpha$, $\mu(0)$, $\Theta$, $\beta$, $i_{\text{max}}$

**Initialization step:**

$$X^{(0)} = Y, U^{(0)} = 0, i = 1$$

**Projection step on the modeling constraint:**

$$Z^{(i)} = S_{\mu(i-1)}(AX^{(i-1)} + U^{(i-1)})$$

**Projection step on declipping constraint:**

$$X^{(i)} = \arg\min_{X \in \Theta} \|AX - Z^{(i)} + U^{(i-1)}\|_{F}^{2}$$

subject to $X \in \Theta$

**Update step:**

$$\mu^{(i)} = \alpha \mu^{(i-1)}$$

if $\|AX^{(i)} - Z^{(i)}\|_{F} \leq \beta$ or $i \geq i_{\text{max}}$ then
terminate

else

$$U^{(i)} = U^{(i-1)} + A\hat{X}^{(i)} - Z^{(i)}$$

$$i \leftarrow i + 1$$

end if

**return** $\hat{X}^{(i)}$

4.1. Quality improvements

SDR improvement results are presentend in Fig. 2 (for speech only subset) and Fig. 3 (for mixed speech and music). We observe that the CASCADE method outperforms the A-SPADE algorithm by 1 dB to more than 3 dB in all settings. The improvement brought by CASCADE over A-SPADE is even more salient on mixed speech and music data (which is the most difficult subset, with a globally lower performance for both methods, compared to that obtained on speech only data.)

The effect of a redundant DFT transform ($R=2$) appears to be slightly different for each method. We note that except for Fig. 3a twice redundant DFT provides at least as good results than non redundant DFT for A-SPADE. For the CASCADE method, redundant DFT is profitable for mild to high input SDR (15 dB to 25 dB), but detrimental at low input SDR (high saturation).

4.2. Computational Aspects

As DFT can be efficiently implemented with a fast transform, the computational cost of the declipping procedure mainly stems from the sparsifying step and the projection on the declipping constraint.

TABLE 2 Runtime tests numerical results

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>CASCADE</th>
<th>A-SPADE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Redundancy</td>
<td>$R=1$</td>
<td>$R=2$</td>
</tr>
<tr>
<td>5</td>
<td>167</td>
<td>398</td>
</tr>
<tr>
<td>10</td>
<td>120</td>
<td>265</td>
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<tr>
<td>25</td>
<td>37</td>
<td>78</td>
</tr>
</tbody>
</table>

(a) Runtime performance (ratio to realtime processing)

For this runtime comparisons, we choose a subset of 25 excerpts (totalizing 3 minutes of audio) from the dataset and compare the computing time of the CASCADE and the A-SPADE algorithms. The runtime tests are performed on workstations running the Matlab<sup>®</sup> associated code in single-thread mode. The computers are equipped with Intel<sup>®</sup>Xeon<sup>®</sup> CPU 5140 @2.33 GHz with 2 GB available ram memory. Table 2 shows runtime performances and Table 3b the corresponding SDR improvements ($\Delta$SDR) averaged on the eight channels and the 25 excerpts. We clearly note higher computing times for both methods with twice redundant DFT. The A-SPADE method is 2.4 to 2.6 times faster than the non redundant case while CASCADE is between 2.1 to 2.4 faster with this setting. We observe that substantial improvements given by the CASCADE algorithm are achieved at the cost of only slightly lower computational efficiency (1.5 to 2.2 times slower than A-SPADE). These different computation time characteristics might come from the properties of the sparsifying operator when used inside the ADMM framework and the total number of iterations needed to finish or converge.

5. CONCLUSION

We introduced a new algorithm combining a cosparse prior with structure in the frequency-channel domain to address the multichannel audio declipping problem. We showed that adding across-channel structure on top of cosparse modeling was bringing considerable reconstruction improvements compared to a cosparisity-based state-of-the-art method applied channel-wise. In addition, we showed that performance can be improved by the use of a redundant frequency transform when the clipping level is moderate. Finally, we demonstrated that the method implies a very limited runtime overcost. Future studies could include perceptual assessments, and model integration of time-frequency structures on top of structured cosparisity across channels.

6. ACKNOWLEDGMENTS

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7. REFERENCES


