



## Trefftz-DG Approximation for the Elasto-Acoustics

Henri Calandra, Elvira Shishenina, Julien Diaz, Hélène Barucq

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Depth Imaging Partnership



<sup>1</sup> Magique-3D, Inria Bordeaux - Sud-Ouest | <sup>2</sup> Total S.A.

# TREFFTZ-DG APPROXIMATION FOR THE ELASTO-AcouSTICS

WAVES17 | 13th International Conference on Mathematical and Numerical Aspects of Wave Propagation

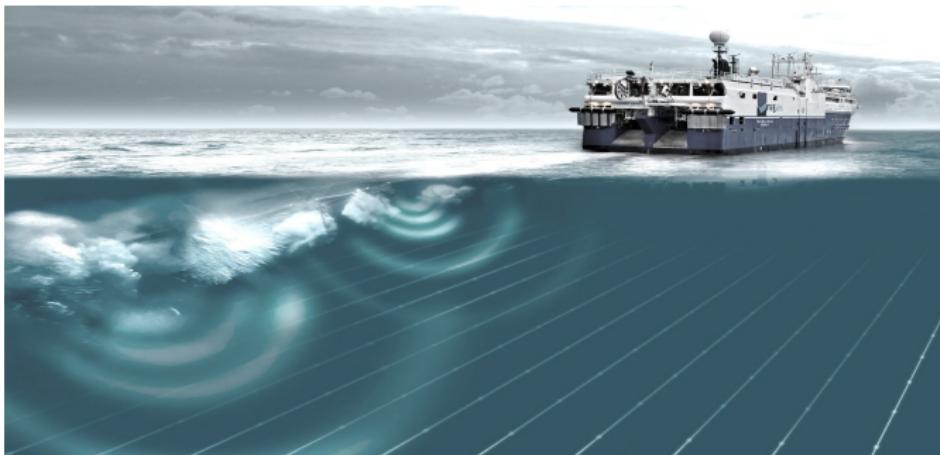
Minneapolis | 2017

# ABSTRACT

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# SEISMIC SURVEY

FIGURE 1: Ocean Bottom Seismic (OBS) data acquisition



# BASIC NUMERICAL METHODS

TABLE 1: Generic properties of the most widely used numerical methods\*

Numerical method	Complex geometries	High-order accuracy and $hp$ -adaptivity	Explicit semi-discrete form	Conservation laws	Elliptic problems
FDM	●	●	●	●	●
FVM					
FEM					
DG-FEM					

\* J.S.Hesthaven T.Warburton. *Nodal DG methods. Algorithms, analysis, and applications.* 2007

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FEM	●	●	●	○	●
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FVM	●	●	●	●	○
FEM	●	●	●	○	●
DG-FEM	●	●	●	●	○

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# DISCONTINUOUS GALERKIN METHODS

## PROS AND CONS



- Adapted to the complex geometries
- High-order accuracy and hp-adaptivity
- Explicit semi-discrete form
- Conservation laws

# DISCONTINUOUS GALERKIN METHODS

## PROS AND CONS



- Adapted to the complex geometries
- High-order accuracy and hp-adaptivity
- Explicit semi-discrete form
- Conservation laws



- Higher number of degrees of freedom  
comparing to the methods with continuous approximation

# TREFFTZ METHOD

## DEFINITION \*

Given a region of an Euclidean space or some partitions of that region,

a **Trefftz method** is any procedure for solving boundary value problems of PDE or systems of PDE, on such region,

**using solutions of that PDE or its adjoint**, defined in its subregions.

\* I.Herrera. *Trefftz method: a general theory*. 2000

# DISCONTINUOUS GALERKIN METHODS

## TIME-HARMONIC FORMULATIONS

O.Cassenat B.Despres. 1998

T.Huttunen P.Monk J.P.Kaipo. 2002

C.Farhat I.Harari U.Hetmaniuk. 2003

R.Tezaur, C.Farhat. 2006

G.Gabard. 2007

## TIME-DOMAIN FORMULATIONS

H.Egger F.Kretzschmar S.M.Schnepp T.Weiland. 2014

F.Kretzschmar S.M.Schnepp I.Tsukerman T.Weiland. 2014

F.Kretzschmar A.Moiola I.Perugia. 2015

# TREFFTZ METHOD

## EXPECTED ADVANTAGES \*



- Better order of convergence
- Flexibility in the choice of basis functions
- Low dispersion
- Adaptivity and local space-time mesh refinement

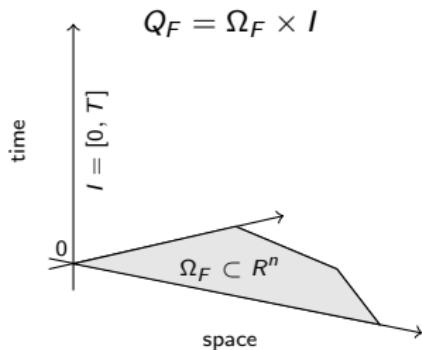
\* I.Herrera. *Trefftz method: a general theory*. 2000

# MATHEMATICAL FORMULATION

## ACOUSTIC SYSTEM

# ACOUSTIC SYSTEM

## PROBLEM EQUATIONS

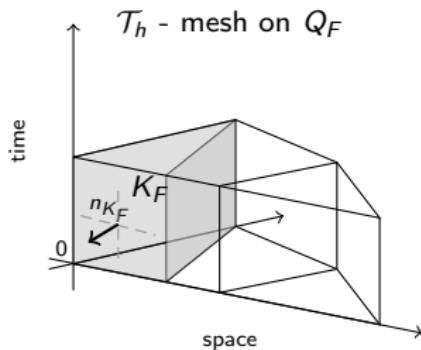


(AS)

$$\begin{cases} \frac{1}{c_F^2 \rho_F} \frac{\partial p}{\partial t} + \operatorname{div} v_F = f, & \text{in } Q_F \\ \rho_F \frac{\partial v_F}{\partial t} + \nabla p = 0, & \text{in } Q_F \\ v_F(\cdot, 0) = v_{F0}, \quad p(\cdot, 0) = p_0, & \text{on } \Omega_F \\ v_F = g_F^D, & \text{on } \partial\Omega_F \times I \end{cases}$$

# ACOUSTIC SYSTEM

## MESHING



$\mathcal{F}_h = \cup_{K_F \in \mathcal{T}_h} \partial K_F$  - mesh skeleton  
 $K_F \in Q_F$  ( $c_F, \rho_F \equiv \text{const}$  in  $K_F$ )

$$\begin{aligned} v_F, p &\in H^1(K_F) \\ \omega_F, q &\in H^1(K_F) \end{aligned}$$

# ACOUSTIC SYSTEM

TREFFTZ-DG FORMULATION IN THE ELEMENT  $K_F$

AS

$(v_F, p)$

# ACOUSTIC SYSTEM

TREFFTZ-DG FORMULATION IN THE ELEMENT  $K_F$



# ACOUSTIC SYSTEM

TREFFTZ-DG FORMULATION IN THE ELEMENT  $K_F$

$$\boxed{\text{AS}}_{(v_F, p)} \times \boxed{\text{test functions}}_{(\omega_F, q)} =$$

SPACE-TIME INTEGRATION ON  $K_F$

# ACOUSTIC SYSTEM

## TREFFTZ-DG FORMULATION IN THE ELEMENT $K_F$

$$\boxed{\text{AS} \quad (\nu_F, p)} \times \boxed{\text{test functions} \quad (\omega_F, q)} = \boxed{\text{VOLUME INTEGRATION TERM}} \\ \boxed{\text{AS} \quad \times (\nu_F, p) \\ (\text{test functions})}$$

SPACE-TIME INTEGRATION ON  $K_F$

# ACOUSTIC SYSTEM

TREFFTZ-DG FORMULATION IN THE ELEMENT  $K_F$

$$\text{AS} \quad (\nu_F, p) \quad \times \quad \begin{matrix} \text{test} \\ \text{functions} \\ (\omega_F, q) \end{matrix} = \quad \begin{matrix} \text{VOLUME INTEGRATION TERM} \\ \text{AS} \quad \times (\nu_F, p) \\ (\text{test functions}) \end{matrix} + \quad \begin{matrix} \text{SURFACE INTEGRATION TERM} \\ (\text{test functions}) \\ (\nu_F, p) \end{matrix}$$

SPACE-TIME INTEGRATION ON  $K_F$

# ACOUSTIC SYSTEM

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SPACE-TIME INTEGRATION ON  $K_F$

# ACOUSTIC SYSTEM

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$$\text{AS} \quad (\nu_F, p) \quad \times \quad \begin{matrix} \text{test} \\ \text{functions} \\ (\omega_F, q) \end{matrix} \quad = \quad \begin{matrix} \text{VOLUME INTEGRATION TERM} \\ \text{test} \\ \text{functions} \quad \text{in} \quad T \\ + \\ \text{SURFACE INTEGRATION TERM} \\ (\text{test functions}) \\ (\nu_F, p) \end{matrix}$$

SPACE-TIME INTEGRATION ON  $K_F$

# ACOUSTIC SYSTEM

TREFFTZ-DG FORMULATION IN THE ELEMENT  $K_F$

$$\text{AS} \quad (\nu_F, p) \quad \times \quad \begin{matrix} \text{test} \\ \text{functions} \\ (\omega_F, q) \end{matrix} \quad = \quad \begin{matrix} \text{VOLUME INTEGRATION TERM} \\ = 0 \end{matrix} + \begin{matrix} \text{SURFACE INTEGRATION TERM} \\ (\text{test functions}) \\ (\nu_F, p) \end{matrix}$$

SPACE-TIME INTEGRATION ON  $K_F$

# ACOUSTIC SYSTEM

TREFFTZ-DG FORMULATION IN THE ELEMENT  $K_F$

$$\begin{array}{c} \text{AS} \\ (\nu_F, p) \end{array} \times \begin{array}{c} \text{test} \\ \text{functions} \\ (\omega_F, q) \end{array} = \begin{array}{c} \text{SURFACE INTEGRATION TERM} \\ \\ \text{(test functions)} \\ (\nu_F, p) \end{array}$$

SPACE-TIME INTEGRATION ON  $K_F$

# ACOUSTIC SYSTEM

## TREFFTZ-DG FORMULATION

$$\underbrace{\text{AS} \quad (\nu_F, p)}_{\text{SPACE-TIME INTEGRATION ON } K_F} \times \underbrace{\text{test functions} \quad (\omega_F, q)}_{\text{SUM OVER ALL } K_F} = \underbrace{\text{(test functions)} \quad (\nu_F, p)}_{\text{SURFACE INTEGRATION TERM}} \underbrace{\text{SUM OVER ALL } \partial K_F}_{}$$

# ACOUSTIC SYSTEM

## SPACE-TIME DISCRETIZATION

$$\begin{aligned} & - \sum_{K_F} \int_{K_F} \left[ p_h \left( \frac{1}{c_F^2 \rho_F} \frac{\partial q}{\partial t} + \operatorname{div} \omega_F \right) + v_F h \cdot (\rho_F \frac{\partial \omega_F}{\partial t} + \nabla q) \right] dv \\ & + \sum_{K_F} \int_{\partial K_F} \left[ \left( \frac{1}{c_F^2 \rho_F} \hat{p}_h q + \rho_F v_F \hat{v}_h \cdot \omega_F \right) n_{K_F}^t + (\hat{p}_h \omega_F + v_F \hat{v}_h q) \cdot n_{K_F}^x \right] ds = \sum_{K_F} \int_{K_F} f q dv \end{aligned}$$

# ACOUSTIC SYSTEM

## SPACE-TIME DISCRETIZATION

$$\begin{aligned} & - \sum_{K_F} \int_{K_F} \left[ p_h \left( \frac{1}{c_F^2 \rho_F} \frac{\partial q}{\partial t} + \operatorname{div} \omega_F \right) + v_F h \cdot (\rho_F \frac{\partial \omega_F}{\partial t} + \nabla q) \right] dv \\ & + \sum_{K_F} \int_{\partial K_F} \left[ \left( \frac{1}{c_F^2 \rho_F} \hat{p}_h q + \rho_F v_F \hat{v}_h \cdot \omega_F \right) n_{K_F}^t + (\hat{p}_h \omega_F + v_F \hat{v}_h q) \cdot n_{K_F}^x \right] ds = \sum_{K_F} \int_{K_F} f q dv \end{aligned}$$

# ACOUSTIC SYSTEM

## SPACE-TIME DISCRETIZATION

$$\begin{aligned}
 & - \sum_{K_F} \int_{K_F} \left[ p_h \left( \frac{1}{c_F^2 \rho_F} \frac{\partial q}{\partial t} + \operatorname{div} \omega_F \right) + v_{Fh} \left( \rho_F \frac{\partial \omega_F}{\partial t} + \nabla q \right) \right] dv \\
 & + \sum_{K_F} \int_{\partial K_F} \left[ \left( \frac{1}{c_F^2 \rho_F} \hat{p}_h q + \rho_F v_{Fh} \omega_F \right) n_{K_F}^t + (\hat{p}_h \omega_F + v_{Fh} q) n_{K_F}^x \right] ds = \sum_{K_F} \int_{K_F} f q d\mathbf{v}
 \end{aligned}$$

## TREFFTZ SPACE

$$\mathbf{T}_F(\mathcal{T}_h) := \left\{ (\omega_F, q) \in H^1(\mathcal{T}_h)^2 : \forall K_F \in \mathcal{T}_h, \rho_F \frac{\partial \omega_F}{\partial t} + \nabla q = \frac{1}{c_F^2 \rho_F} \frac{\partial q}{\partial t} + \operatorname{div} \omega_F = 0 \right\}$$

# ACOUSTIC SYSTEM

## SPACE-TIME DISCRETIZATION

$$\begin{aligned}
 & - \sum_{K_F} \int_{K_F} \left[ p_h \left( \frac{1}{c_F^2 \rho_F} \frac{\partial q}{\partial t} + \operatorname{div} \omega_F \right) + v_F h \left( \rho_F \frac{\partial \omega_F}{\partial t} + \nabla q \right) \right] dv \\
 & + \sum_{K_F} \int_{\partial K_F} \left[ \left( \frac{1}{c_F^2 \rho_F} \hat{p}_h q + \rho_F v_F \hat{\omega}_h \omega_F \right) n_{K_F}^t + (\hat{p}_h \omega_F + v_F \hat{\omega}_h q) n_{K_F}^x \right] ds = \sum_{K_F} \int_{K_F} f q dv
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## SPACE-TIME DISCRETIZATION IN $\mathbf{T}_F$

$$\sum_{K_F} \int_{\partial K_F} \left[ \left( \frac{1}{c_F^2 \rho_F} \hat{p}_h q + \rho_F v_F \hat{\omega}_h \cdot \omega_F \right) n_{K_F}^t + (\hat{p}_h \omega_F + v_F \hat{\omega}_h q) \cdot n_{K_F}^x \right] ds = \sum_{K_F} \int_{K_F} f q dv$$

# ACOUSTIC SYSTEM

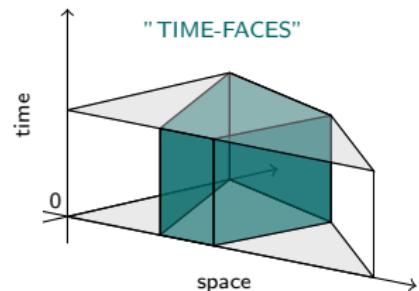
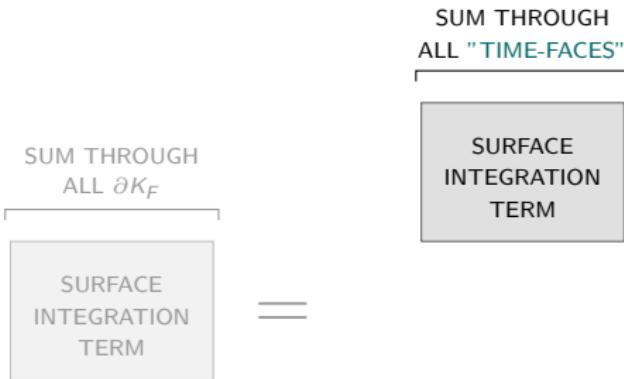
## TREFFTZ-DG FORMULATION

SUM OVER  
ALL  $\partial K_F$

SURFACE  
INTEGRATION  
TERM

# ACOUSTIC SYSTEM

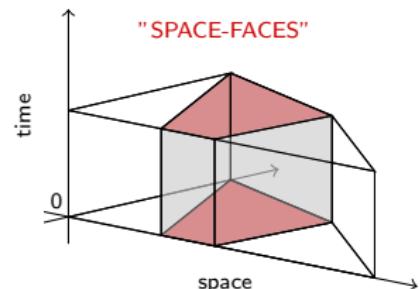
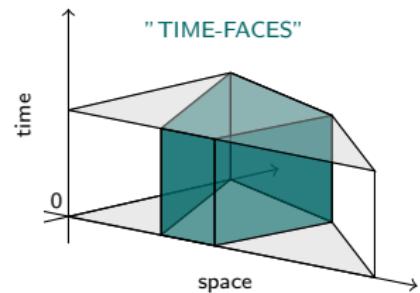
## TREFFTZ-DG FORMULATION



# ACOUSTIC SYSTEM

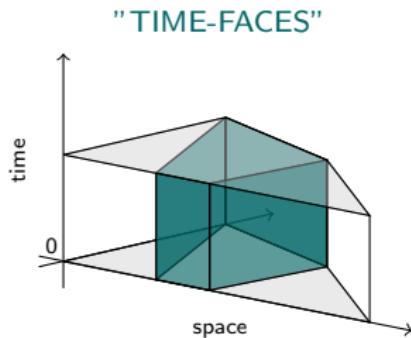
## TREFFTZ-DG FORMULATION

$$\begin{aligned}
 & \text{SUM THROUGH} \\
 & \text{ALL } \partial K_F \\
 & \boxed{\text{SURFACE INTEGRATION TERM}} \\
 & = + \\
 & \boxed{\text{SURFACE INTEGRATION TERM}} \\
 & \quad \text{SUM THROUGH} \\
 & \quad \text{ALL "TIME-FACES"} \\
 & \quad \boxed{\text{SURFACE INTEGRATION TERM}} \\
 & \quad + \\
 & \quad \text{SUM THROUGH} \\
 & \quad \text{ALL "SPACE-FACES"} \\
 & \quad \boxed{\text{SURFACE INTEGRATION TERM}}
 \end{aligned}$$

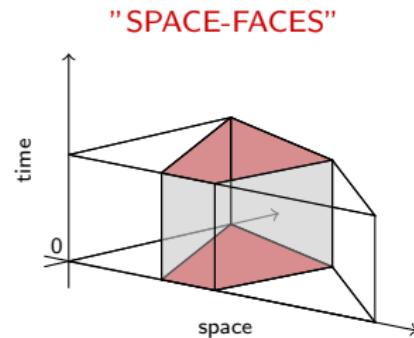


# ACOUSTIC SYSTEM

## MESH NOTATIONS



internal     $\mathcal{F}_h^{I_F}$   
 boundary     $\mathcal{F}_h^{DF} (\partial\Omega_F \times [0, T])$



internal     $\mathcal{F}_h^{\Omega_F} (t \equiv const.)$   
 initial     $\mathcal{F}_h^{0_F} (\Omega_F \times \{0\})$   
 final     $\mathcal{F}_h^{T_F} (\Omega_F \times \{T\})$

# ACOUSTIC SYSTEM

## TREFFTZ-DG FORMULATION

$$\sum_{\partial K_F} \int_{\partial K_F} \left[ \left( \frac{1}{c_F^2 \rho_F} \hat{p}_h q + \rho_F v_F^\wedge h \omega_F \right) n_{K_F}^t + (\hat{p}_h \omega_F + v_F^\wedge h q) n_{K_F}^x \right] ds = \sum_{K_F} \int_{K_F} f q dV$$

## NUMERICAL FLUXES THROUGH THE ELEMENT FACES

$$\begin{aligned}
 \mathcal{F}_h^{I_F} : \quad & \begin{pmatrix} v_F^\wedge h \\ \hat{p}_h \end{pmatrix} := \begin{pmatrix} \{v_F h\} + \beta [p_h]_x \\ \{p_h\} + \alpha [v_F h]_x \end{pmatrix} \\
 \mathcal{F}_h^{D_F} : \quad & \begin{pmatrix} v_F^\wedge h \\ \hat{p}_h \end{pmatrix} := \begin{pmatrix} g_D^\wedge_F \\ p_h + \alpha (v_F h \cdot n_{K_F}^x - g_D^\wedge_F) \end{pmatrix} \\
 \mathcal{F}_h^{\Omega_F} : \quad & \begin{pmatrix} v_F^\wedge h \\ \hat{p}_h \end{pmatrix} := \begin{pmatrix} v_F h^- \\ p_h \end{pmatrix} \\
 \mathcal{F}_h^{T_F} : \quad & \begin{pmatrix} v_F^\wedge h \\ \hat{p}_h \end{pmatrix} := \begin{pmatrix} v_F h \\ p_h \end{pmatrix} \\
 \mathcal{F}_h^{0_F} : \quad & \begin{pmatrix} v_F^\wedge h \\ \hat{p}_h \end{pmatrix} := \begin{pmatrix} v_F 0 \\ p_0 \end{pmatrix}
 \end{aligned}$$

# ACOUSTIC SYSTEM

## TREFFTZ-DG FORMULATION

Seek  $(v_{Fh}, p_h) \subset \mathbf{V}(\mathcal{T}_h) \subset \mathbf{T}_F(\mathcal{T}_h)$  s.t. for all  $(\omega_F, q) \subset \mathbf{V}(\mathcal{T}_h)$  it holds:

$$\mathcal{A}_{TDG_F}((v_{Fh}, p_h); (\omega_F, q)) = \ell_{TDG_F}(\omega_F, q)$$

# ACOUSTIC SYSTEM

## $L^2(\mathcal{T}_h)$ NORMS IN $\mathbf{T}_F$

$$\begin{aligned} |||(\omega_F, q)|||^2_{TDG_F} := & \frac{1}{2} \left\| \left( \frac{1}{c_F^2 \rho_F} \right)^{1/2} [\![q]\!]_t \right\|_{L^2(\mathcal{F}_h^{\Omega_F})}^2 + \frac{1}{2} \left\| \rho_F^{1/2} [\![\omega_F]\!]_t \right\|_{L^2(\mathcal{F}_h^{\Omega_F})}^2 + \left\| \alpha^{1/2} [\![\omega_F]\!]_x \right\|_{L^2(\mathcal{F}_h^{I_F})}^2 \\ & + \left\| \beta^{1/2} [\![q]\!]_x \right\|_{L^2(\mathcal{F}_h^{I_F})}^2 + \frac{1}{2} \left\| \left( \frac{1}{c_F^2 \rho_F} \right)^{1/2} q \right\|_{L^2(\mathcal{F}_h^{T_F})}^2 + \frac{1}{2} \left\| \rho_F^{1/2} \omega_F \right\|_{L^2(\mathcal{F}_h^{T_F})}^2 \\ & + \left\| \alpha^{1/2} \omega_F \right\|_{L^2(\mathcal{F}_h^{D_F})}^2 \end{aligned}$$

# ACOUSTIC SYSTEM

## $L^2(\mathcal{T}_h)$ NORMS IN $\mathcal{T}_F$

$$\begin{aligned} |||(\omega_F, q)|||_{TDG_F}^2 &:= \frac{1}{2} \left\| \left( \frac{1}{c_F^2 \rho_F} \right)^{1/2} [q]_t \right\|_{L^2(\mathcal{F}_h^{\Omega_F})}^2 + \frac{1}{2} \left\| \rho_F^{1/2} [\omega_F]_t \right\|_{L^2(\mathcal{F}_h^{\Omega_F})}^2 + \left\| \alpha^{1/2} [\omega_F]_x \right\|_{L^2(\mathcal{F}_h^{I_F})}^2 \\ &\quad + \left\| \beta^{1/2} [q]_x \right\|_{L^2(\mathcal{F}_h^{I_F})}^2 + \frac{1}{2} \left\| \left( \frac{1}{c_F^2 \rho_F} \right)^{1/2} q \right\|_{L^2(\mathcal{F}_h^{T_F})}^2 + \frac{1}{2} \left\| \rho_F^{1/2} \omega_F \right\|_{L^2(\mathcal{F}_h^{T_F})}^2 \\ &\quad + \left\| \alpha^{1/2} \omega_F \right\|_{L^2(\mathcal{F}_h^{D_F})}^2 \end{aligned}$$

$$\begin{aligned} |||(\omega_F, q)|||_{TDG_F^*}^2 &:= |||(\omega_F, q)|||_{TDG_F}^2 \\ &\quad + \left\| \rho_F^{1/2} \omega_F^- \right\|_{L^2(\mathcal{F}_h^{\Omega_F})}^2 + \left\| \left( \frac{1}{c_F^2 \rho_F} \right)^{1/2} q^- \right\|_{L^2(\mathcal{F}_h^{\Omega_F})}^2 + \left\| \beta^{-1/2} \{\omega_F\} \right\|_{L^2(\mathcal{F}_h^{I_F})}^2 \\ &\quad + \left\| \alpha^{-1/2} \{q\} \right\|_{L^2(\mathcal{F}_h^{I_F})}^2 + \left\| \alpha^{-1/2} \omega_F \right\|_{L^2(\mathcal{F}_h^{D_F})}^2 \end{aligned}$$

# ACOUSTIC SYSTEM

## COERCIVITY OF TREFFTZ-DG FORMULATION

$$\mathcal{A}_{TDG_F}((\omega_F, q); (\omega_F, q)) = |||(\omega_F, q)|||_{TDG_F}^2, \quad \forall (\omega_F, q) \in \mathbf{T}_F(\mathcal{T}_h)$$

# ACOUSTIC SYSTEM

## COERCIVITY OF TREFFTZ-DG FORMULATION

$$\mathcal{A}_{TDG_F}((\omega_F, q); (\omega_F, q)) = |||(\omega_F, q)|||_{TDG_F}^2, \quad \forall (\omega_F, q) \in \mathbf{T}_F(\mathcal{T}_h)$$

## CONTINUITY OF TREFFTZ-DG FORMULATION

$$|\mathcal{A}_{TDG_F}((v_F, p); (\omega_F, q))| \leq 2 ||| (v_F, p) |||_{TDG_F^*} ||| (\omega_F, q) |||_{TDG_F},$$

$$|\ell_{TDG_F}(\omega_F, q)| \leq \sqrt{2} \left[ \|\rho_F^{1/2} v_{f0}\|_{L^2(\mathcal{F}_h^{0F})}^2 + \left\| \left( \frac{1}{c_F^2 \rho_F} \right)^{1/2} p_0 \right\|_{L^2(\mathcal{F}_h^{0F})}^2 \right]^{1/2}$$

# ACOUSTIC SYSTEM

## WELL-POSEDNESS

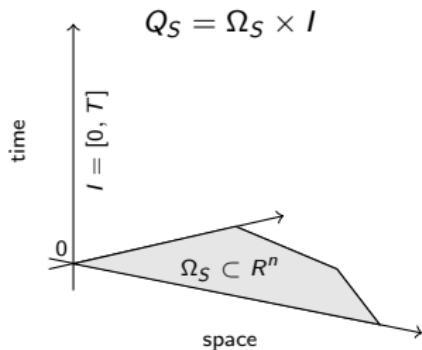
$$|||(\boldsymbol{v}_F - \boldsymbol{v}_{F,h}, p - p_h)|||_{TDG_F} \leq 3 \inf_{(\omega_F, q) \in \mathbf{V}(T_h)} |||(\boldsymbol{v}_F - \omega_F, p - q)|||_{TDG_F^*}$$

# MATHEMATICAL FORMULATION

## ELASTODYNAMIC SYSTEM

# ELASTODYNAMIC SYSTEM

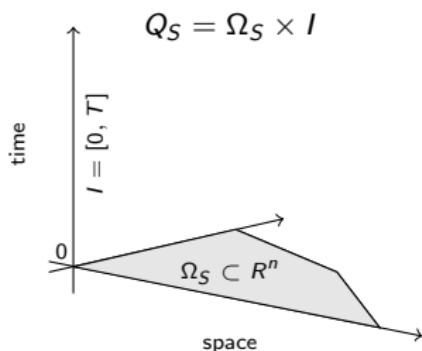
## PROBLEM EQUATIONS



$$\begin{cases} \frac{\partial \underline{\underline{\sigma}}}{\partial t} - \underline{\underline{C}} \underline{\underline{\varepsilon}}(\underline{\underline{v}}_S) = 0 & \text{in } Q_S \\ \rho_S \frac{\partial \underline{\underline{v}}_S}{\partial t} - \operatorname{div} \underline{\underline{\sigma}} = 0 & \text{in } Q_S \\ \underline{\underline{v}}_S(\cdot, 0) = \underline{\underline{v}}_{S0}, \quad \underline{\underline{\sigma}}(\cdot, 0) = \underline{\underline{\sigma}}_0 & \text{on } \Omega_S \\ \underline{\underline{v}}_S = g_{D_S} & \text{on } \partial\Omega_S \times I \end{cases}$$

# ELASTODYNAMIC SYSTEM

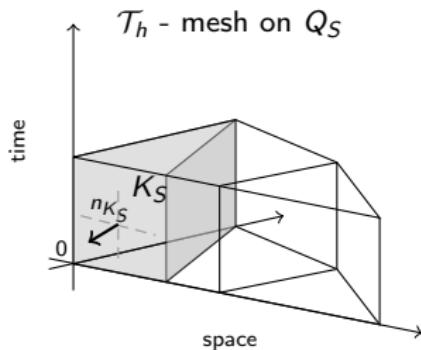
## PROBLEM EQUATIONS



$$\begin{cases} A \frac{\partial \underline{\underline{\sigma}}}{\partial t} - \underline{\underline{\varepsilon}}(\underline{\underline{v}}_S) = 0 & \text{in } Q_S \\ \rho_S \frac{\partial \underline{\underline{v}}_S}{\partial t} - \text{div} \underline{\underline{\sigma}} = 0 & \text{in } Q_S \\ \underline{\underline{v}}_S(\cdot, 0) = \underline{\underline{v}}_{S0}, \quad \underline{\underline{\sigma}}(\cdot, 0) = \underline{\underline{\sigma}}_0 & \text{on } \Omega_S \\ \underline{\underline{v}}_S = g_{D_S} & \text{on } \partial\Omega_S \times I \end{cases}$$

# ELASTODYNAMIC SYSTEM

## MESHING



$\mathcal{F}_h = \cup_{K_S \in T_h} \partial K_S$  - mesh skeleton

$K_S \in Q_S$  ( $A, \rho_S \equiv \text{const}$  in  $K_S$ )

$$v_S, \underline{\underline{\sigma}} \in H^1(K_S)$$

$$\omega_S, \underline{\xi} \in H^1(K_S)$$

# ELASTODYNAMIC SYSTEM

## TREFFTZ SPACE

$$\mathbf{T}_S(\mathcal{T}_h) := \left\{ (\omega_S, \underline{\xi}) \in H^1(\mathcal{T}_h)^2 : \forall K_S \in \mathcal{T}_h, \rho_S \frac{\partial \omega_S}{\partial t} - \operatorname{div} \underline{\xi} = A \frac{\partial \underline{\xi}}{\partial t} - \underline{\varepsilon}(\omega_S) = 0 \right\}$$

# ELASTODYNAMIC SYSTEM

## TREFFTZ SPACE

$$\mathbf{T}_S(\mathcal{T}_h) := \left\{ (\omega_S, \underline{\xi}) \subset H^1(\mathcal{T}_h)^2 : \forall K_S \in \mathcal{T}_h, \rho_S \frac{\partial \omega_S}{\partial t} - \operatorname{div} \underline{\xi} = \underline{\underline{A}} \frac{\partial \underline{\xi}}{\partial t} - \underline{\underline{\varepsilon}}(\omega_S) = 0 \right\}$$

## SPACE-TIME DISCRETIZATION IN $\mathbf{T}_S$

$$\sum_{K_S} \int_{\partial K_S} \left[ (\underline{\underline{A}} \hat{\underline{\underline{\sigma}}}_h : \underline{\xi} + \rho_S v_S \hat{v}_h \omega_S) n_{K_S}^t - (v_S \hat{v}_h \underline{\xi} + \hat{\underline{\sigma}}_h \omega_S) \cdot n_{K_S}^x \right] ds = 0$$

# ELASTODYNAMIC SYSTEM

## TREFFTZ-DG FORMULATION

$$\sum_{K_S} \int_{\partial K_S} \left[ (\underline{\underline{A}} \hat{\underline{\underline{\sigma}}}_h : \underline{\underline{\xi}} + \rho_S \hat{v}_S h \omega_S) n_{K_S}^t - (\hat{v}_S h \underline{\underline{\xi}} + \hat{\underline{\underline{\sigma}}}_h \omega_S) \cdot n_{K_S}^x \right] ds = 0$$

## NUMERICAL FLUXES THROUGH THE ELEMENT FACES

$$\begin{aligned}
 \mathcal{F}_h^{Is}: \quad & \begin{pmatrix} \hat{v}_S h \\ \hat{\underline{\underline{\sigma}}}_h \end{pmatrix} := \begin{pmatrix} \{v_S h\} - \delta [\underline{\underline{\sigma}}_h]_x \\ \{\underline{\underline{\sigma}}_h\} - \gamma [v_S h]_x \end{pmatrix} \\
 \mathcal{F}_h^{Ds}: \quad & \begin{pmatrix} \hat{v}_S h \\ \hat{\underline{\underline{\sigma}}}_h \end{pmatrix} := \begin{pmatrix} v_S h - \delta (\underline{\underline{\sigma}}_h \cdot n_{K_S}^x - g_{Ds}) \\ g_{Ds} \end{pmatrix} \\
 \mathcal{F}_h^{\Omega_S}: \quad & \begin{pmatrix} \hat{v}_S h \\ \hat{\underline{\underline{\sigma}}}_h \end{pmatrix} := \begin{pmatrix} v_S h^- \\ \underline{\underline{\sigma}}_h^- \end{pmatrix} \\
 \mathcal{F}_h^{Ts}: \quad & \begin{pmatrix} \hat{v}_S h \\ \hat{\underline{\underline{\sigma}}}_h \end{pmatrix} := \begin{pmatrix} v_S h \\ \underline{\underline{\sigma}}_h \end{pmatrix} \\
 \mathcal{F}_h^{0s}: \quad & \begin{pmatrix} \hat{v}_S h \\ \hat{\underline{\underline{\sigma}}}_h \end{pmatrix} := \begin{pmatrix} v_{s0} \\ \underline{\underline{\sigma}}_0 \end{pmatrix}
 \end{aligned}$$

# ELASTODYNAMIC SYSTEM

## TREFFTZ-DG FORMULATION

Seek  $(v_{Sh}, \underline{\underline{\sigma}}_h) \subset \mathbf{V}(\mathcal{T}_h) \subset \mathbf{T}_S(\mathcal{T}_h)$ , s.t. for all  $K_S \in \mathcal{T}_h$ ,  $(\omega_S, \underline{\xi}) \subset \mathbf{V}(\mathcal{T}_h)$  it holds:

$$\mathcal{A}_{TDG_S}((v_{Sh}, \underline{\underline{\sigma}}_h); (\omega_S, \underline{\xi})) = \ell_{TDG_S}(\omega_S, \underline{\xi})$$

# ELASTODYNAMIC SYSTEM

## $L^2(\mathcal{T}_h)$ NORMS IN $\mathsf{T}_S$

$$\|(\omega_S, \underline{\xi})\|_{TDG_S}^2 := \frac{1}{2} \|(\underline{A})^{1/2} [\underline{\xi}]_t\|_{L^2(\mathcal{F}_h^{\Omega_S})}^2 + \frac{1}{2} \|\rho_S^{1/2} [\omega_S]_t\|_{L^2(\mathcal{F}_h^{\Omega_S})}^2 + \|\gamma^{1/2} [\omega_S]_x\|_{L^2(\mathcal{F}_h^{I_S})}^2 \\ + \|\delta^{1/2} [\underline{\xi}]_x\|_{L^2(\mathcal{F}_h^{I_S})}^2 + \frac{1}{2} \|(\underline{A})^{1/2} \underline{\xi}\|_{L^2(\mathcal{F}_h^{T_S})}^2 + \frac{1}{2} \|\rho_S^{1/2} \omega_S\|_{L^2(\mathcal{F}_h^{T_S})}^2 \\ + \|\delta^{1/2} \underline{\xi}\|_{L^2(\mathcal{F}_h^{D_S})}^2$$

# ELASTODYNAMIC SYSTEM

## $L^2(\mathcal{T}_h)$ NORMS IN $\mathsf{T}_S$

$$\|(\omega_S, \underline{\xi})\|_{TDG_S}^2 := \frac{1}{2} \|(\underline{A})^{1/2} [\underline{\xi}]_t\|_{L^2(\mathcal{F}_h^{\Omega_S})}^2 + \frac{1}{2} \|\rho_S^{1/2} [\omega_S]_t\|_{L^2(\mathcal{F}_h^{\Omega_S})}^2 + \|\gamma^{1/2} [\omega_S]_x\|_{L^2(\mathcal{F}_h^{I_S})}^2 \\ + \|\delta^{1/2} [\underline{\xi}]_x\|_{L^2(\mathcal{F}_h^{I_S})}^2 + \frac{1}{2} \|(\underline{A})^{1/2} \underline{\xi}\|_{L^2(\mathcal{F}_h^{T_S})}^2 + \frac{1}{2} \|\rho_S^{1/2} \omega_S\|_{L^2(\mathcal{F}_h^{T_S})}^2 \\ + \|\delta^{1/2} \underline{\xi}\|_{L^2(\mathcal{F}_h^{D_S})}^2$$

$$\|(\omega_S, \underline{\xi})\|_{TDG_S^*}^2 := \|(\omega_S, \underline{\xi})\|_{TDG_S}^2 \\ + \|\rho_S^{1/2} \omega_S^-\|_{L^2(\mathcal{F}_h^{\Omega_S})}^2 + \|(\underline{A})^{1/2} \underline{\xi}^-\|_{L^2(\mathcal{F}_h^{\Omega_S})}^2 + \|\delta^{-1/2} \{\omega_S\}\|_{L^2(\mathcal{F}_h^{I_S})}^2 \\ + \|\gamma^{-1/2} \{\underline{\xi}\}\|_{L^2(\mathcal{F}_h^{I_S})}^2 + \|\delta^{-1/2} \underline{\xi}\|_{L^2(\mathcal{F}_h^{D_S})}^2.$$

# ELASTODYNAMIC SYSTEM

## COERCIVITY OF TREFFTZ-DG FORMULATION

$$\mathcal{A}_{TDG_S}((\omega_S, \underline{\xi}); (\omega_S, \underline{\xi})) = |||(\omega_S, \underline{\xi})|||_{TDG_S}^2, \quad \forall (\omega_S, \underline{\xi}) \in \mathbf{T}_S(\mathcal{T}_h)$$

# ELASTODYNAMIC SYSTEM

## COERCIVITY OF TREFFTZ-DG FORMULATION

$$\mathcal{A}_{TDG_S}((\omega_S, \underline{\xi}); (\omega_S, \underline{\xi})) = |||(\omega_S, \underline{\xi})|||_{TDG_S}^2, \quad \forall (\omega_S, \underline{\xi}) \in \mathbf{T}_S(\mathcal{T}_h)$$

## CONTINUITY OF TREFFTZ-DG FORMULATION

$$|\mathcal{A}_{TDG_S}((v_S, \underline{\sigma}); (\omega_S, \underline{\xi}))| \leq 2 |||(\omega_S, \underline{\xi})|||_{TDG_S^*} |||(\omega_S, \underline{\xi})|||_{TDG_S}$$

$$|\ell_{TDG_S}(\omega_S, \underline{\xi})| \leq \sqrt{2} \left[ \|\rho_S^{1/2} v_{s0}\|_{L^2(\mathcal{F}_h^{0S})}^2 + \|\underline{A}^{1/2} \underline{\sigma}_0\|_{L^2(\mathcal{F}_h^{0S})}^2 \right]^{1/2}$$

# ELASTODYNAMIC SYSTEM

## WELL-POSEDNESS

$$|||(\nu_S - \nu_{S,h}, \underline{\sigma} - \underline{\sigma}_h)|||_{TDG_S} \leq 3 \inf_{\substack{(\omega_S, \underline{\xi}) \in \mathbf{V}(\mathcal{T}_h) \\ \underline{\underline{\xi}}}} |||(\nu_S - \omega_S, \underline{\sigma} - \underline{\xi})|||_{TDG_S^*}$$

# MATHEMATICAL FORMULATION

## ELASTO-ACOUSTIC SYSTEM

# ELASTO-AcouSTIC SYSTEM

## NUMERICAL COUPLING ADVANTAGES



Computing 1 unknown scalar pressure instead of 6 components  
of stress in fluid

# ELASTO-AcouSTIC SYSTEM

## NUMERICAL COUPLING ADVANTAGES

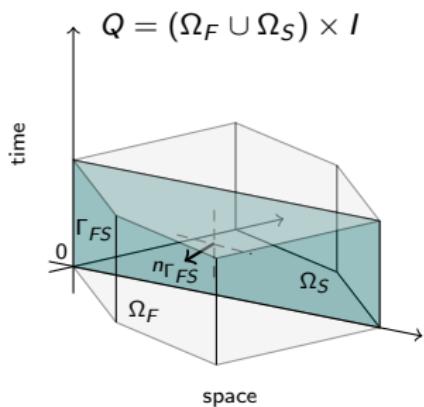


Computing 1 unknown scalar pressure instead of 6 components of stress in fluid

Avoiding the numerical artefacts caused by the slow  $S$ -waves

# ELASTO-AcouSTIC SYSTEM

## TRANSMISSION CONDITIONS



$$(TC) \begin{cases} v_F n_{\Gamma_{FS}} = v_S n_{\Gamma_{FS}} & \text{on } \Gamma_{FS} \\ \sigma \cdot n_{\Gamma_{FS}} = -p n_{\Gamma_{FS}} & \text{on } \Gamma_{FS} \end{cases}$$

# ELASTO-AcouSTIC SYSTEM

## TREFFTZ SPACE

$$\begin{aligned} \mathbf{T}(\mathcal{T}_h) := \left\{ (\omega_F, q, \omega_S, \underline{\xi}) \in H^1(\mathcal{T}_h)^4 : \right. & \rho_F \frac{\partial \omega_F}{\partial t} + \nabla q = \frac{1}{c_F^2 \rho_F} \frac{\partial q}{\partial t} + \operatorname{div} \omega_F = 0, \\ & \left. \rho_S \frac{\partial \omega_S}{\partial t} - \operatorname{div} \underline{\xi} = \frac{\partial \underline{\underline{A}} \underline{\xi}}{\partial t} - \underline{\varepsilon}(\omega_S) = 0, \quad \forall K_F, K_S \in \mathcal{T}_h \right\} \end{aligned}$$

# ELASTO-Acoustic SYSTEM

## TREFFTZ SPACE

$$\begin{aligned} \mathbf{T}(\mathcal{T}_h) := \left\{ (\omega_F, q, \omega_S, \underline{\xi}) \in H^1(\mathcal{T}_h)^4 : \right. & \rho_F \frac{\partial \omega_F}{\partial t} + \nabla q = \frac{1}{c_F^2 \rho_F} \frac{\partial q}{\partial t} + \operatorname{div} \omega_F = 0, \\ & \rho_S \frac{\partial \omega_S}{\partial t} - \operatorname{div} \underline{\xi} = \frac{\partial \underline{\underline{A}} \underline{\xi}}{\partial t} - \underline{\underline{\varepsilon}}(\omega_S) = 0, \quad \forall K_F, K_S \in \mathcal{T}_h \left. \right\} \end{aligned}$$

## SPACE-TIME DISCRETIZATION IN T

$$\begin{aligned} & \sum_{K_F} \int_{\partial K_F} \left[ \left( \frac{1}{c_F^2 \rho_F} \hat{p}_h q + \rho_F v_F \hat{v}_h \omega_F \right) n_{K_F}^t + (\hat{p}_h \omega_F + v_F \hat{v}_h q) n_{K_F}^x \right] ds \\ & + \sum_{K_S} \int_{\partial K_S} \left[ (\underline{\underline{A}} \hat{\underline{\xi}}_h : \underline{\underline{\xi}} + \rho_S v_S \hat{v}_h \omega_S) n_{K_S}^t - (v_S \hat{v}_h \underline{\underline{\xi}} + \hat{\underline{\sigma}}_h \omega_S) \cdot n_{K_S}^x \right] ds = \sum_{K_F} \int_{K_F} f q d\mathbf{v} \end{aligned}$$

# ELASTO-Acoustic SYSTEM

## TREFFTZ-DG FORMULATION

$$\begin{aligned} & \sum_{K_F} \int_{\partial K_F} \left[ \left( \frac{1}{c_F^2 \rho_F} \hat{\rho}_h q + \rho_F v_F \hat{v}_h \cdot \omega_F \right) n_{K_F}^t + (\hat{\rho}_h \omega_F + v_F \hat{v}_h q) \cdot n_{K_F}^x \right] ds, \\ & + \sum_{K_S} \int_{\partial K_S} \left[ (A \hat{\sigma}_h : \underline{\underline{\xi}} + \rho_S v_S \hat{v}_h \cdot \omega_S) n_{K_S}^t - (\underline{\underline{\xi}} v_S \hat{v}_h + \hat{\sigma}_h \omega_S) \cdot n_{K_S}^x \right] ds = \sum_{K_F} \int_{K_F} f q d\nu \end{aligned}$$

## NUMERICAL FLUXES THROUGH THE FLUID-SOLID INTERFACE

$$\mathcal{F}_h^{FS}: \quad \begin{pmatrix} v_F \hat{v}_h n_{K_F}^x \\ \hat{\rho}_h \\ v_S \hat{v}_h \\ \hat{\sigma}_h \cdot n_{K_S}^x \end{pmatrix} := \quad \begin{pmatrix} v_S \hat{v}_h n_{K_F}^x \\ \rho_h + \alpha(v_F \hat{v}_h n_{K_F}^x - v_S \hat{v}_h n_{K_F}^x) \\ v_S \hat{v}_h - \delta(\hat{\sigma}_h \cdot n_{K_S}^x - \rho_h n_{K_S}^x) \\ -\rho_h n_{K_S}^x \end{pmatrix}$$

# ELASTO-AcouSTIC SYSTEM

## TREFFTZ-DG FORMULATION

Seek  $(v_{Fh}, p_h, v_{Sh}, \underline{\underline{\sigma}}_h) \subset \mathbf{V}(\mathcal{T}_h)$ :  $\forall K_F, K_S \in \mathcal{T}_h$ ,  $(\omega_F, q, \omega_S, \underline{\xi}) \subset \mathbf{T}(\mathcal{T}_h)$  it holds:

$$\mathcal{A}_{TDG}((v_{Fh}, p_h, v_{Sh}, \underline{\underline{\sigma}}_h); (\omega_F, q, \omega_S, \underline{\xi})) = \ell_{TDG}(\omega_F, q, \omega_S, \underline{\xi})$$

# ELASTO-AcouSTIC SYSTEM

## $L^2(\mathcal{T}_h)$ NORMS IN T

$$\| | | (\omega_F, q, \omega_S, \underline{\xi}) ) \| | |_{TDG}^2 := \| | | (\omega_F, q) \| | |_{TDG_F}^2 + \| | | (\omega_S, \underline{\xi}) \| | |_{TDG_S}^2 + 2\|\delta^{1/2}\underline{\xi}\|_{L^2(\mathcal{F}_h^{FS})}^2$$

# ELASTO-AcouSTIC SYSTEM

## $L^2(\mathcal{T}_h)$ NORMS IN T

$$|||(\omega_F, q, \omega_S, \underline{\xi}))|||^2_{TDG} := |||(\omega_F, q)|||^2_{TDG_F} + |||(\omega_S, \underline{\xi})|||^2_{TDG_S} + 2\delta^{1/2} \|\underline{\xi}\|_{L^2(\mathcal{F}_h^{FS})}^2$$

$$|||(\omega_F, q, \omega_S, \underline{\xi})|||^2_{TDG^*} := |||(\omega_F, q)|||^2_{TDG_F^*} + |||(\omega_S, \underline{\xi})|||^2_{TDG_S^*} + \frac{1}{2} \|\delta^{-1/2} \underline{\xi}\|_{L^2(\mathcal{F}_h^{FS})}^2$$

# ELASTO-AcouSTIC SYSTEM

## COERCIVITY OF TREFFTZ-DG FORMULATION

$$\mathcal{A}_{TDG}((\omega_F, q, \underline{\omega_S}, \underline{\xi}); (\omega_F, q, \underline{\omega_S}, \underline{\xi})) = |||(\omega_F, q, \underline{\omega_S}, \underline{\xi})|||_{TDG}^2$$

# ELASTO-Acoustic SYSTEM

## COERCIVITY OF TREFFTZ-DG FORMULATION

$$\mathcal{A}_{TDG}((\omega_F, q, \omega_S, \underline{\xi}); (\omega_F, q, \omega_S, \underline{\xi})) = |||(\omega_F, q, \omega_S, \underline{\xi})|||_{TDG}^2$$

## CONTINUITY OF TREFFTZ-DG FORMULATION

$$|\mathcal{A}_{TDG}((v_F, p, v_S, \underline{\sigma}); (\omega_F, q, \omega_S, \underline{\xi}))| \leq 2 ||| (v_F, p, v_S, \underline{\sigma}) |||_{TDG^*} ||| (\omega_F, q, \omega_S, \underline{\xi}) |||_{TDG}$$

$$\begin{aligned} |\ell_{TDG}(\omega_F, q, \omega_S, \underline{\xi})| &\leq \sqrt{2} \left[ \|\rho_F^{1/2} v_{F0}\|_{L^2(\mathcal{F}_h^{0F})}^2 + \|(\frac{1}{c_F^2 \rho_F})^{1/2} p_0\|_{L^2(\mathcal{F}_h^{0F})}^2 \right. \\ &\quad \left. + \|\rho_S^{1/2} v_{S0}\|_{L^2(\mathcal{F}_h^{0S})}^2 + \|\underline{A}^{1/2} \underline{\sigma}_0\|_{L^2(\mathcal{F}_h^{0S})}^2 \right]^{1/2} \end{aligned}$$

# ELASTO-AcouSTIC SYSTEM

## WELL-POSEDNESS

$$|||(\nu_F - \nu_{Fh}, p - p_h, \nu_S - \nu_{Sh}, \underline{\underline{\sigma}} - \underline{\underline{\sigma}}_h)|||_{TDG}$$

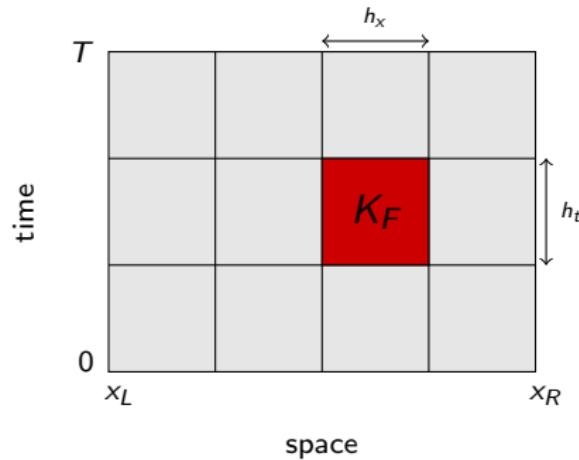
$$\leq 3 \inf_{(\omega_F, q, \omega_S, \underline{\underline{\xi}}) \in \mathbf{V}(\mathcal{T}_h)} |||(\nu_F - \omega_F, p - q, \nu_S - \omega_S, \underline{\underline{\sigma}} - \underline{\underline{\xi}})|||_{TDG^*}.$$

# IMPLEMENTATION OF THE ALGORITHM

## 1D ACOUSTIC SYSTEM EXAMPLE

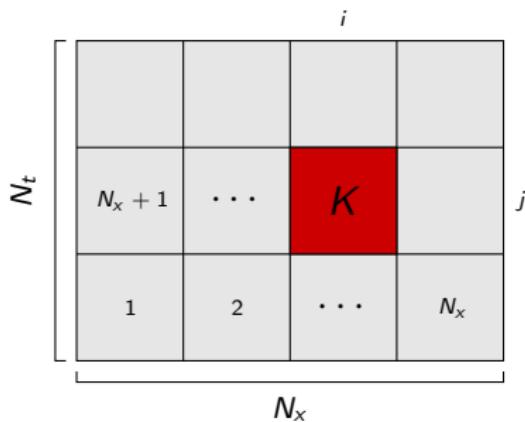
# IMPLEMENTATION OF THE ALGORITHM

## MESH AND ELEMENT NUMBERING



# IMPLEMENTATION OF THE ALGORITHM

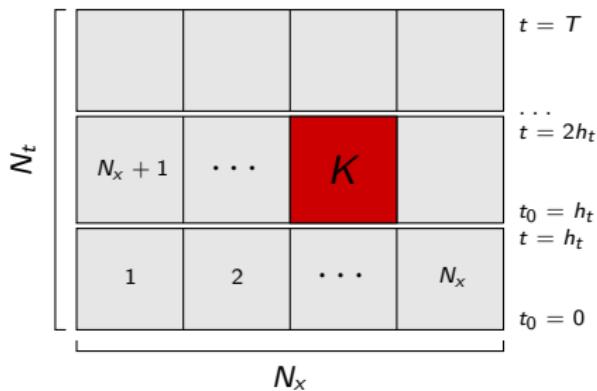
## MESH AND ELEMENT NUMBERING



$$K = (j - 1) \times N_x + i$$

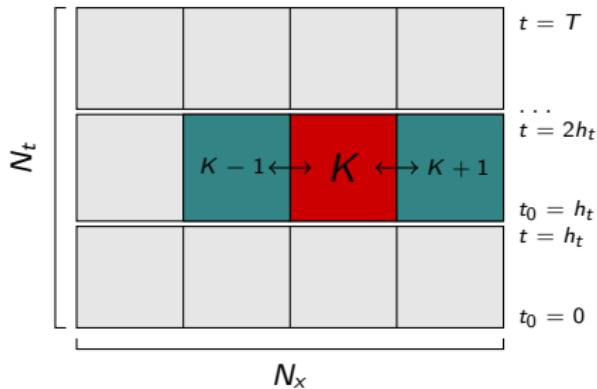
# IMPLEMENTATION OF THE ALGORITHM

## MESH AND ELEMENT NUMBERING



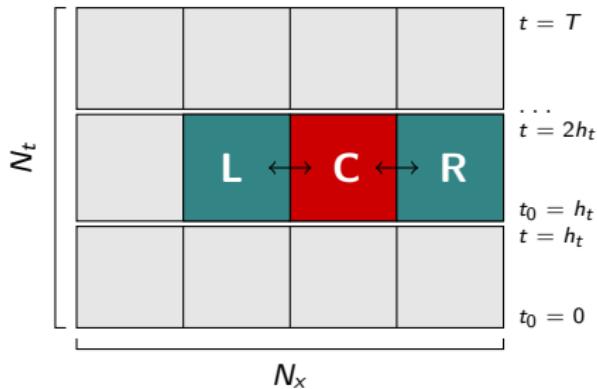
# IMPLEMENTATION OF THE ALGORITHM

## MESH AND ELEMENT NUMBERING



# IMPLEMENTATION OF THE ALGORITHM

## MESH AND ELEMENT NUMBERING



# IMPLEMENTATION OF THE ALGORITHM

GLOBAL MATRIX (1<sup>ST</sup> LAYER)

$$\begin{bmatrix} \text{C} & \text{R} \\ \text{L} & \text{C} & \text{R} \\ \text{L} & \text{C} & \text{R} \\ \vdots & & \ddots \\ \text{R} & & \dots & \text{L} & \text{C} & \text{R} \\ & & & \text{L} & \text{C} & \end{bmatrix} \times \begin{bmatrix} \mathbf{U}_1 \\ \mathbf{U}_2 \\ \mathbf{U}_3 \\ \vdots \\ \mathbf{U}_{N_x-1} \\ \mathbf{U}_{N_x} \end{bmatrix} = \begin{bmatrix} \mathbf{U}_1^0 \\ \mathbf{U}_2^0 \\ \mathbf{U}_3^0 \\ \vdots \\ \mathbf{U}_{N_x-1}^0 \\ \mathbf{U}_{N_x}^0 \end{bmatrix}$$

# IMPLEMENTATION OF THE ALGORITHM

GLOBAL MATRIX (1<sup>ST</sup> LAYER)

$$\begin{bmatrix} \mathbf{M} \end{bmatrix} \times \begin{bmatrix} \mathbf{U} \end{bmatrix} = \begin{bmatrix} \mathbf{U}^0 \end{bmatrix}$$

# IMPLEMENTATION OF THE ALGORITHM

## 1. INITIATION

COMPUTE

$$\begin{bmatrix} \mathbf{M} \end{bmatrix}; \begin{bmatrix} \mathbf{M}^{-1} \end{bmatrix}; \begin{bmatrix} \mathbf{U}^0 \end{bmatrix}$$

## 2. PROPAGATION

FOR  $t = 1 : N_t$

$$\begin{bmatrix} \mathbf{U} \end{bmatrix} = \begin{bmatrix} \mathbf{M}^{-1} \end{bmatrix} \times \begin{bmatrix} \mathbf{U}^0 \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{U}^0 \end{bmatrix} = \begin{bmatrix} \text{projection operator} \end{bmatrix} \times \begin{bmatrix} \mathbf{U} \end{bmatrix}$$

ENDFOR

# IMPLEMENTATION OF THE ALGORITHM

## POLYNOMIAL BASIS (ORDER 3)

$$\phi_1^v = 0$$

$$\phi_1^p = -c_F$$

$$\phi_2^v = 1$$

$$\phi_2^p = 0$$

$$\phi_3^v = x$$

$$\phi_3^p = -c_F^2 t$$

$$\phi_4^v = c_F t$$

$$\phi_4^p = -c_F x$$

$$\phi_5^v = -\frac{x^2}{2} - \frac{c_F^2 t^2}{2}$$

$$\phi_5^p = c_F^2 x t$$

$$\phi_6^v = -c_F x t$$

$$\phi_6^p = c_F \left( \frac{x^2}{2} + \frac{c_F^2 t^2}{2} \right)$$

$$\phi_7^v = -\frac{x^3}{6} - \frac{x c_F^2 t^2}{2}$$

$$\phi_7^p = c_F \left( \frac{c_F^3 t^3}{6} + \frac{x^2 c_F t}{2} \right)$$

$$\phi_8^v = -\frac{c_F^3 t^3}{6} - \frac{x^2 c_F t}{2}$$

$$\phi_8^p = c_F \left( \frac{x^3}{6} + \frac{x c_F^2 t^2}{2} \right)$$

# IMPLEMENTATION OF THE ALGORITHM

## NUMERICAL TESTS

FIGURE 1: 1D Acoustic system.

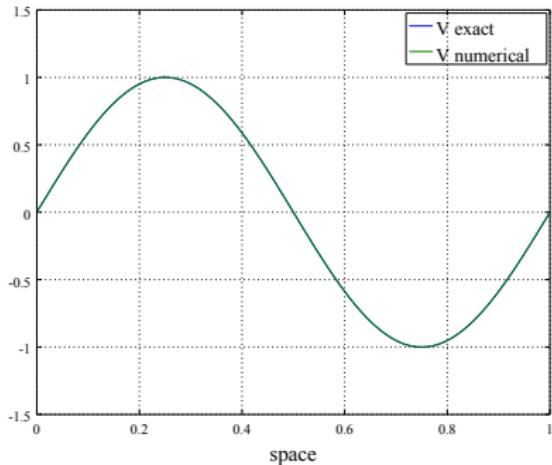
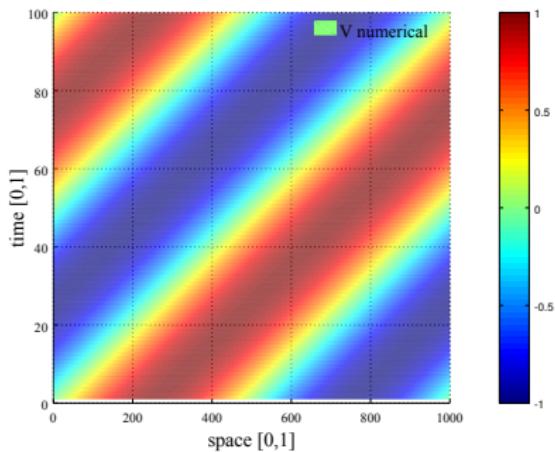
Exact and numerical velocities  $v_F(x, t = 1)$ .Periodical boundaries.  $\alpha = \beta = 0$ 

FIGURE 2: 1D Acoustic system.

Numerical velocity  $v_F(x, t)$ .Periodical boundaries.  $\alpha = \beta = 0$ 

# IMPLEMENTATION OF THE ALGORITHM

## NUMERICAL TESTS

FIGURE 3: 1D Acoustic system.

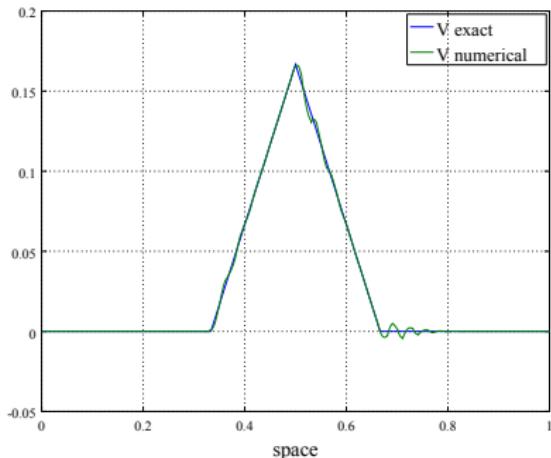
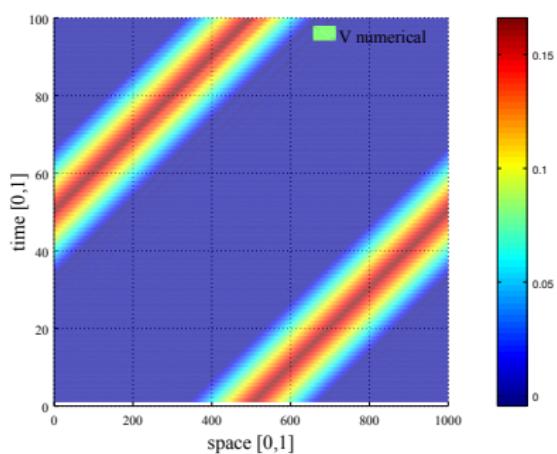
Exact and numerical velocities  $v_F(x, t = 1)$ .Periodical boundaries.  $\alpha = \beta = 0$ 

FIGURE 4: 1D Acoustic system.

Numerical velocity  $v_F(x, t)$ .Periodical boundaries.  $\alpha = \beta = 0$ 

# IMPLEMENTATION OF THE ALGORITHM

## NUMERICAL TESTS

FIGURE 5: 1D Acoustic system.

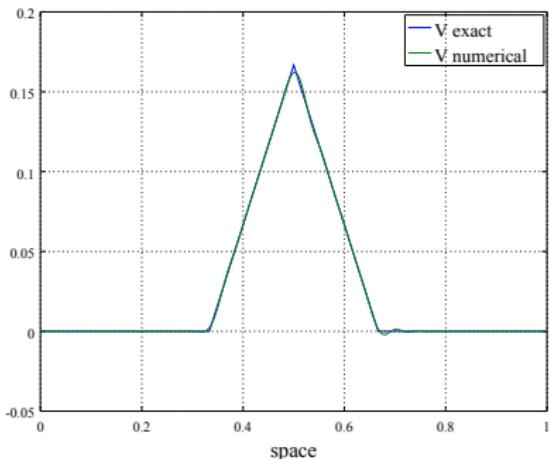
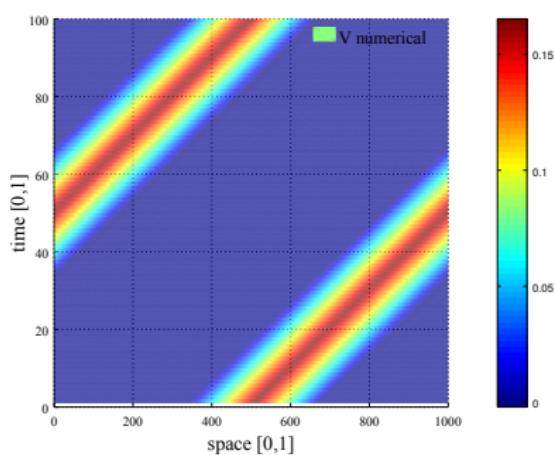
Exact and numerical velocities  $v_F(x, t = 1)$ .Periodical boundaries.  $\alpha = \beta = 0.5$ 

FIGURE 6: 1D Acoustic system.

Numerical velocity  $v_F(x, t)$ .Periodical boundaries.  $\alpha = \beta = 0.5$ 

# IMPLEMENTATION OF THE ALGORITHM

## NUMERICAL TESTS

FIGURE 7: 1D Acoustic system.

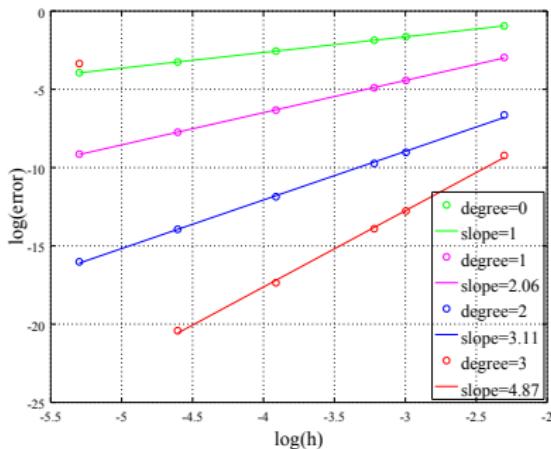
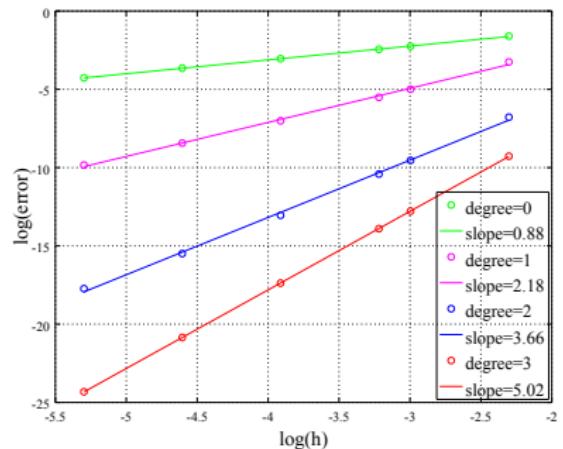
Convergence of velocity  $v_F$  in function of cell size  $h$ .Periodical boundaries.  $\alpha = \beta = 0.0$ 

FIGURE 8: 1D Acoustic system.

Convergence of velocity  $v_F$  in function of cell size  $h$ .Periodical boundaries.  $\alpha = \beta = 0.5$ 

# IMPLEMENTATION OF THE ALGORITHM

## NUMERICAL TESTS

FIGURE 9: 1D Acoustic system.

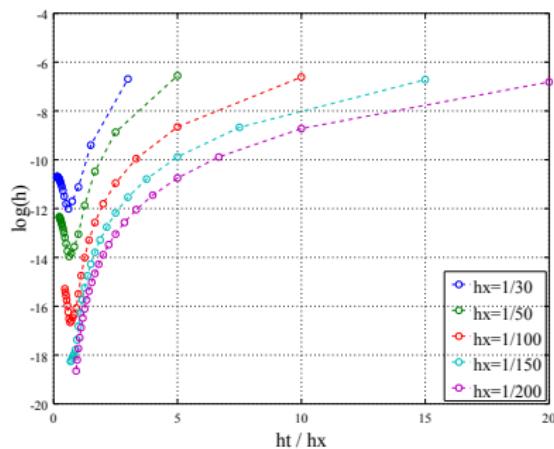
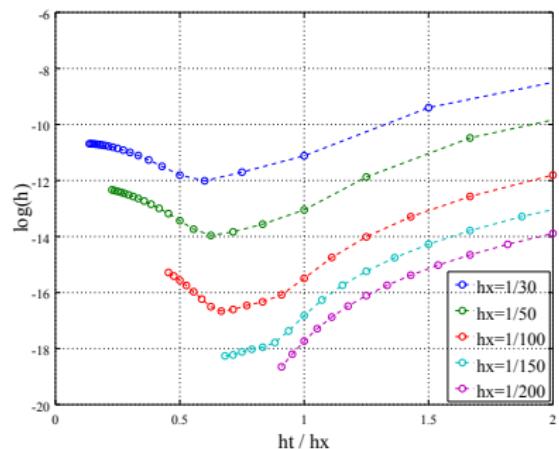
Numerical error in function of ratio  $h_t/h_x$ .Periodical boundaries.  $\alpha = \beta = 0.5$ 

FIGURE 10: 1D Acoustic system.

Numerical error in function of ratio  $h_t/h_x$ .Error minimum corresponds to  $h_t/h_x = 1/(1 + \alpha)$ 

# NUMERICAL RESULTS

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2D SIMULATION

# NUMERICAL RESULTS

## 2D ACOUSTIC AND ELASTODYNAMIC SYSTEMS

FIGURE 11: 2D Acoustic system.

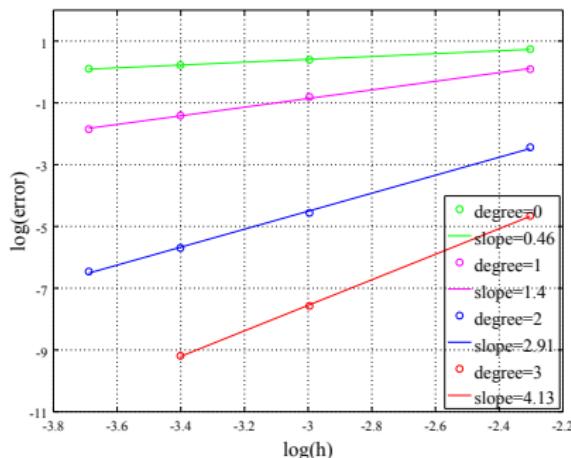
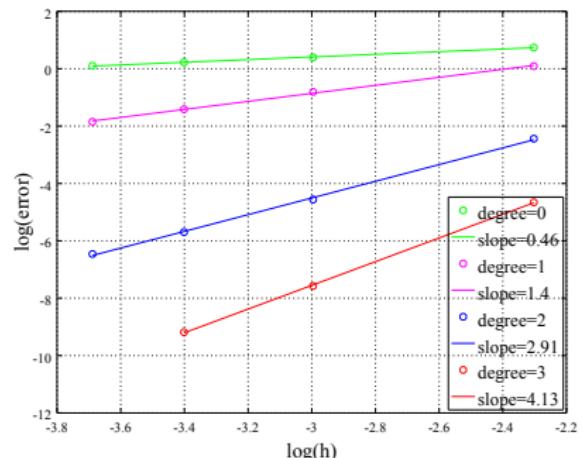
Convergence of velocity  $v_F$  in function of cell size  $h$ 

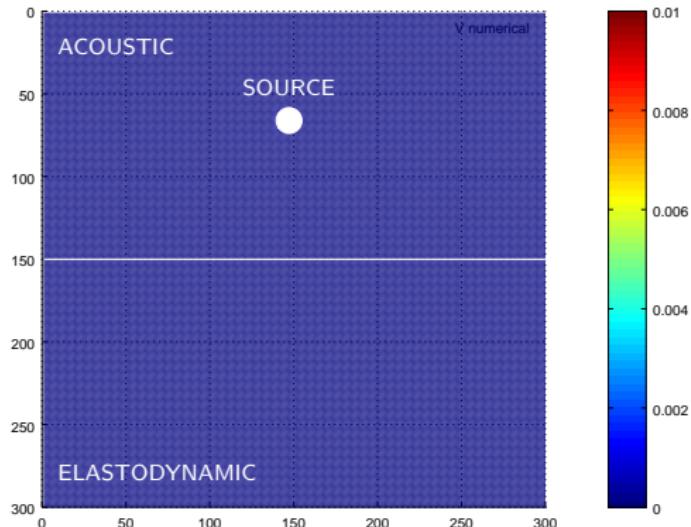
FIGURE 12: 2D Elastodynamic system.

Convergence of velocity  $v_S$  in function of cell size  $h$ 

# NUMERICAL RESULTS

## 2D ELASTO-ACOUSTIC SYSTEM

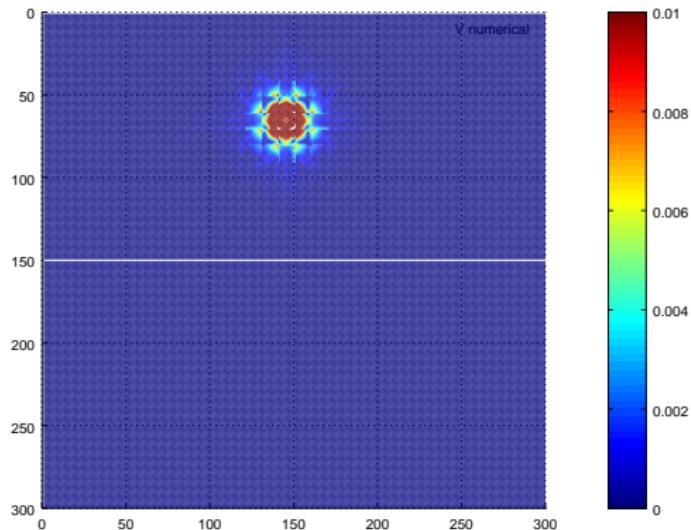
FIGURE 13: 2D Elasto-acoustic system.  
Numerical velocity  $v(x, 0.0)$ . Dirichlet boundaries



# NUMERICAL RESULTS

## 2D ELASTO-ACOUSTIC SYSTEM

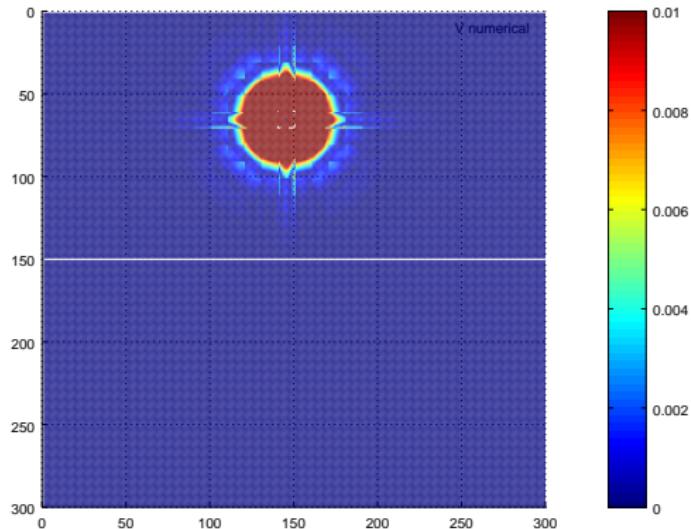
FIGURE 13: 2D Elasto-acoustic system.  
Numerical velocity  $v(x, 0.233)$ . Dirichlet boundaries



# NUMERICAL RESULTS

## 2D ELASTO-ACOUSTIC SYSTEM

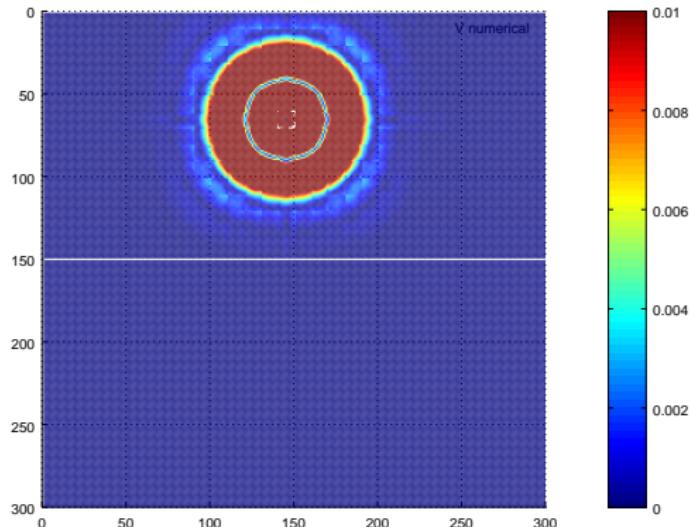
FIGURE 13: 2D Elasto-acoustic system.  
Numerical velocity  $v(x, 0.300)$ . Dirichlet boundaries



# NUMERICAL RESULTS

## 2D ELASTO-AcouSTIC SYSTEM

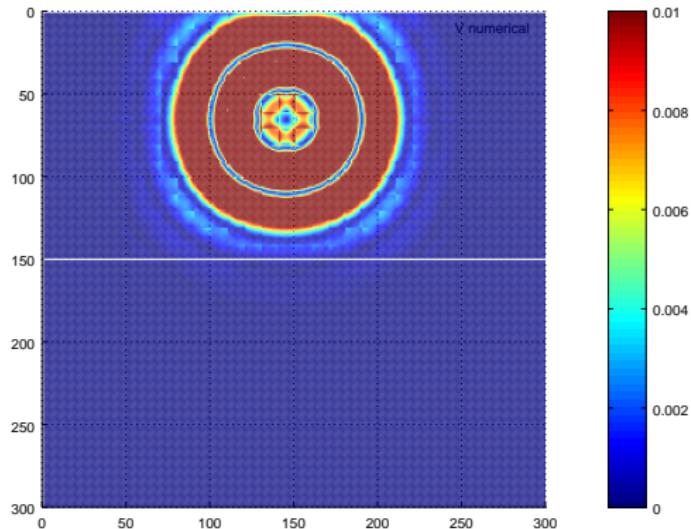
FIGURE 13: 2D Elasto-acoustic system.  
Numerical velocity  $v(x, 0.367)$ . Dirichlet boundaries



# NUMERICAL RESULTS

## 2D ELASTO-ACOUSTIC SYSTEM

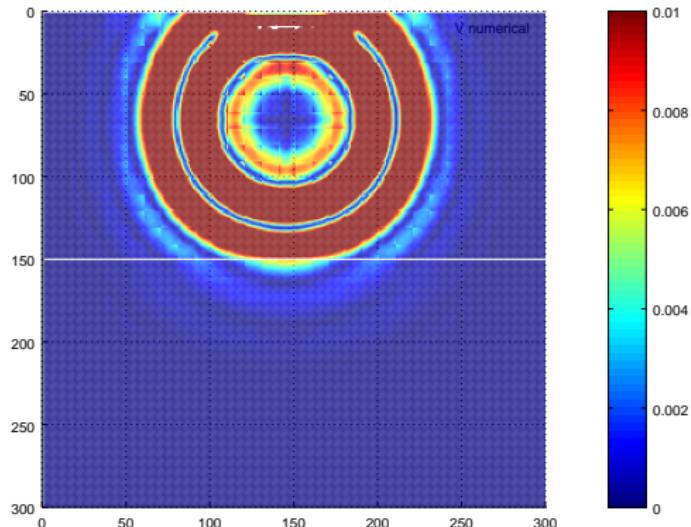
FIGURE 13: 2D Elasto-acoustic system.  
Numerical velocity  $v(x, 0.433)$ . Dirichlet boundaries



# NUMERICAL RESULTS

## 2D ELASTO-AcouSTIC SYSTEM

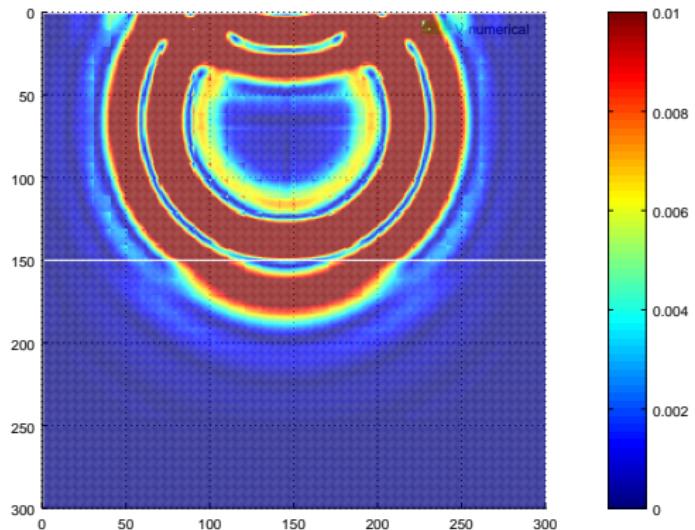
FIGURE 13: 2D Elasto-acoustic system.  
Numerical velocity  $v(x, 0.500)$ . Dirichlet boundaries



# NUMERICAL RESULTS

## 2D ELASTO-AcouSTIC SYSTEM

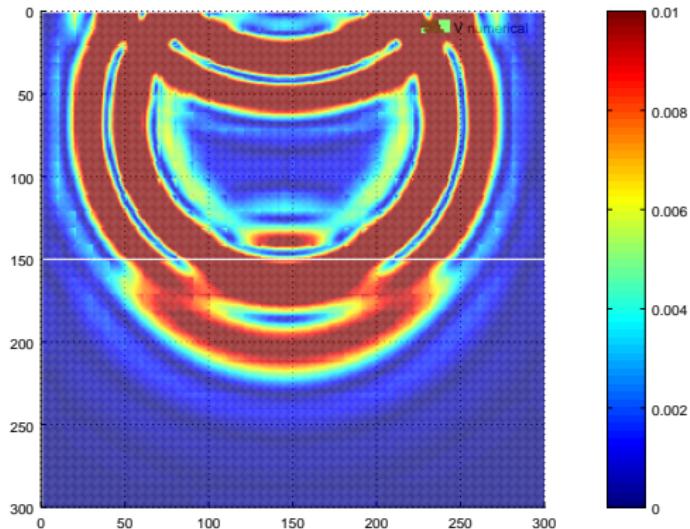
FIGURE 13: 2D Elasto-acoustic system.  
Numerical velocity  $v(x, 0.567)$ . Dirichlet boundaries



# NUMERICAL RESULTS

## 2D ELASTO-AcouSTIC SYSTEM

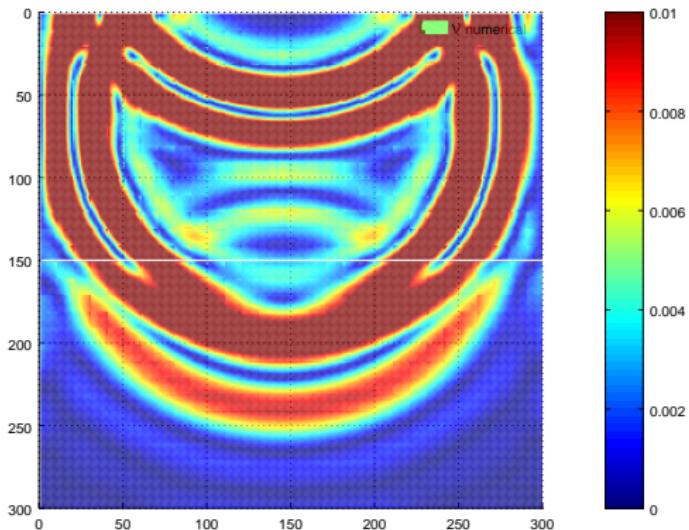
FIGURE 13: 2D Elasto-acoustic system.  
Numerical velocity  $v(x, 0.633)$ . Dirichlet boundaries



# NUMERICAL RESULTS

## 2D ELASTO-ACOUSTIC SYSTEM

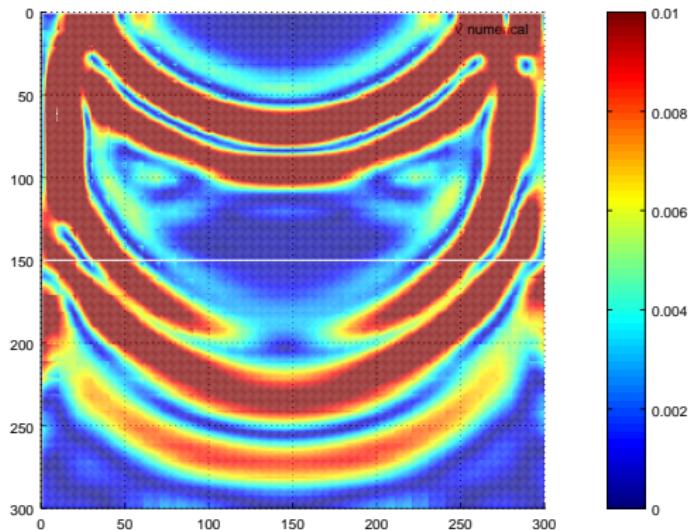
FIGURE 13: 2D Elasto-acoustic system.  
Numerical velocity  $v(x, 0.700)$ . Dirichlet boundaries



# NUMERICAL RESULTS

## 2D ELASTO-AcouSTIC SYSTEM

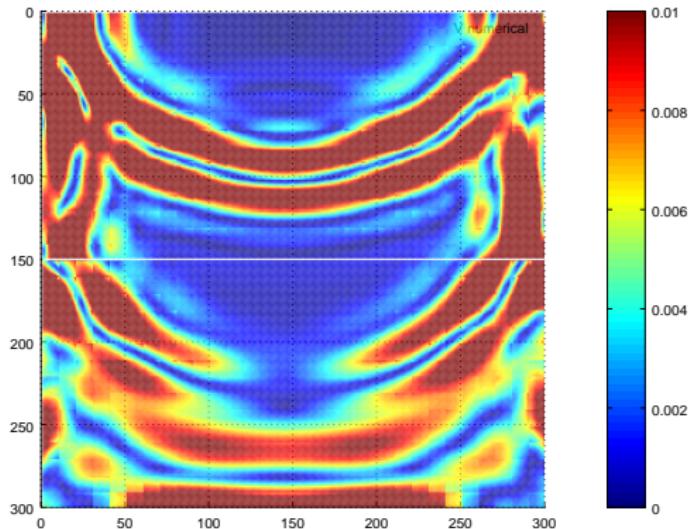
FIGURE 13: 2D Elasto-acoustic system.  
Numerical velocity  $v(x, 0.767)$ . Dirichlet boundaries



# NUMERICAL RESULTS

## 2D ELASTO-AcouSTIC SYSTEM

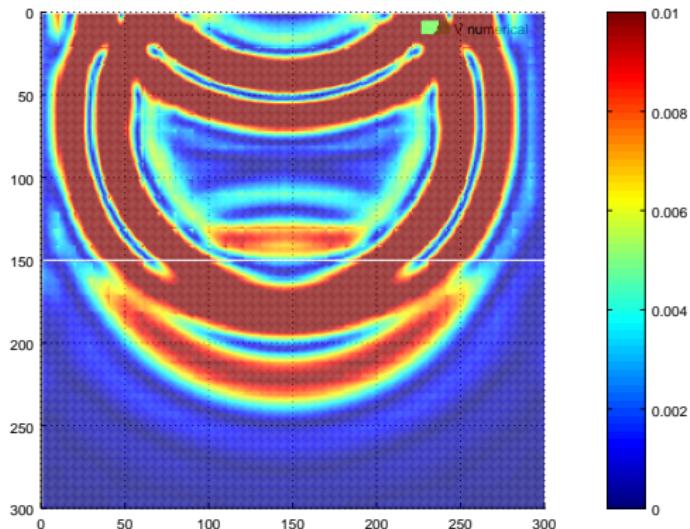
FIGURE 13: 2D Elasto-acoustic system.  
Numerical velocity  $v(x, 0.833)$ . Dirichlet boundaries



# NUMERICAL RESULTS

## 2D ELASTO-AcouSTIC SYSTEM

FIGURE 13: 2D Elasto-acoustic system.  
Numerical velocity  $v(x, 0.667)$ . Dirichlet boundaries



# NUMERICAL RESULTS

## 2D ELASTO-ACOUSTIC SYSTEM

FIGURE 14: 2D Elasto-acoustic system.

Numerical and analytical\* seismograms for  $v_x$ .  
Source (150,75), receiver (195, 210)

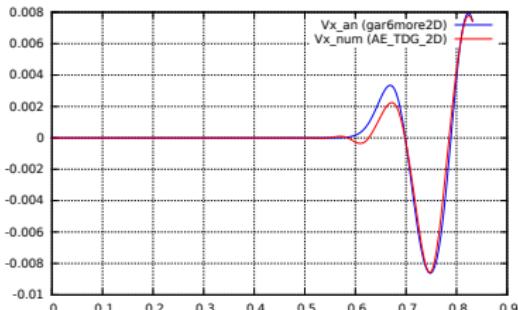
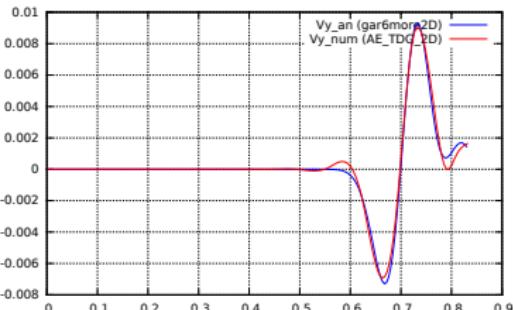


FIGURE 15: 2D Elasto-acoustic system.

Numerical and analytical\* seismograms for  $v_y$ .  
Source (150,75), receiver (195, 210)



\* J.Diaz. *Gar6more2D*.

# CONCLUSION

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# CONCLUSION



- Implementation of the method for 2D acoustic, elastic, elasto-acoustic systems
- Numerical tests, validation of the results by analytical solutions
- Analytical and numerical study of efficiency of the method
- Change-over between time-layers study (initial conditions)

# CONCLUSION

## ON-GOING WORK AND PERSPECTIVES



- Change-over between time-layers study (source)
- Alternative for the global matrix inversion
- Boundary conditions
- Hybrid Trefftz-DG + FVM coupling for the elasto-acoustics

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A.Maciag. *Wave polynomials in elasticity problems.* 2006

THANK YOU FOR ATTENTION!

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