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## Asymptotic Modelling for 3D Eddy Current Problems with a Conductive Thin Layer

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### Abstract

In this work we derive and analyze an equivalent model for 3D Eddy Current problems with a conductive thin layer of small thickness  $\epsilon$ . In our model, the conductive sheet is replaced by its mid-surface and their shielding behavior is satisfied by an equivalent transmission conditions on this interface. The transmission conditions are derived asymptotically for vanishing sheet thickness  $\epsilon$ .

*Key words:* Asymptotic Expansions, Eddy-Current Problems, Thin Conducting Layers, Transmission Conditions

## 1 Introduction

We denote by  $\Omega = \Omega_-^\epsilon \cup \overline{\Omega_0^\epsilon} \cup \Omega_+^\epsilon \subset \mathbb{R}^3$  the domain of study, where  $\Omega_-^\epsilon$  corresponds to a non-conductive linear material,  $\Omega_+^\epsilon$  the exterior of the structure domain, and  $\Omega_0^\epsilon$  a conductive thin layer of constant thickness  $\epsilon$  (see figure 1). The discretisation of the conducting sheet by FEM needs a very fine mesh due to the rapid decay of the field under high conductivity. For this, we approximate a new model defined in  $\epsilon$ -independent domains. Let  $\Sigma$  be a smooth surface, we denote by  $[v]_\Sigma$  and  $\{v\}_\Sigma$  the jump and mean of  $v$  respectively across  $\Sigma$

$$[v]_\Sigma = v|_{\Sigma^+} - v|_{\Sigma^-}, \quad \{v\}_\Sigma = \frac{1}{2}(v|_{\Sigma^+} + v|_{\Sigma^-}), \quad \text{for } v \in (C^\infty(\Omega_\pm))^3.$$

We consider the eddy current problem as follows

$$\begin{cases} \operatorname{curl} H^\epsilon &= \sigma^\epsilon E^\epsilon + J_0 & \text{in } \Omega \\ \operatorname{curl} E^\epsilon &= i\omega\mu_0 H^\epsilon & \text{in } \Omega \\ \operatorname{div}(H^\epsilon) &= 0 & \text{in } \mathbb{R}^3 \\ [E \times n] = [H \times n] &= 0 & \text{on } \Gamma_\pm^\epsilon \end{cases}$$

$$\text{where } \sigma^\epsilon = \begin{cases} 0 & \text{in } \Omega_\pm^\epsilon \\ \sigma_0 = \epsilon^{-2} \bar{\sigma} & \text{in } \Omega_0^\epsilon \end{cases}$$

Let  $u$  be a vector field on  $\Gamma$ , then we denote by  $\gamma_D u = n \times (u \times n)$ , and  $\gamma_N u = \operatorname{curl} u \times n$ , the Dirichlet and Neumann data, respectively.

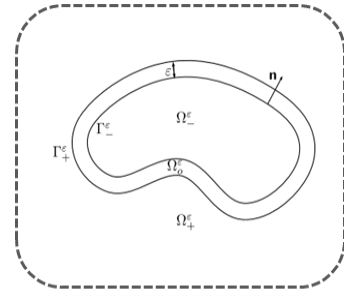


Figure 1: A cross Section of the domain  $\Omega$

## 2 Multiscale Expansion

Assuming that  $\Gamma$  is a smooth surface, then  $E^\epsilon$  and  $H^\epsilon$  can be expanded with an asymptotic expansion in power series of the small parameter  $\epsilon$ . [1]

$$\begin{aligned} E^\epsilon(x) &\approx E_0(x) + \epsilon E_1(x) + \epsilon^2 E_2(x) + \dots + O(\epsilon^k) \quad \text{in } \Omega_\pm^\epsilon \\ H^\epsilon(x) &\approx \mathcal{H}_0(y_\alpha, \frac{h}{\epsilon}) + \epsilon \mathcal{H}_1(y_\alpha, \frac{h}{\epsilon}) + \dots + O(\epsilon^k) \quad \text{in } \Omega_0^\epsilon \end{aligned}$$

Here,  $x \in \mathbb{R}^3$  are the cartesian coordinated, and  $(y_\alpha, h)$  is the local normal coordinate system,  $h \in (-\frac{\epsilon}{2}, \frac{\epsilon}{2})$  is the normal coordinate to  $\Gamma$ . The term  $\mathcal{H}_j$  is a profile defined on  $\Gamma \times (-\frac{1}{2}, \frac{1}{2})$ . The derivation is based on the expansion of the differential operators inside the thin layer  $\Omega_0^\epsilon$ , and the Taylor expansion of  $E_j|_{\Gamma_\pm^\epsilon}$  around the mid-surface  $\Gamma$ .

## 3 Equivalent Model of Order 2

We introduce a problem satisfied by an approximation  $E_\epsilon^k$  of the expression  $E_0(x) + \epsilon E_1(x) + \epsilon^2 E_2(x) + \dots + \epsilon^k E_k(x)$  up to a residual term  $O(\epsilon^{k+1})$ .

The second order approximate solution  $E_\epsilon^1$ , solves

$$\begin{aligned} \text{curl}(\text{curl} E_\epsilon^1) &= i\omega\mu J_0 && \text{in } \Omega_\pm \\ \begin{pmatrix} [\gamma_D E_\epsilon^1]_\Gamma \\ \{\gamma_D E_\epsilon^1\}_\Gamma \end{pmatrix} &= \epsilon \begin{pmatrix} C_1 & 0 \\ 0 & C_2 \end{pmatrix} \begin{pmatrix} \{\gamma_N E_\epsilon^1\}_\Gamma \\ [\gamma_N E_\epsilon^1]_\Gamma \end{pmatrix} && \text{on } \Gamma \end{aligned}$$

where

$$\begin{aligned} C_1 &= -1 + \frac{2 \tanh(\frac{\gamma}{2})}{\gamma}, \quad C_2 = -\frac{1}{4} + \frac{\coth(\frac{\gamma}{2})}{2\gamma} \\ \gamma &= \exp(\frac{3i\pi}{4}) \sqrt{\omega\mu_0\sigma}. \end{aligned}$$

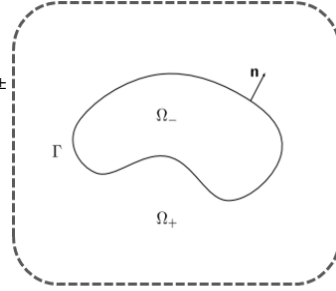


Figure 2: A cross section of the  $\epsilon$ -independent subdomains

## 4 Numerical Results

Numerical experiments are performed to assess the accuracy of our model. The results are in particular compared to the model given in [2]. Complementary simulations will be conducted to study the robustness with respect to the sheet conductivity and the convergence of the modelling error.

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## References

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