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Spur-free MASH DDSM with Eliminable Dither

Yilong Liao ^(✉), and Xiangning Fan

Institute of RF&OE-ICs, School of Information Science and Engineering, Southeast University,
Nanjing, China

{yilongliao, xnfan}@seu.edu.cn

Abstract. This paper presents a new dither adding method for multistage-noise-shaping (MASH) digital delta-sigma modulator (DDSM), to reduce the output spurious tones. In this method, the additive dither signal is added to two different nodes of any preceding stage of the MASH DDSM, without using dither shaping filter. Then, those dither signals are sent to the error cancellation logic (ECL), where they are eliminated by each other, avoiding appearing in the output spectrum. Simulation results show that, for the MASH DDSM, the new dither adding method can guarantee a spur-free spectrum for all digital constant inputs without raising the noise floor.

Keywords: Digital delta-sigma modulator (DDSM) · Dither · Eliminable · Multistage-noise-shaping (MASH) · Spur-free

1 Introduction

In order to obtain a good performance and spectral purity, digital delta-sigma modulator (DDSM) is widely used in modern wireless communication subsystems, such as fractional- N frequency synthesizers and digital to analogue converters [1, 2]. Functionally, a DDSM oversamples a discrete-time discrete-amplitude input signal and produces a lower-resolution and noise-shaped output [3]. In general, there are two types of DDSM, which are multistage-noise-shaping (MASH) DDSM and single loop DDSM. However, due to the inherent stability, the MASH structure is much preferred in most applications [4].

In the case of constant input, MASH DDSM is a deterministic finite state machine (FSM), whose output is periodic [5]. When the cycle length is not long enough, its output spectrum shows high-power spurious tones, which will deteriorate the performance of the whole communication system.

In order to reduce the spurious tones, deterministic technique and stochastic technique were both presented in the past decades to extend the cycle length effectively.

For the deterministic technique, it can inherently extend the output cycle length of MASH DDSM. The most commonly used methods include setting the odd initial conditions [6, 7], using prime modulus quantizer [8, 9], and some other modified structures, such as HK-MASH [10] and SP-MASH [11].

As to the stochastic technique, it is realized by injecting an additive random dither signal to DDSM to break up the periodic output cycles [12, 13]. In the traditional dither

adding method, the additive dither signal added to the circuit cannot be eliminated and the order of dither shaping filter cannot go beyond $(l - 2)$ for a conventional l -th-order MASH DDSM [13]. Thus, though this technique reduces high-power spurious tones effectively, the noise floor of the output spectrum is also raised inevitably. Then, a modified second-order error feedback modulator (EFM2) was introduced in [14, 15]. Compared with the conventional first-order error feedback modulator (EFM1), it contains an extra delay in the feedback loop, as shown in Fig. 1. Using this modified structure, a MASH 1-2-2 DDSM was constructed in [14], which can increase the order of the dither shaping filter to that of the modulator, without causing spurious tones in the spectrum. Therefore, adopting the EFM2 can alleviate the noise raising problem to some extent. However, as pointed out in [15], compared with the conventional MASH 1-1-1 DDSM without EFM2, the output spectrum power of MASH 1-2-2 DDSM is also raised about 12 dB in the lower frequency band.

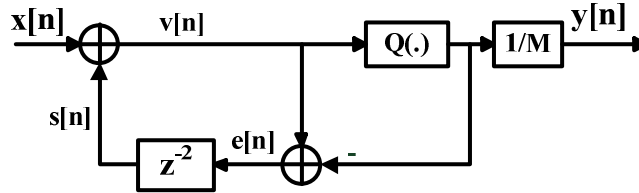


Fig. 1. Block diagram of modified second-order error feedback modulator (EFM2)

In this paper, we consider the stochastic technique and propose a new dither adding method for MASH DDSM to produce a spur-free spectrum, without raising the shaped quantization noise power in the output spectrum.

This paper is organized as follows. In section 2, we review and analyse the structure of the previous dithered MASH DDSM briefly. Section 3 introduces the new method of the dither adding. Simulation results of MASH DDSM with the new dither adding method are presented in section 4. At last, we summarise the whole work in section 5.

2 Previous dithered MASH DDSM

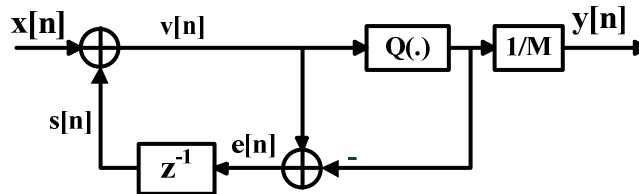


Fig. 2. Block diagram of a first-order error feedback modulator (EFM1)

In a MASH DDSM, the most basic block is the first-order error feedback modulator, which quantises the summation of input and the quantization error of preceding moment

to yield the output, as shown in Fig. 2. Mathematically, we write the quantization function as

$$y[n] = \frac{1}{M} Q(v[n]) = \begin{cases} 1, & v[n] \geq M \\ 0, & v[n] < M \end{cases} \quad (1)$$

where $v[n]$ is the input to the quantizer and M is the quantization interval. When the input word length is set as n_0 , the corresponding quantization interval is $M = 2^{n_0}$.

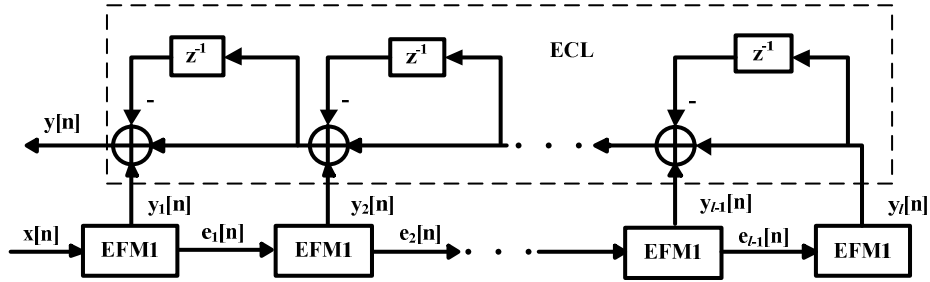


Fig. 3. Block diagram of an l -th-order MASH DDSM

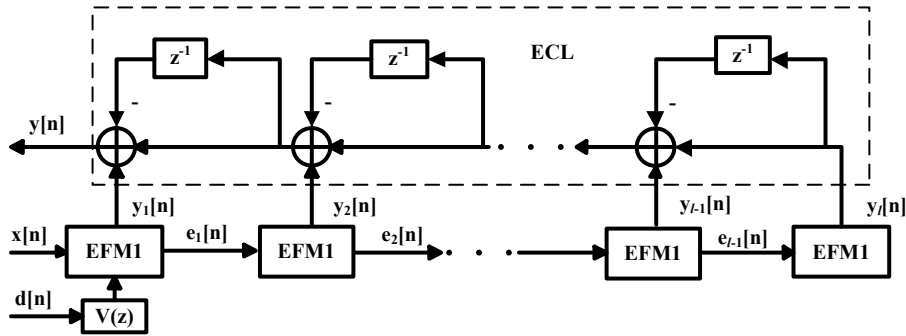


Fig. 4. Block diagram of MASH DDSM with additive filtered dither

Cascading l EFMI blocks and combining their outputs to the error cancellation logic (ECL) circuit, we can construct an l -th-order MASH DDSM, as presented in Fig. 3. The role of ECL is to eliminate the intermedia stage noise and yields the final output. In the z -domain, the transfer function of l -th-order MASH DDSM can be expressed as

$$Y(z) = \frac{X(z)}{M} - \frac{E_l(z)}{M} (1 - z^{-1})^l, \quad (2)$$

where $X(z)$, $Y(z)$, and $E_l(z)$ represent the input signal, output signal, and the quantization error of the last stage in the z -domain, respectively. From (2), we can see that, since the quantization error of the intermedia stages are eliminated by ECL, only the input signal with an all-pass filter and the quantization error of the last stage with an l -th-order high-pass filter are remained. The filtering functions related to the input signal and the quantization error are called signal transfer function (STF) and noise

transfer function (NTF), respectively. In (2), they are $1/M$ and $(1 - z^{-1})^l/M$, respectively.

When the filtered dither is added, as shown in Fig. 4, the output can be derived as

$$Y(z) = \frac{X(z)}{M} - \frac{E_l(z)}{M} (1 - z^{-1})^l + \frac{D(z)}{M} V(z), \quad (3)$$

where $V(z)$ is normally in the form of $(1 - z^{-1})^{l_d}$, with $l_d \leq (l - 2)$ [13], and $D(z)$ is the dither signal in the z-domain.

In order to illustrate the effectiveness of the stochastic technique, the output PSD plots of the MASH 1-1-1 DDSM with first-order shaped dither and the un-dithered MASH 1-1-1 DDSM are presented in Fig. 5. The simulation conditions are input word length $n_0 = 9$, quantization interval $M = 2^9$ and input $x[n] = M/2$.

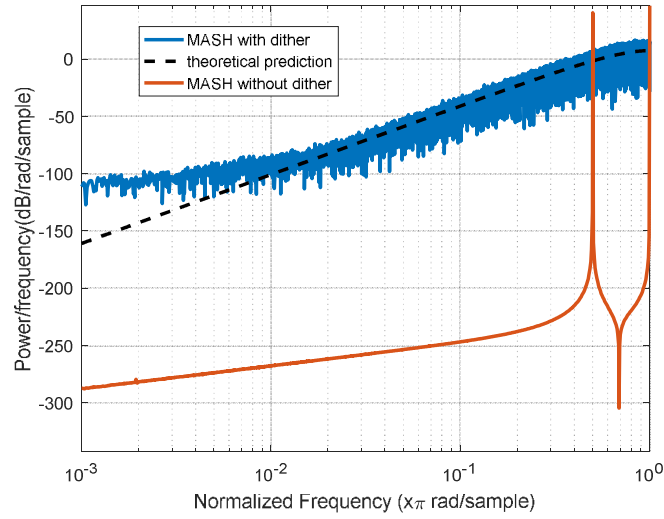


Fig. 5. Output PSD of 9-bit MASH 1-1-1 DDSM with first-order shaped dither and un-dithered MASH 1-1-1 DDSM, with constant input 256

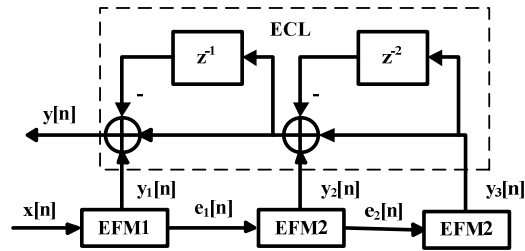


Fig. 6. Block diagram of MASH 1-2-2 DDSM

Fig. 5 shows that, for an un-dithered MASH 1-1-1 DDSM, when the input is $M/2$, its output cycle length is very small and the output spectrum has only two high-power

spurious tones. However, when it comes to the MASH 1-1-1 DDSM with first-order shaped dither, a smooth output spectrum is obtained and no obvious spurious tones can be observed. Note that, compared with the theoretical prediction result of the output spectrum, we can find the MASH 1-1-1 DDSM with first-order shaped dither raised the noise floor inevitably.

The MASH 1-2-2 DDSM presented in [14] is redrawn in Fig. 6. With the same reasoning for the traditional MASH DDSM, its transfer function can be derived as

$$Y(z) = \frac{X(z)}{M} - \frac{E_3(z)}{M}(1 - z^{-2})^2(1 - z^{-1}). \quad (4)$$

Similarly, with the shaped dither, the transfer function becomes

$$Y(z) = \frac{X(z)}{M} - \frac{E_3(z)}{M}(1 - z^{-2})^2(1 - z^{-1}) + \frac{D(z)}{M}V(z). \quad (5)$$

Observing equation (3) and (5), we can easily find that, in the traditional dither adding method, the dither signal appears in the output, raising the noise floor inevitably. In order to avoid this problem, we introduce a new dither adding method in the following section. Without losing generality, the new method is mainly analysed in MASH 1-1-1 DDSM.

3 MASH DDSM with new dither adding method

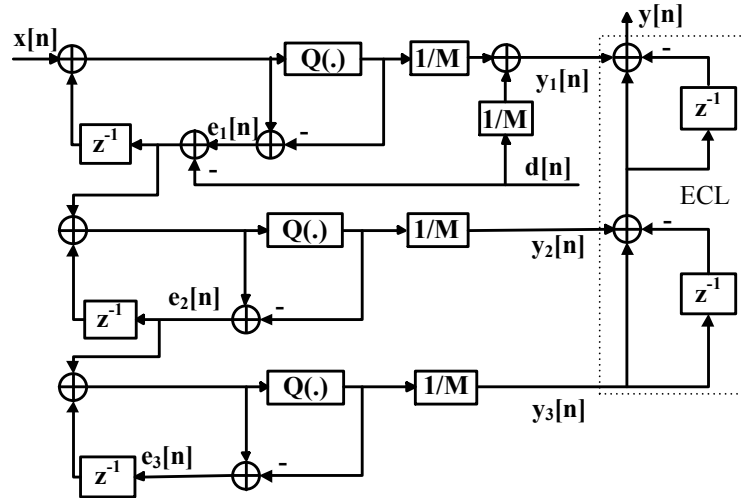


Fig. 7. MASH 1-1-1 DDSM with the new dither adding method in the first stage (MASH 1D-1-1 DDSM)

The MASH 1-1-1 DDSM with the new dither adding method in the first stage (referred to as MASH 1D-1-1 DDSM) is illustrated in Fig. 7, which shows that the dither signal is added to two different nodes of the first stage of the MASH 1-1-1 DDSM, without dither shaping filter. Thereby, the input-output relationship of the first stage in the z -

domain can be derived as

$$Y_1(z) = \frac{X(z)}{M} - (1 - z^{-1}) \frac{E_1(z) - D(z)}{M}, \quad (6)$$

where $Y_1(z)$ and $E_1(z)$ represent the output $y_1[n]$ and quantization error $e_1[n]$ of the first stage in the z -domain, respectively.

Since the negative dither signal is also sent to the next stage, the transfer function of the second stage can be written as

$$Y_2(z) = \frac{E_1(z) - D(z)}{M} - (1 - z^{-1}) \frac{E_2(z)}{M}, \quad (7)$$

where $Y_2(z)$ and $E_2(z)$ represent the output $y_2[n]$ and quantization error $e_2[n]$ of the second stage in the z -domain, respectively. Then, the input-output relationship of the third stage and ECL can be expressed as

$$Y_3(z) = \frac{E_2(z)}{M} - (1 - z^{-1}) \frac{E_3(z)}{M}, \quad (8)$$

$$Y(z) = Y_1(z) + Y_2(z)(1 - z^{-1}) + Y_3(z)(1 - z^{-1})^2, \quad (9)$$

where $Y_3(z)$ and $Y(z)$ represent the output of the third stage $y_3[n]$ and the final output $y[n]$ in the z -domain, respectively. Substituting equation (6), (7), and (8) into (9), the transfer function of MASH 1D-1-1 DDSM is derived as

$$Y(z) = \frac{X(z)}{M} - \frac{E_3(z)}{M} (1 - z^{-1})^3. \quad (10)$$

Compared with (3) and (5), (10) states that, with the new dither adding method, the output of MASH 1D-1-1 DDSM is free from the dither signals. Equivalently, since the additive dither signals are eliminated in ECL, no noise floor rising phenomenon can be observed in the output of MASH 1D-1-1 DDSM.

Furthermore, from (10), we know that the shaped quantization noise of MASH 1D-1-1 DDSM is $\frac{E_3(z)}{M} (1 - z^{-1})^3$. For ease of reading and comparison, the shaped quantization noise of dithered MASH 1-2-2 DDSM is written as $\frac{E_3(z)}{M} (1 - z^{-1})(1 - z^{-2})^2$ [14]. The power of the each shaped quantization noise can be calculated as

$$P_{D,dB} = 20 \log \left| \frac{E_3(z)}{M} (1 - z^{-1})^3 \right|, \quad (11)$$

$$P_{T,dB} = 20 \log \left| \frac{E_3(z)}{M} (1 - z^{-1})(1 - z^{-2})^2 \right|, \quad (12)$$

where $P_{D,dB}$ and $P_{T,dB}$ represent the power of shaped quantization noise of MASH 1D-1-1 DDSM and dithered MASH 1-2-2 DDSM, respectively. Substituting $z = \exp(j2\pi f)$ into (11) and (12), when f is small enough, we can get

$$P_{T,dB} - P_{D,dB} = 20 \log \left(\frac{2\pi f}{\pi f} \right)^2 \approx 12 \text{ dB}, \quad (13)$$

where f is the normalized frequency. Obviously, the computing result in (13) is consistent with the simulation result in [15]. In other words, compared with dithered MASH 1-2-2 DDSM, the shaped quantization noise power of MASH 1D-1-1 DDSM is reduced by 12 dB in the lower band.

It is worth noting that, the proposed dither adding method can be applied in any preceding except the last stage of a MASH DDSM. Since (6) has clearly stated that, in the new dither adding method, the dither signal can be treated as the quantization noise of the corresponding stage, and it always can be eliminated in ECL.

4 Simulation results

The output PSD plots of MASH 1-1-1 DDSM with first-order shaped dither, MASH 1-2-2 DDSM with third-order shaped dither, and MASH 1D-1-1 DDSM are simulated in Simulink, which are illustrated in Fig. 8. In addition, the theoretical prediction results are also labelled in the output PSD plots. The simulation parameters are listed as follows: input word length $n_0 = 9$, quantization interval $M = 2^9$ and input $x[n] = M/2$.

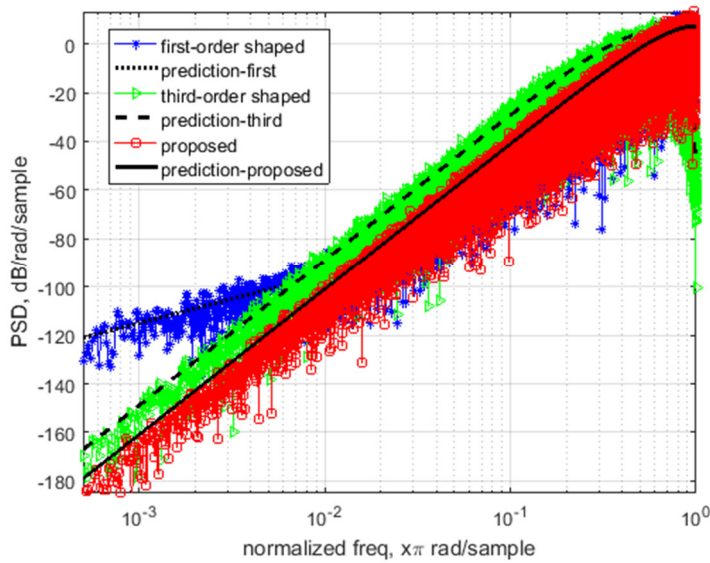


Fig. 8. Output PSD of 9-bit MASH 1-1-1 DDSM with first-order shaped dither, MASH 1-2-2 DDSM with third-order shaped dither, and MASH 1D-1-1 DDSM, with constant input 256

Fig. 8 presents that the spur-free spectra are achieved in all those three structures. But MASH 1-1-1 DDSM with first-order shaped dither raises the noise floor obviously. Moreover, in the lower band, though a slope of 60 dB/decade is both obtained in MASH 1-2-2 DDSM with third-order shaped dither and MASH 1D-1-1 DDSM, the latter one further reduces the shaped quantization noise power about 12 dB, as expected.

Besides, For all 512 DC inputs with $n_0 = 9$, the output PSD plots of MASH 1D-1-

1 DDSM and MASH 1-1-1 DDSM with the proposed dither adding method in the second stage are shown in Fig. 9 and Fig. 10, respectively. As can be seen, the spur-free spectra are achieved for all digital constant inputs in both cases.

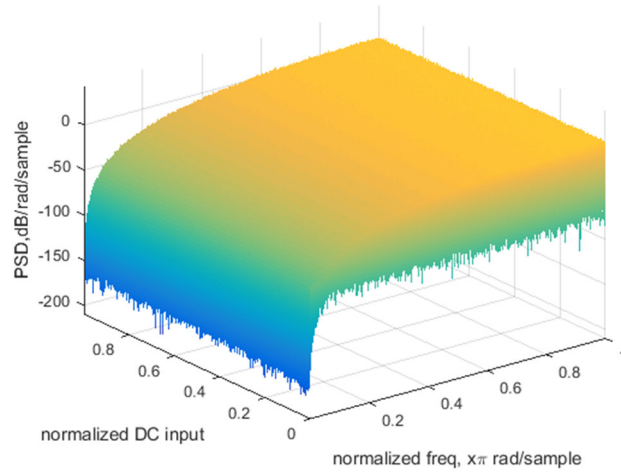


Fig. 9. Output spectra of MASH 1D-1-1 DDSM for all 512 DC inputs with $n_0 = 9$

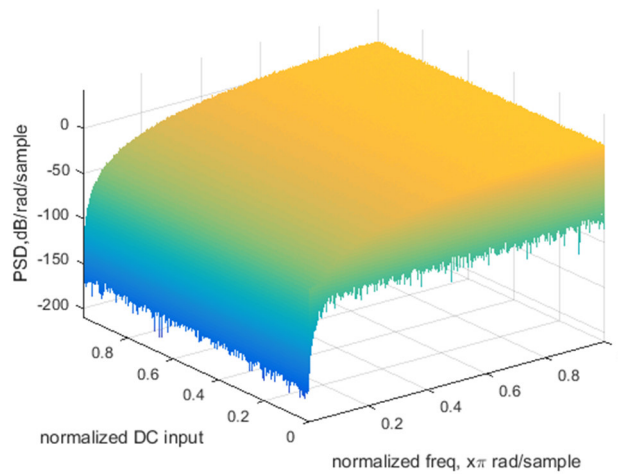


Fig. 10. Output spectra of MASH 1-1-1 DDSM, with the proposed dither adding method applied in the second stage, for all 512 DC inputs with $n_0 = 9$

5 Conclusion

A new dither adding method which can be applied in any preceding except the last stage of a MASH DDSM is proposed in this paper. Theoretical analysis states that, in the new

method, the additive dither signal can be treated as the quantization noise of the corresponding stage. Thus, it can be eliminated in the ECL. Simulation results show that spur-free spectra are achieved for all constant digital inputs. Furthermore, compared with conventional MASH 1-1-1 DDSM with first-order shaped dither and MASH 1-2-2 DDSM with third-order shaped dither, the MASH 1-1-1 DDSM applying the new dither adding method can avoid noise floor rising problem and acquire better spectrum performance.

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