Asymptotic Modelling for 3D Eddy Current Problems with a Conductive Thin Layer

Mohammad ISSA¹, Victor PÉRON² and Ronan PERRUSSEL¹

¹ Laboratoire Plasma et Conversion d’Énergie, UMR 5213 CNRS/INPT/UPS, Université de Toulouse, 31071 France
² LMAP CNRS UMR 5142 et Team MAGIQUE 3D INRIA, Université de Pau et des Pays de l’Adour, 64013 France

e-mails: mohammad.issa@laplace.univ-tlse.fr, victor.peron@univ-pau.fr, perrussel@laplace.univ-tlse.fr

Abstract

In this work we derive and analyze an equivalent model for 3D Eddy Current problems with a conductive thin layer of small thickness ε. In our model, the conductive sheet is replaced by its mid-surface and their shielding behavior is satisfied by an equivalent transmission conditions on this interface. The transmission conditions are derived asymptotically for vanishing sheet thickness ε.

Key words: Asymptotic Expansions, Eddy-Current Problems, Thin Conducting Layers, Transmission Conditions

1 Introduction

We denote by \( \Omega = \Omega^c \cup \Omega_0^- \cup \Omega_0^+ \subset \mathbb{R}^3 \) the domain of study, where \( \Omega^c \) corresponds to a non-conductive linear material, \( \Omega_0^- \) the exterior of the structure domain, and \( \Omega_0^+ \) a conductive thin layer of constant thickness ε (see figure 1). The discretisation of the conducting sheet by FEM needs a very fine mesh due to the rapid decay of the field under high conductivity. For this, we approximate a new model defined in ε-independent domains. Let \( \Sigma \) be a smooth surface, we denote by \( [v]_\Sigma \) and \( \{v\}_\Sigma \) the jump and mean of \( v \) respectively across \( \Sigma \)

\[
[v]_\Sigma = v_\Sigma^+ - v_\Sigma^-, \quad \{v\}_\Sigma = \frac{1}{2}(v_\Sigma^+ + v_\Sigma^-), \quad \text{for} \quad v \in (C^\infty(\Omega_\pm))^3.
\]

We consider the eddy current problem as follows

\[
\begin{align*}
\text{curl} H^e &= \sigma^e E^e + J_0 \quad \text{in} \quad \Omega \\
\text{curl} E^e &= i\omega \mu_0 H^e \quad \text{in} \quad \Omega \\
\text{div}(H^e) &= 0 \quad \text{in} \quad \mathbb{R}^3 \\
[E \times n] = [H \times n] &= 0 \quad \text{on} \quad \Gamma^e_\pm
\end{align*}
\]

where \( \sigma^e = \begin{cases} 
0 & \text{in} \quad \Omega^c \\
\sigma_0 = \epsilon^{-2} \bar{\sigma} & \text{in} \quad \Omega_0^+ 
\end{cases} \)

Figure 1: A cross Section of the domain \( \Omega \)

Let \( u \) be a vector field on \( \Gamma \), then we denote by \( Y_D u = n \times (u \times n) \), and \( Y_N u = \text{curl} u \times n \), the Dirichlet and Neumann data, respectively.
2 Multiscale Expansion

Assuming that $\Gamma$ is a smooth surface, then $E^\varepsilon$ and $H^\varepsilon$ can be expanded with an asymptotic expansion in power series of the small parameter $\varepsilon$. [1]

$$
E^\varepsilon(x) \approx E_0(x) + \varepsilon E_1(x) + \varepsilon^2 E_2(x) + \ldots + O(\varepsilon^k) \text{ in } \Omega^\varepsilon \\
H^\varepsilon(x) \approx \mathcal{H}_0(y_{a\varepsilon}, \frac{h}{\varepsilon}) + \varepsilon \mathcal{H}_1(y_{a\varepsilon}, \frac{h}{\varepsilon}) + \ldots + O(\varepsilon^k) \text{ in } \Omega^\varepsilon_0
$$

Here, $x \in \mathbb{R}^3$ are the cartesian coordinated, and $(y_{a\varepsilon}, h)$ is the local normal coordinate system, $h \in (-\frac{1}{2}, \frac{1}{2})$ is the normal coordinate to $\Gamma$. The term $\mathcal{H}_j$ is a profile defined on $\Gamma \times (-\frac{1}{2}, \frac{1}{2})$.

The derivation is based on the expansion of the differential operators inside the thin layer $\Omega_0^\varepsilon$, and the Taylor expansion of $E_j|_{\Gamma^\varepsilon}$ around the mid-surface $\Gamma$.

3 Equivalent Model of Order 2

We introduce a problem satisfied by an approximation $E^k_\varepsilon$ of the expression $E_0(x) + \varepsilon E_1(x) + \varepsilon^2 E_2(x) + \ldots + \varepsilon^k E_k(x)$ up to a residual term $O(\varepsilon^{k+1})$.

The second order approximate solution $E_1^\varepsilon$, solves

$$
curl(curl E_1^\varepsilon) = i\omega \mu_0 J_0 \\
\begin{pmatrix}
[y_D E_1^\varepsilon]_\Gamma \\
[y_D E_1^\varepsilon]_\Gamma
\end{pmatrix} = \varepsilon \begin{pmatrix}
C_1 & 0 \\
0 & C_2
\end{pmatrix} \begin{pmatrix}
\{Y_N E_1^\varepsilon\}_\Gamma \\
\{Y_N E_1^\varepsilon\}_\Gamma
\end{pmatrix} \text{ on } \Gamma
$$

where

$$
C_1 = -1 + \frac{2\tanh\left(\frac{y_0\gamma}{4}\right)}{y_0\gamma}, \quad C_2 = -\frac{1}{4} + \frac{\coth\left(\frac{y_0\gamma}{2}\right)}{2y_0\gamma}
$$

4 Numerical Results

Numerical experiments are performed to assess the accuracy of our model. The results are in particular compared to the model given in [2]. Complementary simulations will be conducted to study the robustness with respect to the sheet conductivity and the convergence of the modelling error.

Acknowledgements

We acknowledge the collaboration of B. BANNWARTH, O. CHADEBEC, and G. MEUNIER who participated in the implementation of the model, and the GDR SEEDS for the financial support.

References
