

# BIGnav: Bayesian Information Gain for Guiding Multiscale Navigation

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## ABSTRACT

This paper introduces BIGnav, a new multiscale navigation technique based on Bayesian Experimental Design where the criterion is to maximize the information-theoretic concept of mutual information, also known as information gain. Rather than simply executing user navigation commands, BIGnav interprets user input to update its knowledge about the user's intended target. Then it navigates to a new view that maximizes the information gain provided by the user's expected subsequent input. We conducted a controlled experiment demonstrating that BIGnav is significantly faster than conventional pan and zoom and requires fewer commands for distant targets, especially in non-uniform information spaces. We also applied BIGnav to a realistic application and showed that users can navigate to highly probable points of interest on a map with only a few steps. We then discuss the tradeoffs of BIGnav—including efficiency vs. increased cognitive load—and its application to other interaction tasks.

## ACM Classification Keywords

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Multiscale navigation; Guided navigation; Bayesian experimental design; Mutual information

## INTRODUCTION

Multiscale interfaces are a powerful way to represent large datasets such as maps and deep hierarchies. However, navigating these spaces can be frustrating and inefficient. Most applications, such as Google Maps, only support pan-and-zoom [22]. Others, such as the DragMag [32], use focus+context techniques. In both cases they leave the user in complete control of navigation, leading to frustrating situations such as getting “lost in desert fog” [17].

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A few techniques assist navigation by taking advantage of the system's knowledge of the information space: topology-aware navigation [21], visual saliency mediated navigation [16], object pointing [11] and semantic pointing [3] “steer” users towards potential targets, therefore reducing the risk of getting lost. Other techniques interpret users' intentions to guide navigation: SDAZ [14], for example, adjusts the zoom level according to the user-controlled velocity. While these approaches have proven effective, we believe we can do better by combining them into a more general framework.

We introduce *BIGnav*, a guided navigation technique that uses both the a priori knowledge of the information space and the progressively acquired knowledge of the user's intention. *BIGnav* guides navigation through a three-step process:

1. The system interprets user input as an intention revealing what the user is and is not interested in;
2. The system then updates a probabilistic model of the information space to take into account this intention;
3. Finally the system navigates to a new view such that the subsequent user input will maximize the expected information gain of the system.

*BIGnav* uses Bayesian Experimental Design [19], an approach where the system “runs experiments” on the user to maximize an expected utility. This utility is information gain, a concept from information theory [28, 29] that represents the amount of information obtained about a variable, here the intended target, from another variable, here the user's input. In other words, *BIGnav* is a form of human-computer partnership where user and system cooperate to achieve a common objective.

The rest of the paper is organized as follows. After a review of related work and a short introduction to Bayesian Experimental Design, we introduce *BIGnav* in detail. We describe a 1D simulation to show how the system can guide navigation to a target using only a few commands. We then report on a controlled experiment with 16 participants comparing standard pan-and-zoom navigation with *BIGnav*. Finally, we describe a real-world application for navigating maps and conclude with directions for future work.

## RELATED WORK

*BIGnav* uses a human-computer partnership to guide navigation. We therefore discuss related work in assisted multiscale navigation and human-computer collaboration and adaptation.

## Assisted Multiscale Navigation

Pan and zoom are the canonical navigation commands in multiscale interfaces. Panning lets users change the position of the view while zooming lets them modify the magnification of the viewport [8, 22]. Navigation tasks consist in acquiring or pointing a specific target, characterized by a position and size. It generally involves view navigation, whereby the user must first bring the target into view, at a scale where it can be selected. Despite being quite different from traditional pointing, Guiard & Beaudouin-Lafon [10] have shown that Fitts' law [6] applies to multiscale pointing. Therefore we adopt Fitts' paradigm and use the index of difficulty (ID) in our controlled experiment to assess pointing performance.

Much work has explored effective pan-and-zoom navigation. Furnas & Bederson [7, 8] introduced space-scale diagrams to better understand navigation trajectories in multiscale interfaces. Igarashi & Hinckley [14] proposed Speed-Dependent Automatic Zooming (SDAZ), where the zooming factor depends on the user-controlled velocity so that users do not need to directly control zooming. By contrast, Appert & Fekete [1] created Orthozoom, a 1D scroller where users control zooming by moving along the scroller's orthogonal dimension, improving navigation in very large documents. *BIGnav* uses a different approach: it assists navigation by interpreting classical pan-and-zoom commands to calculate a new view according to an information gain criterion.

Other approaches assist navigation by exploiting features of the information space. GravNav [16] uses visual saliency to combine standard pan and zoom with an attention vector. Topology-aware navigation [21] guides panning movements along the links of a node-link structure. Content-aware scrolling [15] uses the content of the document to control the direction, speed and zooming factor. In a similar vein, Galyean [9] assists 3D navigation by pulling users along a pre-computed path, while still letting them deviate slightly from it. Similar to these approaches, *BIGnav* takes advantage of the structure of the information space to guide navigation, but uses a probabilistic model to represent its knowledge.

## Collaboration and Adaptation

While the notion of cooperative interaction between humans and computers is quite ancient [18], the concept of human-computer collaboration has emerged in HCI more recently [31]. In human-computer collaboration, two agents, a human and a computer, work together to achieve shared goals. A key aspect of collaboration is communication, for example to define goals, negotiate how to proceed and determine who will do what. Similar ideas can be found in *mixed-initiative design* [12], *human-computer partnership* [27] and *continuous uncertain interaction* [33]. All these approaches suggest that great opportunities lie between systems that provide automatic services [13] and systems where users are in direct control [30]. *BIGnav* applies this general framework to multiscale navigation, which to our knowledge has not been done before.

Several approaches have used the notion of information to guide navigation and pointing. Pirolli & Card [25, 26] have introduced the concept of *information foraging*, whereby people adapt their navigation strategies to increase information

in an information seeking task. An important difference with our work is that in information foraging, users do not know where the information is located. In addition, information foraging is based on Optimal Foraging Theory while *BIGnav* is based on Bayesian Experimental Design [19]. Williamson & Murray-Smith [34] use control theory and an entropy-based method where agents "run 'experiments' on the user" to let the user select a target without pointing. This work is closest to ours, however the proposed system used randomized stimuli while *BIGnav* maximizes the expected information gain.

Finally, Mackay [20] has proposed the concept of *co-adaptation* between people and technology to describe how users adapt to technology but also adapt technology to their needs. However, as with information foraging, co-adaptation comes from the users, not the system. In *BIGnav*, the system adapts to user's input and navigates to the view such that the user's subsequent input will provide maximum information. In other words, the system both adapts to the user and seeks valuable information from the user.

## BACKGROUND: BAYESIAN EXPERIMENTAL DESIGN

This section provides background on Bayesian Experimental Design and Information Theory. Readers familiar with these two concepts can skip this section. Only formulas (1) and (4) are used in the rest of the paper.

Consider a scientist who wants to determine some parameter  $\theta$  of nature. He can choose to perform an experiment  $x$  that will provide an observation  $y$ . A probabilistic model is used where  $\Theta$ ,  $X$  and  $Y$  are the random variables corresponding to  $\theta$ ,  $x$  and  $y$ , respectively. Bayesian Experimental Design [19] provides a framework to optimize the choice of the experiment  $x$  by maximizing an expected utility, commonly defined in terms of the information gained about the parameter  $\theta$  by the experiment  $x$ . The utility may also involve factors such as the financial (or other) cost of performing the experiment.

To optimize the choice of the experiment, the scientist needs two pieces of information, or *priors*:

1. A prior probability distribution  $P(\Theta = \theta)$  for all values of  $\theta$ , which expresses the scientist's knowledge about the random variable  $\Theta$  before the experiment; and
2. A conditional probability distribution<sup>1</sup>  $P(Y = y | \Theta = \theta, X = x)$  of the observation  $Y$  given the actual value of the parameter  $\theta$  and the chosen experiment  $x$ .

After an experiment  $x$  is performed and an observation  $y$  is obtained, the scientist updates his knowledge about the parameter  $\theta$  through Bayes' theorem:

$$P(\Theta = \theta | X = x, Y = y) = \frac{P(Y = y | \Theta = \theta, X = x)P(\Theta = \theta)}{P(Y = y | X = x)} \quad (1)$$

where  $P(Y = y | X = x) = \sum_{\theta'} P(Y = y | \Theta = \theta', X = x)P(\Theta = \theta')$ .

This new probability distribution serves as the new prior, on which the scientist can perform another experiment.

<sup>1</sup>The conditional probability  $P(A = a | B = b)$  reads "the probability of  $A = a$  given  $B = b$ ".

The goal of an experiment is to reduce the uncertainty about  $\Theta$ . As a measure of this uncertainty we use Shannon’s entropy function<sup>2</sup> [28, 29]. Initially, the scientist’s uncertainty about  $\Theta$  is given by  $H(\Theta)$ . After performing an experiment  $X = x$  and having observed  $Y = y$ , the scientist’s uncertainty about  $\Theta$  is given by  $H(\Theta|X = x, Y = y)$ . The *information gain* is the difference between these two uncertainties:

$$IG(\Theta|X = x, Y = y) = H(\Theta) - H(\Theta|X = x, Y = y). \quad (2)$$

It is generally not possible to know a priori how much information a specific experiment will give<sup>3</sup>. However, for each experiment one can calculate the *expected* information gain<sup>4</sup>:

$$IG(\Theta|X = x, Y) = H(\Theta) - H(\Theta|X = x, Y). \quad (3)$$

To calculate the expected information gain, the scientist uses Bayes’ theorem for entropies to convert equation (3) to:

$$IG(\Theta|X = x, Y) = H(Y|X = x) - H(Y|\Theta, X = x). \quad (4)$$

where the first term is given by

$$\sum_y P(Y = y|X = x) \log_2 P(Y = y|X = x).$$

and the second one by

$$\sum_{y, \theta} P(\Theta = \theta) P(Y = y|\Theta = \theta, X = x) \log_2 P(Y = y|\Theta = \theta, X = x).$$

All the elements in these terms are given by the two priors given to scientist. The scientist can therefore calculate the expected information gain for each possible experiment and choose the experiment that he expects to be most informative.

### BIGnav: BAYESIAN INFORMATION GAIN NAVIGATION

The key idea of our approach is to have the system “run experiments” on the user in order to gain information about the user’s goal, i.e., the intended target. *BIGnav* uses Bayesian Experimental Design as follows (Fig. 1a):

- The system plays the role of the scientist;
- The unknown parameter  $\theta$  is the intended target (known only to the user);
- The experiment  $x$  is the view that the system shows to the user after each input; and
- The observation  $y$  is the user input after seeing  $x$ .

### Scenario

Before describing the process in detail, we illustrate how it works through a brief scenario introducing Lucy.

*Lucy is a HCI student exploring how BIGnav works on a 1D map featuring isolated islands. To navigate to a particular island, she issues a series of commands to go left, go right or zoom in. With only two islands on the map, a single input*

<sup>2</sup>The Shannon entropy of a random variable  $V$  that takes  $n$  possible values, the  $i$ -th value of which has probability  $p_i$ , is given by  $H(V) = -\sum_{i=1}^n p_i \log_2 p_i$ . Entropy is usually measured in bits and can be interpreted as the level of uncertainty about a variable. It is maximal when all possible values of the variable have the same probability.

<sup>3</sup>Information gain might be negative but is positive on average.

<sup>4</sup>Also known as the mutual information  $I(\Theta; Y|X = x)$ , which in contrast to equation (2), is always positive.

*is sufficient for BIGnav to determine the intended target and update its probability distribution of most likely target. With three islands, all to the left of Lucy, the first input will go to the most likely island if BIGnav has some prior knowledge about what Lucy wants. Otherwise, it will show the island in the middle and wait for Lucy’s next input, because in this configuration the next input (left, right or zoom in) will uniquely specify the target. If Lucy makes a mistake, her next input will let BIGnav dynamically update its knowledge of Lucy’s interest. With the more difficult task of navigating among 50 islands, BIGnav interprets each command, updates its knowledge and shows Lucy a view where her next command is most likely to maximize the reduction of uncertainty (or information gain) about the intended target. Figure 1(b) shows a case where BIGnav guides Lucy to her target in 4 steps.*

### Detailed Description

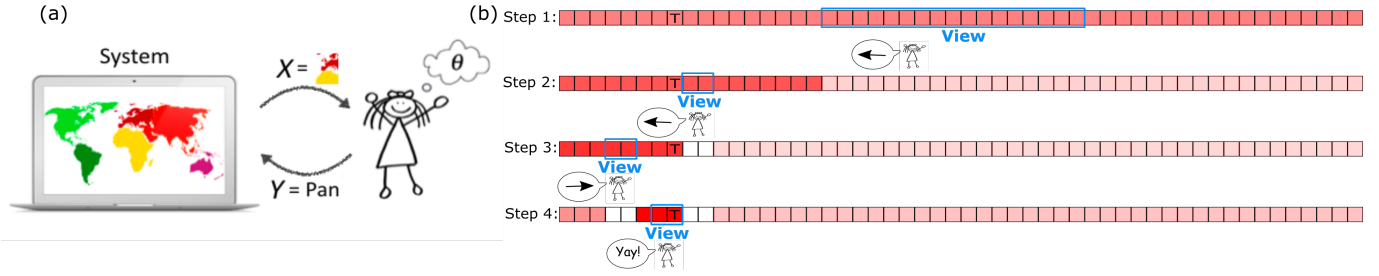
We now describe in detail how *BIGnav* uses Bayesian Experimental Design and information gain to guide navigation. The three key random variables are:

- $\Theta$  represents any point of interest, i.e., possible intended target. For each target  $\theta$ , and the probability that it is the actual intended target is  $P(\Theta = \theta)$ . These probabilities constitute the a priori knowledge that the system has about the user’s interest, and is updated as the user navigates.
- $X$  represents any possible view provided by the system.  $X = x$  is a particular view shown to the user. Note that the number of possible views is potentially very large.
- $Y$  represents any particular command  $y$  issued by the user. The possible input commands are: move towards a direction, zoom in or click on the target when it is big enough to be clickable. Note that zooming out is not required in this framework: if the target is out of view, the user should indicate in which direction it is rather than zooming out.

We now describe the three-stage navigation process.

(1) *Interpreting user input*: Given the view  $x$  shown to the user and the user’s intended target  $\theta$ ,  $P(Y = y|\Theta = \theta, X = x)$  is the probability that the user provides an input command  $Y = y$  given  $\theta$  and  $x$ . This probability distribution is the system’s interpretation of the user’s intention when giving this command. For instance, if island B is to the left of Lucy, what is the probability of Lucy giving the left command when knowing that island B is located to her left?  $P(\text{go left} \mid \text{island B is the intended target, island B is located to the left of the current view}) = 1$  if Lucy is completely confident about what she is doing. But maybe Lucy is not accurate all the time. Say she is only correct 95% of time, then we need to consider that she makes errors. For instance,  $P(\text{go left} \mid \text{island B is the intended target, island B is located to the left of the current view}) = 0.95$  and  $P(\text{go right} \mid \text{island B is the intended target, island B is located to the left of the current view}) = 0.05$ .  $P(Y|\Theta = \theta, X = x)$  is a priori knowledge that must be given to the system. In the implementation section, we describe how we define it in 1D and 2D situations respectively.

(2) *Updating system’s knowledge*: Given the view  $x$  shown to the user and the user reaction  $y$  to that view, the system can update its estimate  $P(\Theta|X = x, Y = y)$  of the user’s interest with equation (1). If the system has no prior knowledge about the



**Figure 1.** (a) *BIGnav*: The system is a scientist experimenting on the user (Lucy).  $\theta$  is the intended target in Lucy’s mind.  $X$  is the view provided by the system. Lucy provides an input  $Y$  given what she sees in the view ( $X$ ) and what she wants ( $\theta$ ). (b) Lucy navigates to a particular island ( $T$ ) among 50 others with *BIGnav*. The color gradient shows the probability of each island being Lucy’s target. The redder, the higher the probability.

user’s intended target, e.g., at the beginning, each  $\theta$  has the same probability of being the target and  $P(\Theta)$  is uniform. As the user issues commands, the system gains knowledge about the likelihood that each point of interest be the target, reflected by the changes to the probability distribution. This is done, for each point of interest, by taking its previous probability, multiplying by the above user input function  $P(Y = y | \Theta = \theta, X = x)$ , and normalizing it so that the sum of the new probabilities over all the points of interest equals one.

(3) *Navigating to a new view*: With the new probability distribution after receiving user input, *BIGnav* then goes over each view  $x \in X$ , calculates its expected information gain with equation (4) and picks the view for which it is maximal. To maximize equation (4), *BIGnav* looks for a trade-off between two entropies. To maximize the first term, the view should be such that all user commands given that view are equally probable (for the system). To minimize the second term, the view should provide the user with meaningful information about the points of interest. Maximizing a difference does not necessarily mean to maximize the first term and minimize the second, so the maximum information gain is a trade-off between these two goals. For example, showing only ocean will increase the first term but will also increase the second term. After locating the view with maximal information gain, *BIGnav* navigates there and waits for user’s next input.

## IMPLEMENTATION

We now describe our implementation of *BIGnav*. We first go through the 1D case with the last example in scenario (Fig. 1b), then show the implementation of the 2D case.

### BIGnav in 1D

The 50 islands are the points of interest, therefore  $\Theta = \{1, 2, \dots, 50\}$ . The system does not have prior knowledge about Lucy’s intended target island, so the initial distribution is  $P(\Theta_1 = i) = \frac{1}{50}$ .

The view presented to Lucy at each step is defined by  $X = \{[a, b] \subseteq [1, 50]\}$ . The maximum zoom factor is such that a view cannot be smaller than two blocks ( $b - a \leq 2$ ). Since it is a 1D map, Lucy can go to the left, go to the right, zoom in or select the target if the view is at the maximum scale. We note these commands  $Y = \{\leftarrow, \rightarrow, +$  (zoom in),  $\bullet$  (click target  $i$ )\}.

We start by modeling Lucy’s behavior. We consider that Lucy makes some mistakes when panning and zooming, but will

not miss the target when it is shown in the view and clickable:

$$P(Y = \rightarrow | \Theta = \theta, X = [a, b]) = \begin{cases} 0.9 & b < \theta \\ 0.05 & a < \theta \\ 0.05 & a \leq \theta \leq b \text{ and } b - a > 2 \\ 0 & a \leq \theta \leq b \text{ and } b - a \leq 2 \end{cases}$$

$$P(Y = \leftarrow | \Theta = \theta, X = [a, b]) = \begin{cases} 0.05 & b < \theta \\ 0.9 & a < \theta \\ 0.05 & a \leq \theta \leq b \text{ and } b - a > 2 \\ 0 & a \leq \theta \leq b \text{ and } b - a \leq 2 \end{cases}$$

$$P(Y = + | \Theta = \theta, X = [a, b]) = \begin{cases} 0.05 & b < \theta \\ 0.05 & a < \theta \\ 0.9 & a \leq \theta \leq b \text{ and } b - a > 2 \\ 0 & a \leq \theta \leq b \text{ and } b - a \leq 2 \end{cases}$$

$$P(Y = \bullet | \Theta = \theta, X = [a, b]) = \begin{cases} 1 & a \leq \theta \leq b \text{ and } b - a \leq 2 \\ 0 & \text{otherwise.} \end{cases}$$

In Figure 1b, the islands are represented by square boxes and colored in shades of red indicating the degrees to which the system believes the island is the target, i.e., island  $i$  is darker than  $j$  if  $P(\Theta = i) > P(\Theta = j)$ . Island 8 has a **T** indicating that it is the target. The blue rectangle is the view that the system shows to Lucy. After seeing the view, Lucy provides an input command  $y$  to the system.

We can now show *BIGnav* in action.

**Step 1:** Since the initial distribution is uniform, the system’s uncertainty about Lucy’s target is  $H_1 = H(\Theta_1) = \log_2 50 = 5.64$  bits.

The system then goes over every image  $[a, b]$ , finds that  $[18, 34]$  maximizes the expected information gain and displays the corresponding initial view to Lucy. In this case the expected information gain from Lucy’s next action is  $IG(\Theta_1 | X = [18, 34], Y) = 1.08$  bits.

Lucy inputs  $\leftarrow$  after seeing  $[18, 34]$ . The system then updates its knowledge with equation (1) and ends up with a new distribution  $\Theta_2$  given by  $P(\Theta_2) = P(\Theta_1 | X = [18, 34], Y = \leftarrow)$ . Using Bayes’ theorem we have:

$$P(\Theta_2 = i) = \begin{cases} 0.05 & i < 18 \\ 0.002 & i \geq 18. \end{cases}$$

The updated uncertainty is  $H_2 = H(\Theta_2) = 4.65$  bits, resulting in an actual information gain  $H_1 - H_2 = 0.99$  bits, very close to the expected information gain of 1.08 bits.

**Step 2:** The system now searches for the best view using the new distribution  $P(\Theta_2)$ , finds that it is  $[9, 10]$  with an expected information gain of  $IG(\Theta_2 | X = [9, 10], Y) = 1.24$  bits and displays it to Lucy. She then inputs  $\leftarrow$  after seeing  $[9, 10]$ . The system then updates  $\Theta_2$  to  $\Theta_3$  as follows:

$$P(\Theta_3 = i) = \begin{cases} 0.12 & i < 9 \\ 0 & 9 \leq i \leq 10 \\ 0.006 & 10 < i < 18 \\ 0.0003 & i \geq 18. \end{cases}$$

The entropy of  $\Theta_3$  is  $H_3 = 3.36$  bits, so the actual information gain for this step is  $H_2 - H_3 = 1.29$  bits, higher than the expected information gain of 1.24 bits.

**Step 3:** With the same process, the best view is now  $[4, 5]$  with an expected information gain of  $IG(\Theta_3 | X = [4, 5], Y) = 1.58$  bits. Lucy inputs  $\rightarrow$ , leading to the updated distribution

$$P(\Theta_4 = i) = \begin{cases} 0.01 & i < 4 \\ 0 & 4 \leq i \leq 5 \\ 0.28 & 5 < i < 9 \\ 0 & 9 \leq i \leq 10 \\ 0.015 & 10 < i < 18 \\ 0.0007 & i \geq 18. \end{cases}$$

The entropy of  $\Theta_4$  is  $H_4 = 2.70$  bits, so the actual information gain is  $H_3 - H_4 = 0.66$  bits, compared to the expected information gain of 1.58 bits.

**Step 4:** The best view is now  $[7, 8]$  with an expected information gain of  $IG(\Theta_4 | X = [7, 8], Y) = 1.84$  bits. Lucy sees that the target island is in the view and happily clicks on it. The updated distribution is updated to

$$P(\Theta_5 = i) = \begin{cases} 1 & i = 8 \\ 0 & \text{otherwise.} \end{cases}$$

The entropy of  $\Theta_5$  is  $H_5 = 0$  bits since there is no more uncertainty about the target. The actual information gain is  $H_4 - H_5 = 2.7$  bits, while the expected gain was 1.84 bits.

Lucy finds her target island in only 4 steps. At step 1, *BIGnav* divides the map in 3 so that the three commands (left, right and zoom in) have equal probability. It does not consider a click as the view is still far from being fully zoomed-in to select the target. At step 2, one would expect it to divide the left third of the map in 3 again so that the view would be about 5 boxes wide. However, since it is close to the maximum scale, and it knows that Lucy never misses her target when it is in the view and is clickable, showing a 2-box zoomed-in view will give *BIGnav* extra information: if this is the target, Lucy will click on it; if it is not and Lucy moves away, the probabilities of these two boxes become 0. Step 3 and step 4 work similarly.

We ran 200 simulations with 50 islands and a uniform initial distribution and found that it required 3.3 steps on average.

### BIGnav in 2D

We implemented *BIGnav* in a 2D application using Java 1.8 and the open source ZVTM toolkit [23]. As for the 1D case, we need to define  $\Theta$ ,  $X$  and  $Y$ :

$\Theta$  represents points of interest in the multiscale information spaces. Each point  $\theta_i$  is defined by a triplet  $(x_i, y_i, p_i)$  where

$(x_i, y_i)$  is the coordinate of point  $i$  and  $p_i$  is the dynamic probability that point  $i$  is the user's intended target.

$X$  represents views that the system can show to the user. A view is defined by a triplet  $(v_x, v_y, z)$  where  $(v_x, v_y)$  is the center of the view and the zoom level  $z$  determines the view size. A view is fully zoomed in when  $z = 1$ . In traditional multiscale navigation, the system can pan and zoom continuously, leading to a huge number of possible views. With *BIGnav*, we need to calculate the information gain corresponding to every single view  $X = x$ , which would incur an enormous computational cost if views could be centered at any pixel and have any size. We therefore discretize the set of views by using tiles and discrete zoom factors. The tiles are  $200 \times 150$  pixels each, and each tile can contain at most one point of interest. When  $z = 1$ , the view is composed of  $4 \times 4$  tiles. Each successive value of  $z$  increases the number of tiles ( $5 \times 5$ ,  $6 \times 6$ , etc.).

$Y$  represents input commands that the user can provide. In many pan-and-zoom applications, users can pan in any direction by a range of distances, and zoom in and out by fixed amounts. As for the views, we reduce this set of commands to make computation tractable in our prototype. We slice the view into nine regions representing eight panning directions and a central zooming region (Fig. 2). The eight panning regions have a  $45^\circ$  angle, and the zooming region is half the size of the view. A single movement of the mouse wheel movement triggers a zoom while a drag action triggers a pan. The angle between the mouse-down and mouse-up points of the drag determines the panning direction. The last input is a click on the target, available only when zoom level  $z = 1$ .

We now describe our implementation of the navigation steps.

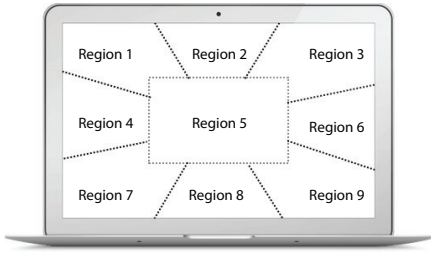
(1) *Interpreting user input:* To interpret user input, we need to define  $P(Y = y | \Theta = \theta, X = x)$ , i.e., the probability of each command given a target and a view, e.g.,  $P(\text{pan East} | \text{target } (5, 7), \text{view } (4, 4, 2))$ . If the user were perfectly reliable, we could assign a probability of 1 to the correct command for each target  $\theta$  and each view  $x$ , and 0 to the others. But we know that users make errors. To model the error rate, we collected data during a calibration session. The goal was to determine how confident participants were when issuing commands. The task was to indicate in which direction the target was in a set of views. A set of concentric circles, identical to those used in the experiment below, showed the direction of the target when it was not within the view (Fig. 3a).

We tested all ten input commands  $Y$  (8 pan operations, zoom in and click on the target) with 5 repetitions each, resulting in 50 trials per participant ( $N = 16$ ). The results (Table 1) show that 90% of panning commands are correct and 4% are in one of the adjacent directions (Fig. 2). For zooming commands, 95% of the commands are correct while for clicking on the target, 100% of the commands are correct.

(2) *Updating system's knowledge:* We use equation (1) to update the probabilities  $p_i$  of each point of interest being the target given the current view  $x$ :

For all points of interest  $\theta_i$ , the new  $p'_i$  is the previous  $p_i$  multiplied by the user expected behavior  $P(Y = y | \Theta = \theta_i, X = x)$  divided by the normalization over all points of interest.





**Figure 2. Nine regions representing user input, delimited by dotted lines. Panning regions also include the space outside the current view.**

Command	Main Region	Adjacent Regions	Other Regions
Pan	0.90	0.04	0.0033
Zoom	0.95	0.00625	0.00625
Click	1	0	0

**Table 1. Calibration results used as prior knowledge about the user behavior  $P(Y = y | \Theta = \theta, X = x)$ .**

(3) *Navigating to a new view* with maximum expected information gain: For each view  $x$  and each user input  $y$ , the expected information gain is the difference between two uncertainties:

$$\begin{aligned} & \text{Uncertainty before user input } y = \\ & \quad \text{minus the sum of } p_i \times \log_2 p_i \text{ over all points of interest} \\ & \text{Uncertainty after user input } y = \\ & \quad \text{minus the sum of } p'_i \times \log_2 p'_i \text{ over all points of interest} \end{aligned}$$

We then calculate the new view:

For all possible views  $x$ , calculate expected information gain with equation (4). Return the view  $(vx_{max}, vy_{max}, z_{max})$  with maximum information gain and display it.

## EXPERIMENT

Our goal is to study the performance of *BIGnav* in what Javed et al. call *micro-level navigation*, when the user has decided on a destination and needs to navigate to it [16]. This is different from searching [24] or wayfinding [5] tasks where the user does not know where the target is located.

We conducted a controlled experiment where participants have to navigate towards a known target. Based on the theoretic analysis, we formulate four hypotheses:

**H1:** *BIGnav* is faster than *STDnav* for distant targets;

**H2:** *BIGnav* performs better in non-uniform information spaces, i.e., when the system has prior knowledge of users' interest;

**H3:** *BIGnav* outperforms *STDnav* in terms of number of commands and rate of decreasing uncertainty;

**H4:** *STDnav* is preferred by users, more comfortable and intuitive.

### Participants

Sixteen participants (3 female), age 24 to 30 (mean = 25.9,  $\sigma = 1.7$ ), were recruited from our institution and received a handful of candies for their participation. All of them were right-handed, had normal or corrected-to-normal vision, and were familiar with WIMP interfaces. Participants were instructed

to navigate to the target as fast as they could but were not informed of the condition they used.

### Apparatus

The experiment was conducted on a MacBook Air with a 1.4 GHz processor and 4 GB RAM. The software was implemented in Java with the ZVTM toolkit [23]. The window was  $800 \times 600$  pixels, centered on a 13-inch screen set to  $1440 \times 900$  resolution. A standard mouse was used with the same sensitivity for all participants.

### Procedure

We use a full-factorial within-participants design with a main factor: navigation technique (TECH); and two secondary factors: distribution of targets (DISTR), index of difficulty (ID).

#### Navigation Technique (TECH)

We compare *BIGnav* with standard pan-and-zoom:

- *BIGnav*: our guided navigation technique. The ten user commands (eight pan, zoom in, select target) and error rates are as described in the previous section.
- *STDnav*: the standard pan and zoom technique, used as baseline. A left mouse drag pans the view in world space proportional to the number of pixels dragged in screen space, and the mouse wheel zooms around the center of the view.

In order to compare information gains between the two conditions, we make the same computations as for *BIGnav* in the *STDnav* condition, except for the display of the new view.

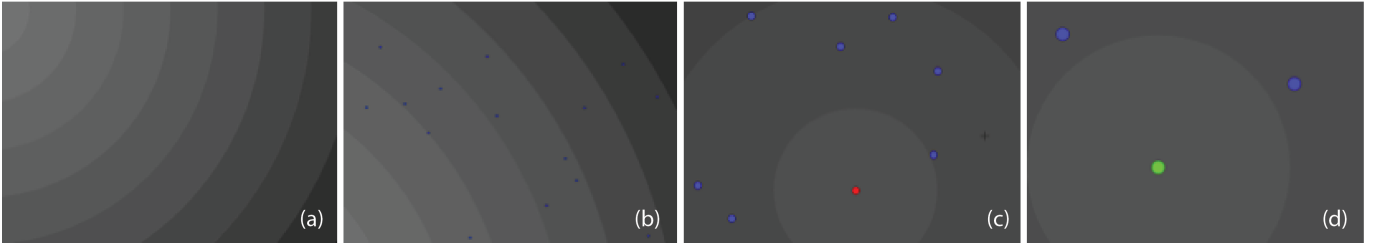
#### Distribution (DISTR)

In order to compare different types of information spaces, we compared six distributions of points of interest by combining three spatial distributions (*Grid*, *Random* and *Cluster*) with three probability distributions (*Uniform*, *Random*, *Cluster*) of the a priori likelihood of each target. Since not all combinations are meaningful, we selected six of them. The first three have a uniform probability distribution, i.e., all points of interest have equal probability of being the target, and different spatial distributions:

- *Grid+Uniform*: points of interest are arranged in a grid, providing a strong visual pattern.
- *Random+Uniform*: points of interest are placed randomly.
- *Cluster+Uniform*: points of interest are organized in clusters, which is typical of geographical maps [4], where a central city is surrounded by smaller towns. We used 5 clusters of 10 targets.

The other three distributions use a non-uniform probability distribution of being the target:

- *Grid+Random*: points of interest are on a grid with random probabilities of being the target. These probabilities are bounded by *Uniform* and *Cluster*.
- *Random+Random*: points of interest are randomly distributed and have random probabilities.
- *Cluster+Cluster*: points of interest are clustered and the probability of the center of each cluster is ten times higher than that of the surrounding points of interest.



**Figure 3.** (a) Calibration: participants were asked to give a direction that indicates where they think the target is located (here: north-west). (b-c) Experimental condition with the target indicated in red. (d) Last step in the experimental condition: the target becomes green when it is clickable.

The first three configurations are meant to demonstrate that *BIGnav* works well even without prior knowledge about potential targets. The other three configurations are meant to assess the added advantage, if any, of using such prior knowledge. In particular, the last distribution is typical of, e.g., maps.

#### Index of Difficulty (ID)

The ID is related to the distance between the initial position of the view and the target to navigate to. Using Fitts' definition of the ID [6], the distance  $D$  to travel is  $D = 2^{ID} \times W$ , where  $W$  is the (constant) target width. We adopted the same large IDs as in other multiscale navigation studies [1, 16]: 10, 15, 20, 25 and 30 bits.

We used a  $[2 \times 6 \times 5]$  within-subject design: we tested 2 TECH for 6 DISTR and 5 ID conditions. Each condition was replicated 5 times, so that each participant performed 300 trials. We blocked the conditions by TECH. Half the participants started with *STDnav* and the other half with *BIGnav*. Within each block, we systematically varied the order of DISTR and ID combinations across participants using a Latin square so as to reduce the influence of learning effects. For each condition, the targets were drawn randomly according to the probability distribution of the DISTR condition. All participants used the same target in the same DISTR  $\times$  ID  $\times$  Replication condition.

#### Task

The task is a multiscale pointing task: starting from a fully zoomed-out view, the participant must navigate towards the target until it is fully zoomed in and click on it. The target is surrounded by concentric circles so that it is always possible to tell in which direction and how far it is (Fig. 3).

The information space contains 50 points of interest: 49 are distractors and displayed in blue, one is the target and displayed in red. The ID is used to compute the scale of the initial view so that it contains all the points of interest. The target becomes green and clickable only when the view is fully zoomed in.

Participants first receive general instructions about the session and performed several practice trials with each technique. After the session, they answer a questionnaire asking them for feedback and comments on the experiment and the techniques. A typical session lasts 60 minutes, including training.

#### Data Collection

For each trial, the program collects the task completion time (TCT), the commands that the participants issued, the uncer-

tainty and position of the view at each step and the information gain after each command. We collected 2 TECH  $\times$  6 DISTR  $\times$  5 ID  $\times$  5 Replications  $\times$  16 Participants = 4800 trials in total.

## RESULTS

For our analyses, we first removed 23 missed trials (about 0.5%) and then 54 outliers (about 1.1%) in which TCT was 3 standard deviations larger than the mean. We verified that misses and outliers were randomly distributed across participants, techniques and conditions.

#### Task Completion Time

Table 2 shows the results of a repeated-measures full factorial ANOVA on TCT. All main effects are significant, as well as two interaction effects: TECH  $\times$  DISTR and TECH  $\times$  ID.

Figure 4 shows the interaction effect between TECH and ID for task completion time (TCT). On average, *BIGnav* is 24.1% faster than *STDnav* across all ID. A post-hoc Tukey HSD test reveals a robust interaction effect: *BIGnav* is significantly faster than *STDnav* when ID  $>$  15 ( $p < 0.0001$ ), significantly slower when ID = 10 ( $p < 0.0001$ ) and not significantly different for ID = 15 ( $p = 0.99$ ). These results support **H1**: *BIGnav* is 22.3% faster than *STDnav* for ID = 25 and 35.8% faster for ID = 30.

The ANOVA also reveals an interaction effect between TECH and DISTR. A post-hoc Tukey analysis shows that for *STDnav*, DISTR does not affect TCT. *BIGnav*, however, shows a larger advantage in non-uniform information spaces (13.7% faster,  $p < 0.01$ ) than in uniform ones, with *Cluster+Cluster* being the fastest (Fig. 5), supporting **H2**. However, there is no significant difference within probability distribution conditions. For non-uniform distributions: *Cluster+Cluster* ( $6.51 \pm 1.71$ s), *Random+Random* ( $6.74 \pm 1.55$ s) and *Grid+Random* ( $6.62 \pm 1.67$ s). For uniform distributions: *Grid+Uniform* ( $7.70 \pm 1.60$ s), *Random+Uniform* ( $7.83 \pm 1.58$ s), and *Cluster+Uniform* ( $7.70 \pm 1.62$ s).

We then further compare *BIGnav* and *STDnav* when ID  $>$  15 in all DISTR conditions with a post-hoc Tukey HSD test. The results indicate that *BIGnav* is significantly faster than *STDnav* for distant targets especially in non-uniform information spaces. Particularly, when ID = 25, *BIGnav* is 16.9% faster than *STDnav* in uniform distributions ( $p < 0.001$ ) and is 27.6% faster in non-uniform ( $p < 0.0001$ ). When ID = 30, *BIGnav* is 31.7% faster than *STDnav* in uniform distributions ( $p < 0.0001$ ) and is 40.0% faster in non-uniform ( $p < 0.0001$ ).

Factors	df, den	F	p
TECH	1, 15	4948.94	< 0.0001
DISTR	5, 75	38.32	< 0.0001
ID	4, 60	5363.35	< 0.0001
TECH × DISTR	5, 75	47.23	< 0.0001
TECH × ID	4, 60	955.19	< 0.0001
DISTR × ID	20, 300	1.26	= 0.2
TECH × DISTR × ID	20, 300	0.72	= 0.8

Table 2. Full-factorial ANOVA on TCT.

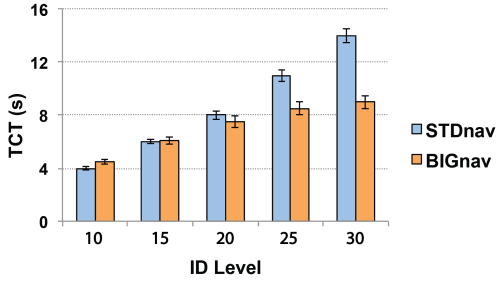


Figure 4. Means and confidence intervals of TCT by ID.

In summary, these results support hypotheses **H1** and **H2**: *BIGnav* is faster than *STDnav* for distant targets and especially in non-uniform information spaces. *BIGnav* is also not significantly different from *STDnav* for close targets (ID = 15).

### Number of Commands

In order to get a sense of the differences in control strategies across conditions, we compare the number of user commands issued by the participants. Because of the continuous control in the *STDnav* conditions, we aggregate the mouse and wheel events as follows: we count one panning command per sequence from a mouse down to a mouse up, and one zooming command per series of mouse wheel with less than 300ms between them.

We perform a TECH × DISTR × ID full-factorial ANOVA on the number of commands issued (Table 3) and find that while TECH and ID significantly affect the number of commands used, DISTR has a non-significant effect. The ANOVA also indicates that the TECH × ID interaction effect is significant. A post-hoc Tukey HSD confirms that while the number of commands progressively increases with ID in *STDnav*, it is barely

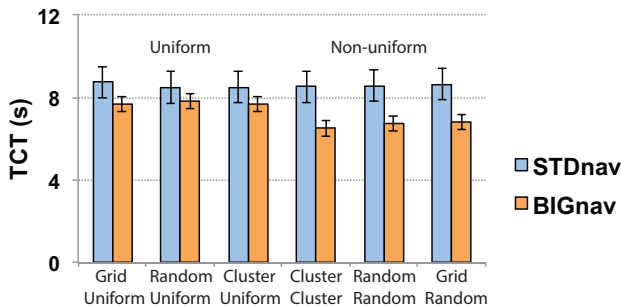


Figure 5. Means and confidence intervals for TCT by DISTR.

Factors	df, den	F	p
TECH	1, 15	364818.2	< 0.0001
DISTR	5, 75	0.99	= 0.4
ID	4, 60	10636.43	< 0.0001
TECH × DISTR	5, 75	0.23	= 0.9
TECH × ID	4, 60	11783.96	< 0.0001
DISTR × ID	20, 300	0.65	= 0.9
TECH × DISTR × ID	20, 300	0.71	= 0.8

Table 3. Full-factorial ANOVA on the number of commands.

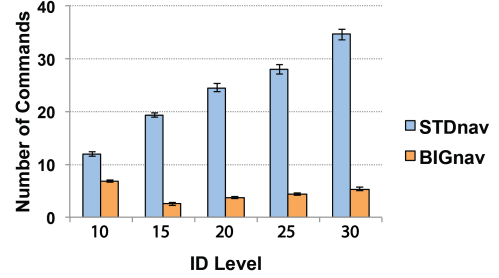


Figure 6. Means and confidence intervals for # of commands by TECH.

affected by ID in *BIGnav* (Fig. 6). For instance, when ID = 25, *STDnav* requires 28.1 commands on average whereas *BIGnav* requires 4.4 commands. When ID grows to 30, *STDnav* requires 34.5 commands on average whereas *BIGnav* requires only 5.8 commands.

Interestingly, *BIGnav* results in more commands for ID = 10 than for larger distances. Although it still outperforms *STDnav*, it requires significantly more commands for ID = 10 than for the other ID except = 30. The reason may be that the target falls into the view very quickly but *BIGnav* tends to move it away to gain more information, because it does not know it is the target. Some participants got frustrated by this behavior and started to issue arbitrary commands.

Regarding the ratio between pan and zoom commands, we find that in *STDnav*,  $79.33 \pm 2.03\%$  of the commands are zooming commands vs.  $20.67 \pm 2.51\%$  for pans, but the proportions are reversed for *BIGnav*:  $26.74 \pm 4.46\%$  for zooming vs.  $73.26 \pm 3.87\%$  for panning. This is because with *BIGnav*, zooming is only needed when the target is within the view, and panning commands most often result in a new view with a different level of zoom.

### Uncertainty and Information Gain

Since the essence of *BIGnav* is to maximize the expected information gain at each command, we compare the actual information gain between *STDnav* and *BIGnav*. In both cases, the uncertainty that the system has about the users' intended target drops to zero gradually.

In *STDnav*, sometimes a command does not make a difference in uncertainty, i.e., the information gain is null. This is typically the case when the system is certain of what users are going to do. For example, when completely zoomed out, users must zoom in. Similarly, if a view contains 99% of the probability distribution, users will almost certainly zoom in.



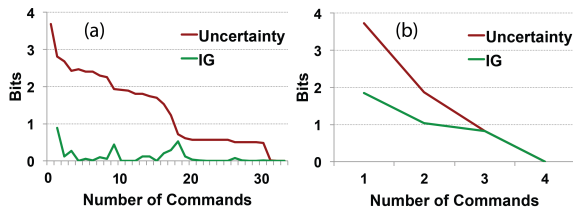


Figure 7. Uncertainty decrease and information gain for each successive command in (a) *STDnav* and (b) *BIGnav*.

By contrast, with *BIGnav*, the system gains information at each step, therefore uncertainty drops to zero much faster and with many fewer commands. In our data, 40.4% of the commands in *STDnav* do not reduce uncertainty. The rest of the commands reduce uncertainty by 0.26 bits on average. In *BIGnav*, all commands reduce uncertainty by 0.88 bits on average. Figure 7 shows typical plots of uncertainty reduction for the two techniques and the same other conditions (*Random+Uniform* and  $ID = 30$ ). These results support **H3** that *BIGnav* outperforms *STDnav* with much lower command usage and much higher rate of decreasing uncertainty.

### Trajectory in Multiscale Worlds

Another way to look at the navigation strategies is to plot the reduction in ID over time. As participants pan and zoom, they get closer (most of the time) to the target and therefore the ID progressively decreases from the initial level to 0. Figure 8 shows typical plots for two trials by the same participant in the same condition (*Random+Uniform* and  $ID = 30$ ). With *STDnav*, the reduction of ID is globally steady while with *BIGnav* we see sudden drops and long plateaus as well as occasional increases of the ID.

ID increases may occur for example when the view is close to the target and there is a cluster of points of interest further away in that direction. To maximize the expected information gain, *BIGnav* may choose to move towards the cluster and end up further away from the target. Another cause for ID increase is when the user makes a mistake.

The long plateaus represent waiting time and confirm the qualitative results reported below: that *BIGnav* incurs a higher cognitive load. This is probably due in part to our implementation of *BIGnav*, which skips to the new view after each input command rather than transition to it with an animation. But it is also probably the case that the user has to interpret the new location to plan the next move, whereas with *STDnav* the user can anticipate the system response.

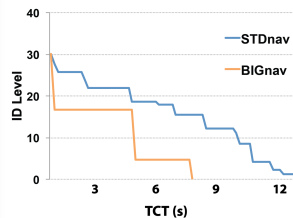


Figure 8. Time plot of the decrease in ID in the *STDnav* and *BIGnav* conditions, for two trials with the same other conditions.

### Qualitative Results

The post-hoc questionnaire provides self-evaluation of performance and comfort level as well as subjective preference for the two techniques. Regarding performance and comfort level, assessed on a five-point Likert scale, we find no significant differences.

While we expected participants to dislike *BIGnav* because of its unusual and possibly counter-intuitive mode of operation despite its efficiency (**H4**), we were surprised that half the participants liked it better than *STDnav*: “with one direction, it combines zoom and pan, which was faster than doing it by hand”, “The way it navigates is quite interesting. For most of the cases, only 2,3 actions are needed to find the target”, and “I like the interaction part. Somebody is guessing what I’m doing”. The eight participants who preferred *STDnav* found it “more comfortable, doesn’t require that much attention”, “more intuitive as I can anticipate what I would see next” and “I’m already used to it”. These results indicate that *BIGnav* can be a practical technique for efficient navigation.

### APPLICATION: BIGMAP

To demonstrate a realistic application of *BIGnav*, we implemented a map application with a  $80000 \times 60000$  high-resolution map of Europe using the ZVTM toolkit [23] (Fig. 9). It features the top 50 largest urban areas<sup>5</sup> and uses their population as probability distribution. This corresponds to the *Random+Random* distribution of the controlled experiment.

We conducted several pilot studies where participants had to navigate to specific cities from a completely zoomed-out view down to the maximum scale, where the city labels are readable ( $ID = 25$ ). Since this task relies on cognitive skills such as the participants’ geographical knowledge, we concentrated on observing users and collecting subjective evaluation feedback.

Most participants could navigate to the target city very quickly, in a few steps, especially for cities with large population, hence higher probability, such as London and Paris. One of the participants referred to *BIGnav* as “3 steps to Paris”. For smaller cities such as Helsinki (rank 50), participants can still navigate efficiently. Most of them feel comfortable with *BIGnav* as they are familiar with the map of Europe and can reorient themselves rapidly. However, they get frustrated when the target is already in the view but *BIGnav* moves away from it in order to gain information. One participant mentioned that “it would be nice if we could change between pan and zoom and this one (*BIGnav*) freely so that it can help us get through all the zooming at the beginning, but once I see the target, maybe I’ll switch back to pan and zoom for the last few steps”.

*BIGmap* illustrates how to derive a probability distribution from external data, here the population of the cities. More generally, the distribution should reflect the targets’ degree of interest, which is typically application-dependent. The distribution can also integrate usage data, such as most popular cities. Finally the results of a search can be turned into a distribution according to the ranking of the results, therefore integrating searching and navigation into a single paradigm.

<sup>5</sup>[https://en.wikipedia.org/wiki/Larger\\_urban\\_zone](https://en.wikipedia.org/wiki/Larger_urban_zone)



Figure 9. (a) Part of the map of Europe used by *BIGmap*. (b) Navigating towards Paris from the previous view.

## DISCUSSION

We have shown that *BIGnav* is an effective technique, especially for distant targets and non-uniform information spaces. The most efficient distribution condition in the experiment was *Cluster+Cluster*, which corresponds to the small-world structures found in many datasets, showing that *BIGnav* is a promising approach for real-world applications. However, both the experiment and the *BIGmap* prototype exhibit some shortcomings, especially for small-ID tasks. We now discuss how to make *BIGnav* more comfortable to use, and then address the wider notion of human-computer partnership.

### Comfort in Navigation

In standard pan-and-zoom interfaces, users can navigate the space in a continuous manner and constantly anticipate the system response. This gives them a sense of control and makes for a smooth user experience. By contrast, *BIGnav* uses discrete steps and the system’s response can be difficult to anticipate and even frustrating, in particular when getting close to the target. This results in long idle times between commands (Fig. 8) and a higher cognitive load as users reorient themselves and decide on their next move. In a sense, this proves the success of the technique, since it is designed to maximally challenge the user at each step.

Yet there must be a way to improve user experience and make navigation smoother. First, we could use animations to smooth transitions and help users stay oriented. Research has shown that one-second animations are sufficient and do not slow down expert users [2]. Second, we could combine *BIGnav* with standard pan-and-zoom according to user input: large panning and zooming movements would use *BIGnav*, smaller ones traditional pan and zoom. Finally we could reduce the size of the grid and increase the number of panning directions to provide finer control, however this requires heuristics or optimizations of the computational cost.

### Navigation Partnership

*BIGnav* is a collaborative approach as system and user work together to achieve a common objective [27, 31]. Whereas current multiscale navigation systems have leveraged either information in the multiscale world [15, 16, 21] or the users’ intentions [14], *BIGnav* combines these two sources of information. Like mixed initiative systems [12], *BIGnav* optimizes decision under uncertainty. However, the use of Bayesian Experimental Design turns these systems on their head: rather than finding the action that best responds to the user, *BIGnav* challenges the user to give it useful information.

*BIGnav* also relates to the notion of adaptation [20, 25]. In *Information foraging* [25], users adapt their strategy to gain valuable information from the system, whereas in *BIGnav*, the systems prompts users for valuable information.

Our approach leads to a reverse form of co-adaptation [20]: While co-adaptation is about users adapting to new technology and also adapting it to their own needs, *BIGnav* adapts to users through its prior knowledge, which could change over time, and adapts users to its needs by prompting them constantly for information. This creates a more balanced partnership than the random changes used by Williamson & Murray-Smith [34], which challenge the user but without the long-term reward of system adaptation to the user.

## CONCLUSION AND FUTURE WORK

*BIGnav* is a new multiscale navigation technique based on Bayesian Experimental Design with the criterion of maximizing the information-theoretic concept of mutual information. At each navigation step, *BIGnav* interprets user input, updates its estimate of the user’s intention, and navigates to a view that maximizes the expected information that will be gained from the user’s subsequent input.

We ran a controlled experiment comparing *BIGnav* with standard pan and zoom for different levels of difficulty and different distributions of the information space. Our main result is that *BIGnav* is up to 40.0% faster than the baseline for distant targets and non-uniform information spaces.

To the best of our knowledge, *BIGnav* is the first attempt at introducing an information-theoretic and Bayesian approach to multiscale navigation. Our next goal is to reduce users’ cognitive load while still ensuring *BIGnav*’s efficiency. We also want to improve the computational cost of the technique in order to support more input commands and a finer grid.

Beyond navigation, we are interested in applying this approach to other tasks, such as searching. Indeed, the model can be framed in terms of human-computer interaction as follows:

- $X$  can be any system feedback, e.g., visual, auditory, haptic;
- $Y$  can be any human input, e.g., touch input or gaze;
- $P(\Theta)$  can model many kinds of prior knowledge about users’ goals, as well as reflect their interaction history.

The paradigm shift from “responding to user input” to “running experiments on the user” is a novel perspective on the notion of human-computer partnerships, and the Bayesian Information Gain model opens the door to a wide range of “BIG” applications.

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