Energy Trade-offs for end-to-end Communications in Urban Vehicular Networks exploiting an Hyperfractal Model

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ABSTRACT

We present results on the trade-offs between the end-to-end communication delay and energy spent for completing a transmission in vehicular communications in urban settings. This study exploits our innovative model called "hyperfractal" that captures the self-similarity of the topology and vehicle locations in cities. We enrich the model by incorporating road-side infrastructure.

We use analytical tools to derive theoretical bounds for the end-to-end communication hop count under two different energy constraints: either total accumulated energy, or maximum energy per node. More precisely, we prove that the hop count is bounded by $O(n^{1-\alpha/(d_m-1)})$ where $\alpha < 1$ and $d_m > 2$ is the precise hyperfractal dimension. This proves that for both constraints the energy decreases as we allow to chose among paths of larger length. In fact the asymptotic limit of the energy becomes significantly small when the number of nodes becomes asymptotically large. A lower bound on the network throughput capacity with constraints on path energy is also given. The results are confirmed through exhaustive simulations using different hyperfractal dimensions and path loss coefficients.

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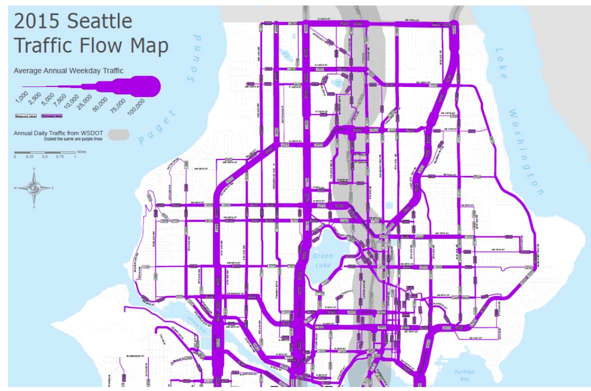

Figure 1: Snapshot of Seattle traffic flow map

1 INTRODUCTION

1.1 Aims and results

Efficient communications for vehicular networks will be a vital part of the 5th generation of communication systems. Vehicular networking emerges as a key area of future communications networks with many innovation opportunities yet significant challenges. In this context, an effective integration of vehicular networks in complex urban environments is paramount.

Vehicular networks have made significant progress and continue to gain momentum. The increase of intelligence of devices and the advance in standards such as 802.11p have contributed to accelerate the development of networks of cars [1]. Companies with fleet of vehicles (such as Uber) also provide a strong demand towards the development of vehicular communications in all contexts: vehicle-to-vehicle (V2V), vehicle-to-infrastructure (V2I), vehicle-to-everything (V2X).

Transforming the cars into communication entities is a compelling and realistic idea with the rise of Internet of Things (IoT). Cooperative Intelligent Transportation Systems (C-ITS), reuniting the technologies that are associated with the concept of vehicular entities, are to provide safer, less congested traffic while minimizing the environmental impact of transportation, as well as other advantages.

Vehicular networks will continue to scale up to reach tremendous network sizes (with diverse hierarchical structures and node types), while vehicular interactions will become more complex. Concurrently, in terms of advanced communication technologies, ultra dense cellular deployments are continuously increasing communications among vehicle units [2] and the Device-to-Device (D2D) based vehicular communications also generate a more complex hybrid communication network [3].

Given the numerous challenges and the important place the vehicular communications hold in the new communications era, a realistic modeling of the topology for accurate estimation of network metrics is highly desirable. The research community has proposed stochastic models that usually fit with high precision cellular networks or ad-hoc networks, yet for vehicles this cannot be

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done without taking into account the crucial fact that vehicular communication resent a tremendous impact from the environmental topology. Cars are located on streets and streets are conditioned by the urban architecture. One of the major features of urban architecture is self-similarity.

While it has been extensively exploited in diverse research fields such as biology and chemistry, self-similarity has been only recently introduced to wireless communications, after understanding that the device to device communication topologies follow the social human topologies. Self-similarity is present in every aspect of the surrounding environment but with an emphasis in the urban environment. The hierarchic organization with different degrees of scaling of cities is a perfect illustration of the fractal structure of human society [4]. Figure 1 is a snapshot of the traffic in a neighborhood of Seattle. Patterns and hierarchical organization can easily be identified in the traffic measurements.

In this paper we extend the "hyperfractal" model that we have introduced in [5, 6] in order to better capture the impact of the network topology on the fundamental performance limits of end-to-end communications over vehicular networks in urban settings. The model consists into assigning decaying traffic densities to city streets, thus avoiding the extremes of regularity (e.g. Manhattan grid) and of uniform randomness (e.g. Poisson point process). In [7] it is proven that the model is realistic, by validation using model fitting on data of real cities. The hyperfractal model shows self-similarity and is characterized by a dimension that is larger than the dimension of the euclidean dimension of the embedding space, e.g., larger than 2 when the whole network lays in a 2-dimensional plan.

Our previous result in [7] showed that, for nodes, the number of hops in a routing path between an arbitrary source-destination pair increases as a power function of the population n of nodes when n tends to infinity. However, we showed that the exponent tends to zero when the hyperfractal dimension tends to infinity. An initial observation through this model, as it is, is that the optimal path may have to go through streets of low density where inter-vehicle distance can become large, therefore the transmission becomes expensive in terms of energy cost. Hence, in this paper, the focus will be on the study of the relationship between efficient communications and energy costs.

Our results:

- We first enhance the hyperfractal model by taking into account radio communication range variations as well as energy costs of transmission. In addition, we enrich the model by incorporating road-side infrastructure with communication relays (with radio communication range variations). We exploit the self-similarity of intersection locations in urban settings.
- We prove that, for an end-to-end transmission in a hyperfractal setup, the energy (either cumulated along the path or bounded for each node) decreases if we allow the path length to increase. In fact, we show that the asymptotic limit of the energy tends to zero when n, the number of nodes, tends to infinity.
- We prove a lower bound on the network throughput capacity with constraints on path energy.
- We validate our analytical results using a discrete time event-based simulator developed in Matlab.

1.2 Related Works

Ad-hoc and vehicular communications aspects have been intensively studied in the past decade with different considerations such as topology modeling, routing optimization, energy consumption estimation, etc. As the problem of energy optimization is critical for wireless nodes, especially sensors as they tend to use small batteries for energy supply that are in many instances non-replenishable, significant attention has been given by the research community to this aspect. In [8] and [9], the authors develop a protocol based on opportunistic routing and asynchronous periods of activity in order to minimize the network energy consumption and end-to-end delay. More specifically, their work looks at sleep-wake periods. By considering an uncoordinated sleep mechanism, [10] provides a queuing analysis of the trade-offs between delay and energy. On the other hand, the authors of [11] optimize the network lifetime (the time until the first node is drained of energy) and optimize it by exploiting the mobility of the sink. The network lifetime is also studied in [12] by considering clustering.

Considerable work has been done on the problem of energy efficiency for broadcast and multicast [13, 14]. In [15] the trade-offs between throughput and delay are computed when each user is allowed to send redundant packets along multiple paths towards its destination. A thorough analysis using queuing is provided. In [16], the authors propose an enhancement for opportunistic routing with the purpose to optimize energy consumption.

In [17], the authors optimize relay placement in a sensor network such that constraints on connectivity, energy and performance are fulfilled.

By describing the interactions between vehicles by means of stochastic geometry, important results were provided in the works of [18–22]. More precisely, the authors model the communication scenarios that arise on the highways or in the street intersections. An extensive study of information propagation speed in vehicular delay tolerant networks in highways is presented in [23]. In [24] an interesting model of Poisson points on Poisson lines is introduced for investigating the speed of packet propagation in the presence of an external field of interference. The model resembles a vehicular topology, yet the authors do not consider the cases with different densities on lines and do not use urban radio propagation conditions.

Self-similarity for the modeling of urban ad-hoc networks has been introduced in [5] and [6], where a hyperfractal model was designed for exploiting the fractal structure of static urban ad-hoc networks with road-side infrastructure. The study we presented in [5] provides results on the minimal path routing using the hyperfractal model for static nodes and the hyperfractal model for the road-side infrastructure. Throughout [5], we make the assumption of infinite radio range, an assumption that may raise the concern of allowed transmission power and network energy consumption. Here, we address this question by adding the constraints on the quantities in discussion and thereby providing insights on the achievable trade-offs between the end-to-end transmission energy and delay. We further prove that the scenario debated in [5] is a particular case of the analysis we will further develop in this paper.

1.3 Organization of the paper

The paper is organized as follows:

- Section 2 describes the hyperfractal model used for the topology of the nodes and the hyperfractal model used for the topology of relays. For completeness, we also remind their basic properties and we develop the communication scenario.
- Section 3 gives the main theoretical results of this paper. We show that either under the constraint of total accumulated energy, or under the constraint of maximum energy per node, the hop count is bounded by $O(n^{1-\alpha/(d_m-1)})$ where $\alpha < 1$ and $d_m > 2$ is the precise hyperfractal dimension. The connection with the network throughput capacity is also made in this section.
- Section 4 confirms the theoretical findings through simulations and gives visual interpretation to results.
- Section 5 provides concluding remarks.

2 SYSTEM MODEL

2.1 Geometric model: Hyperfractals for vehicular networks

For the sake of completeness, let us remind the hyperfractal model and its key properties. For a detailed and comprehensive description, we refer the reader to [5, 6] for static settings and to [7] for mobile settings.

Cities have been proven to be hierarchically organized [4]. The centers which form this hierarchy have many elements in common in functional terms and repeat themselves across several spatial scalings. This is reminiscent of a fractal, described by Lauwerier [25] as a geometrical figure that consists of an identical motif repeating itself on an ever-reduced scale.

The map is assumed to be the unit square and contains n (static or mobile) nodes. The support of the population is a grid of streets similar to a Manhattan grid but with an infinite resolution.

The process of assigning points to lines is performed recursively, in iterations. The two lines of level 0 form a central cross which splits the map in exactly 4 quadrants. According to a uniform distribution, with probability p', a node is placed on the lines of the central cross. Denote by $q' = 1 - p'$ the complementary probability. The central cross splits the map in four quadrants and the nodes are assigned equally likely with $q'/4$ to each of the quadrants. The assignation procedure recursively continues in each quadrant and it stops when the node is assigned to a cross of a level $m \geq 0$. This leads to the fact that the map as a whole is identically reproduced in each of the four quadrants but with a weight of $q'/4$ instead of 1, therefore considering the map in only half of its length on each side consists into considering the same map but with a reduced weight by a factor $q'/4$. One obtains:

$$\left(\frac{1}{2}\right)^{d_m} = q'/4 \qquad \text{thus} \qquad d_m = \frac{\log(\frac{4}{q'})}{\log 2} > 2 \tag{1}$$

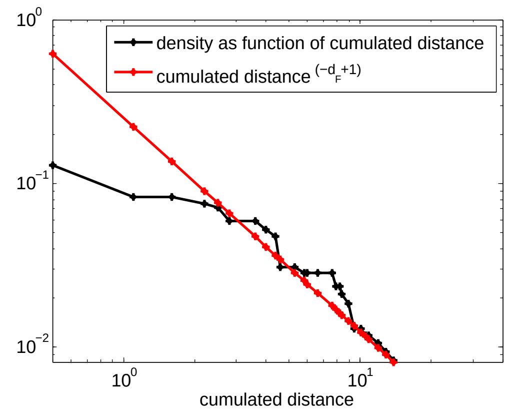


Figure 2: Hyperfractal model fitting with data for the city of Seattle

Observe that, interestingly, the fractal dimension here, d_m, is in fact *greater* than 2, the Euclidean dimension.

Let us further remind the concept of street. A cross of level H consists of two intersecting segments of lines of level H and each segment of the cross is considered to be a segment of level H. Two segments that belong to the same line are necessarily of the same level. A street of level H consists of the union of consecutive segments of level H in the same line. The length of a street is the length of the side of the map.

Denote by λ_H the density of the nodes assigned on a street of level H. It satisfies:

$$\lambda_H = (p'/2)(q'/2)^H \tag{2}$$

Notice that when $p' \to 0$ then $d_F \to 2$. In other words, a hyperfractal with an asymptotic value of $d_m = 2$ is a uniform Poisson point process.

In [7], we presented the hyperfractal model for mobile and delay tolerant vehicular networks. The study proved that the average broadcast time in a hyperfractal setup is of order of $\Theta(n^{1-\frac{1}{d_m-1}})$ where n is the number of mobile vehicles. We have shown how the model can be used to describe real cities traffic measurements (such as Minneapolis) using a fitting procedure. The procedure expresses the density repartition function, $\lambda(l)$, using the cumulated length of the streets, l, and the fractal dimension, d_m as $\lambda(l) = \Theta\left(l^{1-d_m}\right)$. In further support of our model, we present here another example of model fitting to city traffic maps. A snapshot of the data sets from [26] used for fitting is displayed in Figure 1. The snapshot contains the average annual weekday traffic in a neighborhood of Seattle. By applying the described fitting procedure (see Figure 2), we can estimate the fractal dimension for Seattle (i.e., $d_m = 2.3$). Note that our previous work also provides a methodology for extending the model to cities that do not follow a regular hierarchical pattern. In order to advocate in the favor of the hyperfractal model, another fitting example is given in the appendix.

Again, one goal of this paper is to provide an extension of the hyperfractal model for urban vehicular networks with access

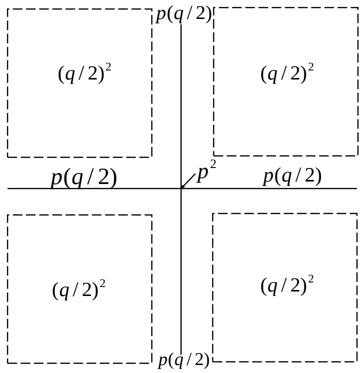


Figure 3: Procedure of assigning relays to intersections

to road-side infrastructure and take into account radio range variations.

2.2 Hyperfractal for Relays

Not surprisingly, locations of communication infrastructure in urban settings also display self-similar behavior. Hence we apply another hyperfractal process for selecting the intersections where a road-side relay is installed. This process has been introduced for the first time in [5]. For the sake of completeness, we remind the model and its basic properties of which we will make use in the development of our main results.

The procedure of assigning relays to intersections is intuitively illustrated in Figure 3.

Denote by p a fixed probability and $q = 1 - p$ the complementary probability. A run for selecting a street crossing requires two processes: the in-quadrant process and the in-segment process. The selection starts with the in-quadrant process: with probability p^2, the selection is the central crossing of the two streets of level 0. With probability $p(q/2)$, the relay is placed in one of the four street segments of level 0 starting at this point: North, South, West or East, and the process continues on the segment with the in-segment process. Otherwise, with probability $(q/2)^2$, the relay is placed in one of the four quadrants delimited by the central cross and the in-quadrant process recursively continues.

The selection run is performed M times. At each run the probability that an intersection between two streets of respective levels H and V is selected is $p(H, V)$ which has expression

$$p(H, V) = p^2 \left(\frac{q}{2}\right)^{H+V} . \tag{3}$$

If one crossing is selected multiple times, only one relay is installed in the respective crossing. To simplify our analysis and to make the intersections independent we set the number of runs M to be a Poisson variable of mean ρ. Consequently, an intersection of level (H, V) has the probability $\exp(-\rho p(H, V))$ of not holding a relay and events relative to each intersection are independent.

Some basics results are further reminded. The relay placement is hyperfractal with dimension d_r:

$$d_r = 2\frac{\log(2/q)}{\log 2} . \tag{4}$$

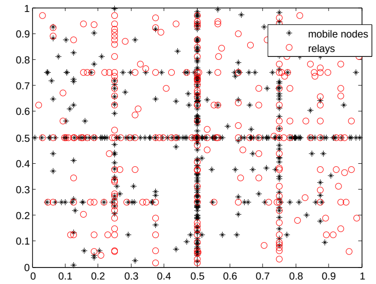


Figure 4: Hyperfractal map with mobiles and relays

The average number of relays on a street of level H, $L_H(\rho)$, satisfies the identity:

$$L_H(\rho) = \sum_{V \geq 0} 2^V \left(1 - \exp(-p(H, V)\rho)\right) . \tag{5}$$

Furthermore:

$$L_H(\rho) = O(((q/2)^H \rho)^{2/d_r} . \tag{6}$$

The total number of relays is $R(\rho) = O(\rho^{2/d_r} \log \rho)$. We consider $\rho = \rho_n$ as a function of n, and, for the sake of simplicity, we take $\rho_n = n$. In this case we notice that the number of relays is, indeed, substantially smaller than the number of mobile nodes.

Note that with the introduction of relays, the graph generated by the nodes and relays may still be partitioned but there will always exist a giant connected component that includes the nodes located on the central cross. A complete Hyperfractal map containing both nodes and relays is presented in Figure 4.

2.3 Canyon Effect

Urban environment is characterized by long streets surrounded by tall buildings, generating the so-called canyon effect. Ideally, a street canyon is a relatively narrow street with tall, continuous buildings on both sides of the road. Geometrical details of the street canyon are used to categorize street canyons. According to [27], the urban canyon can have different characteristics in function of the aspect ratio between the height and width of the canyon. Due to the canyon effect, in urban scenarios, and at the frequency of 5.9 GHz (the frequency band adopted by the 802.11p standard), radio signals are highly directional and will experience a very low depth of penetration [28]. Therefore, buildings absorb radio waves, making communication only possible when vehicles are in line-of-sight. In order to accurately model the propagation of radio signals in urban scenario, one must consider the effect of the signal attenuation due to distance, along with the effect of obstacles blocking the signal propagation.

2.4 Communication model

In this paper, as we primarily seek to understand the relationship between end-to-end communications and energy costs, we do not consider other detailed aspects of the communication protocol, such as the distributed aspects needed to gather position information and construct routing tables in every node.

The considered routing strategy is the nearest neighbor routing strategy. The next hop is always the next neighbor on a street, i.e.

there exists no other node between the transmitter and the receiver. Denote by $d(i,j)$ the euclidean distance between nodes i and j, r_{ij} represents the cost of directly transmitting a packet from node i to node j. Thus:

$$\begin{cases} r_{ij} = 1 & \text{if nodes } i \text{ and } j \text{ are aligned} \\ & \text{and } \nexists k \text{ such that } d(i,j) = d(i,k) + d(k,j) \\ r_{ij} = \infty & \text{otherwise} \end{cases}$$

The end-to-end transmission delay is represented by the total number of hops the packet takes in its path towards the destination.

The transmission is done in a half-duplex way, a node is not allowed to transmit and receive during the same time-slot. The received signal is affected by additive white Gaussian noise (AWGN) noise N and path-loss with pathloss exponent $\delta > 2$.

Let us make the simplified assumption that all nodes on a street transmit the same nominal power P_m which depends only on the number of nodes on the street. We argue that a good approximation is to suppose that:

$$P_m = \frac{P_{\max}}{m^{\delta}}. \tag{7}$$

Where $\delta > 2$ is the path loss and $P_{\max}$ is the transmitting power necessary for a node at one end of the street to transmit a packet directly to a node at the other end of the street. In other words assume a road of infinite length with uniformly distributed nodes with a density $1/L$ where L is the length of the streets of our city. If in this configuration every node has a nominal power of $P_{\max}$, then the nominal power to achieve the same performance with a density m times larger but with the same noise values should be $P_{\max} m^{-\delta}$ in order to cope with the loss effect.

Following this reasoning, the cumulated energy to cover a whole street containing m nodes with uniform distribution via nearest neighbor routing is $mP_m = \frac{P_{\max}}{m^{\delta-1}}$. In this case the larger the population of the street the smaller the nominal power and the smaller the energy to cover the street.

Relays stand in intersections, and thus on two streets with different values of m. In this case, as we are mainly interested in fundamental performance limits, we will assume that a relay is using two different radio interfaces, each with a transmission power according the previously mentioned rule for each of the streets.

3 MAIN RESULTS

In our previous studies, [5, 6], we considered giant components without constraints on energy. Here, given that the transmitting energy is dependent of the average density of the nodes on the streets and that the transmission energy per node is limited by the protocols to a value of $P_{\max}$, the connectivity is much more restricted.

We introduce the following notions and notations. Denote by t a node and by $P(t)$ the nominal transmission power of this node.

Definition 1. *Let $\mathcal{T}$ be a sequence of nodes which constitutes a routing path. The path length is $D(\mathcal{T}) = |\mathcal{T}|$. The relevant energy paths are:*

- *The path cumulated energy is the quantity $C(\mathcal{T}) = \sum_{t\in\mathcal{T}} P(t)$.*
- *The path maximum energy is the quantity $M(\mathcal{T}) = \max_{t\in\mathcal{T}} P(t)$.*

The path cumulated energy is of interest as we want to optimize the quantity of energy expended in the-end-to-end communication, and respectively, the path maximum energy as we want to find the path whose maximum energy does not exceed a given threshold depending on the energy sustainability of the nodes or the protocol. For example, it is likely that no node can sustain a nominal power of $P_{\max}$ which is the energy needed to transmit in a range corresponding to the entire length of a street. In this case it is necessary to find a path which uses streets with enough population to reduce the node nominal power.

Definition 2.

- *Let $G(n, \mathbf{E})$ be the set of all nodes connected to the central cross with a path cumulated energy not exceeding $\mathbf{E}$.*
- *Let $G_k(n, \mathbf{E})$ be the subset of $G(n, \mathbf{E})$, where the path to the central cross should not go through more than k fixed relays.*

Definition 3. *Let $G'(n, \mathbf{E})$ and $G'_k(n, \mathbf{E})$ the respective equivalents of $G(n, \mathbf{E})$ and $G_k(n, \mathbf{E})$ but with the consideration of the path maximum energy instead of cumulated energy.*

3.1 Path cumulated energy

The following theorem shows the asymptotic connectivity properties of the hyperfractal in function of the cumulated energy and in function of the path maximum energy.

Theorem 3.1. *In a hyperfractal with n nodes. The following holds:*

$$\lim_{n\to\infty} \mathbb{E}\left\{\frac{|G_1(n, n^{-\gamma} P_{\max})|}{n}\right\} = 1 \tag{8}$$

for $\gamma < \delta - 1$
and

$$\lim_{n\to\infty} \mathbb{E}\left\{\frac{|G'_1(n, n^{-\gamma} P_{\max})|}{n}\right\} = 1 \tag{9}$$

for $\gamma < \delta$
where δ is the pathloss coefficient.

We make use of the following instrumental lemma that ensures the existence of nodes in a street.

Lemma 3.2. *There exists $a > 0$ such that, for all integers H and n, the probability that a street of level H contains less than $n\lambda_H/2$ nodes or more than $2n\lambda_H$ nodes is smaller than $\exp(-an\lambda_H)$.*

Proof. Let $N_H(n)$ be the number of nodes contained in the street of level H.

Let z be a real number. By Chebyshev's inequality, we have:

$$\mathbb{E}[e^{zN_H(n)}] = \left(1 + (e^z - 1)\lambda_H\right)^n$$

If $z > 0$:

$$\begin{aligned} P\left(N_H(n) < \frac{n\lambda_H}{2}\right) &= P\left(e^{-zN_H(n)} > e^{zn\lambda_H/2}\right) \\ &\leq \frac{\mathbb{E}[e^{-zN_H(n)}]}{e^{-zn\lambda_H/2}} \end{aligned}$$

Therefore

$$\frac{\mathbb{E}[e^{-zN_H(n)}]}{e^{-zn\lambda_H/2}} = \exp\left(n\left(\log\left(1 + (e^{-z} - 1)\lambda_H\right) + z\lambda_H/2\right)\right).$$

For $|z|$ bounded there exists $b > 0$ such that $|e^z - 1| \leq b|z|$ and there exists c such that $e^z - 1 \leq z + cz^2$. For $|x|$ bounded there exists d such that $\log(1+x) \leq x - cx^2$. From these steps we obtain that, for sufficiently small $|z|$, one has:

$$\begin{aligned}\log\left(1 + (e^{-z} - 1)\lambda_H\right) + z\frac{\lambda_H}{2} &\leq -z\frac{\lambda_H}{2} \\ &\quad +b\lambda_H z^2 - c\lambda_H^2 z^2 \\ &\leq -a\lambda_H.\end{aligned}$$

which settles that

$$\frac{\mathbb{E}[e^{-zN_H(n)}]}{e^{-zn\lambda_H/2}} \leq e^{-an\lambda_H}. \tag{10}$$

The proof of the second part of the lemma proceeds via a similar reasoning, by using the inequality:

$$P\left(N_H(n) > 2n\lambda_H\right) \leq \frac{\mathbb{E}[e^{zN_H(n)}]}{e^{2zn\lambda_H}}. \tag{11}$$

□

The following corollary gives a result on the scaling of the number of nodes in a segment of street and the cumulated energy, getting us one step closer to the results we are searching for.

Corollary 1. *Let $0 < \phi \leq 1$, assume an interval corresponding to a fraction ϕ of the street length. If the interval is on a street of level H, the probability that it contains less than $\phi\lambda_H n/2$ nodes and it is covered with a cumulated energy greater than $\phi(n\lambda_H)^{1-\delta}P_{\max}$ is smaller than $e^{-a\phi\lambda_H n}$.*

Proof. This is a slight variation of the previous proof.

Indeed if we denote by $N_H(n,\phi)$ the number of nodes on the segment, we have $\mathbb{E}[e^{tN_H(n,\phi)}] = (1 + \lambda_H\phi(e^z - 1))^n$. Similarly, the previous proof applies by just replacing λ_H by $\phi\lambda_H$.

The cumulated energy has the expression $P_{\max}\frac{N_H(n,\phi)}{N_H^{\delta}(n)}$. By further applying the previous reasoning to each of the random variables $N_H(n)$ and $N_H(n,\phi)$, we obtain the result. □

Throughout the rest of the paper, we only consider the cases where $d_m > 3$ and $d_r < d_m - 1$, *i.e.* $(2/q)^2 < 2/q'$.

The following theorem is the **main result** of our paper and shows that increasing the path length decreases the cumulated energy. In fact, for $n \to \infty$, the limiting energy goes to zero.

Theorem 3.3. *In a hyperfractal with n nodes, with nodes of fractal dimension d_m and relays of fractal dimension d_r, let $c_E > 0$ and $\alpha < 1$, let $E_n = c_E n^{(1-\delta)(1-\alpha)}P_{\max}$. There exists c_E such that the number of hops, D_n, on the shortest path of cumulated energy less than E_n between two nodes belonging to the giant component $G_1(n, E_n)$ is :*

$$D_n = O(n^{1-\alpha/(d_m-1)}). \tag{12}$$

Although the source and the destination belong to $G_1(n, E_n)$, it is not necessary that all the nodes constituting the path also belong to $G_1(n, E_n)$, i.e., the path may include nodes that are more than one hop from the central cross.

Remark 1. *We have the identity*

$$\left(\frac{E_n}{P_{\max}}\right)^{1/(\delta-1)} D_n^{d_m-1} = O(n^{d_m-2}). \tag{13}$$

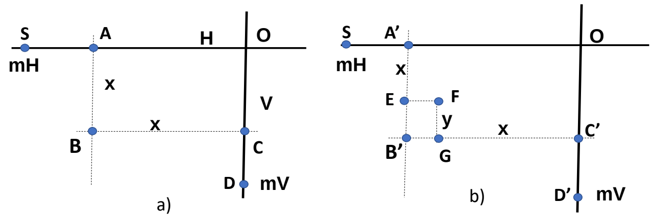


Figure 5: a) Diverted path with three fixed relays (left), b) five fixed relays (right).

Proof. The main part of our proof is to consider the case when the source, denoted by m_H, and the destination, m_V, both stand on two different segments of the central cross. In this case we consider the energy constraint $\frac{1}{3}E_n$. We can then easily extend the result to the case when the source and the destination stand anywhere in the giant component $G_1(n, E_n)$ by taking E_n as energy constraint and the theorem follows.

When m_H and m_V are on the central cross, there exists a direct path which takes the direct route by staying on the central cross, more specifically, in Figure 5 a), the segments $[SA]$,$[AO]$,$[OC]$,$[CD]$. Then, the path length is of order of $\Theta(n)$ while the cumulated energy of order $\Theta(n^{1-\delta})P_{\max}$.

In order to significantly reduce the order of magnitude of the path hop length, one must consider a diverted path with three fixed relays, as indicated in Figure 5 a). The diverted path proceeds into two streets of level x. Let $\mathcal{T}$ be the path. It is considered that $x = \alpha\frac{\log n}{\log(2/q')}$ for $\alpha < 1$.

The path is made of two times two segments: the segment of street $[SA]$ on the central cross which corresponds to the distance from the source to the first fixed relay to a street of level x, and then the segment $[AB]$ between this relay and the fixed relay to the crossing street of level x. The second part of the path is symmetric and corresponds to the connection between this relay and the destination through segment $[BC]$ and $[CD]$.

Denote by $L(x, y)$ the distance from an arbitrary position on a street of level y to the first fixed relay to a street of level x. The probability that a fixed relay exists at a crossing of two streets of respective level x and y is $1 - \exp(-\rho_n p(x, y))$. Since the spacing between streets of level x is 2^{-x}, it is known from [5] that

$$L(x, y) \leq \frac{2^{-x}}{1 - \exp(-\rho_n p(x, y))} \tag{14}$$

where ρ_n is the effective number of relays in the map (reminding that $\rho_n = n$ to simplify. The probability that the two streets of level x have a fixed relay at their crossing is $1 - \exp(-\rho_n p(x, x))$. With the condition $\rho = n$, one gets $\rho_n p(x, x) = n^{1-2\alpha\log(2/q)/\log(2/q')} > n^{1-\alpha}$ since $2\log(2/q) < \log(2/q')$. Therefore the probability that the relay does not exist decays exponentially fast. Since the cumulated energy of the path, $E(\mathcal{T})$, satisfies with high probability

$$E(\mathcal{T}) = O(L(x, 0)n^{1-\delta}P_{\max}) + O((n\lambda_x)^{1-\delta}P_{\max}) \tag{15}$$

and the average number of nodes of the path, $D(\mathcal{T})$, satisfies with probability tending to 1, exponentially fast:

$$D(\mathcal{T}) = O(L(x, 0)n) + O(n\lambda_x). \tag{16}$$

Then, with the value $x = \alpha \frac{\log n}{\log(2/q')}$, we detect that the main contributor of the cumulated energy are the segments $[AB]$ and $[BC]$, namely

$$E(\mathcal{T}) = O\left(n^{(1-\delta)(1-\alpha)}\right). \tag{17}$$

and let c_E such that

$$E(\mathcal{T}) \leq \frac{c_E}{3} n^{(1-\delta)(1-\alpha)}. \tag{18}$$

The main contributor in hop count in the path is in fact in the parts which stands on the central cross, namely $[m_H A]$ and $[m_V C]$:

$$D(\mathcal{T}) = O\left(n^{1-\alpha/(d_m-1)}\right). \tag{19}$$

□

In Theorem 3.3, it is always assumed that $E_n \to 0$, since $\alpha < 1$. In this case D_n spans from $O(n^{1-1/(d_m-1)})$ to $O(n)$ (corresponding to a path staying on the central cross). When the hyperfractal dimension d_m is large it does not make a large span. In fact, if E_n is assumed to be constant, *i.e.* $\alpha = 1$, then we can have a substantial reduction in number of hops, as described in the following theorem.

Theorem 3.4. *If $E_n = c_E P_{\max}$ with $c_E > 6$, then $D_n = O(n^{1-2/(d_r(1+1/d_m))})$.*

Remark 2. *When $d_r \to 2$ then $D_n = O(n^{1/(d_m+1)})$, and the hyperfractal model is behaving like a hypercube of dimension $d_m + 1$. Notice that in this case D_n tends to be $O(1)$ when $d_m \to \infty$.*

Proof. In the proof of Theorem 3.3, it is assumed that $x < \frac{\log n}{\log(2/q')}$ in order to ensure that the number of hops on the route of level x tends to infinity. However, we can rise the parameter x in the range $\frac{\log n}{\log(2/q')} \leq x < \frac{\log n}{2\log(2/q)}$.

We have $n\lambda_x \to 0$. In this case, $E(\mathcal{T}) \to 2P_{\max}$ since the streets of level x are empty of nodes with probability tending to 1. Let us denote $x = \beta \frac{\log n}{2\log(2/q)}$ with $\beta < 1$. We have $D(\mathcal{T}) = O(L(x,0)n) = O(n^{1-\beta/d_r})$. Clearly β cannot be greater than 1 as, in this case, the two streets of level x will not hold a fixed relay with high probability and the packet will not turn at the intersection. Therefore the smallest order one can obtain on the diverted path with three relays is limited to n^{1-1/d_r}, which is not the claimed one.

To obtain the claimed order, one must use the diverted path with five fixed relays, as shown in figure 5 b). The diverted path is composed by the segments: $[SA']$,$[A'E]$,$[EF]$,$[FG]$,$[GC']$ and $[C'D']$. It is shown in [5] that the order can be decreased to $n^{1-2/((1+1/d_m)d_r)}$. □

3.2 Path maximum energy

The next results revisit the previous theorems on the path cumulated energy in the corresponding case of the imposed constraint on the path maximum energy.

Theorem 3.5. *Let $M_n = n^{-\delta(1-\alpha)} P_{\max}$ for $\alpha < 1$. The number of hops D_n on the shortest path of maximum energy less than M_n between two nodes belonging to the giant component $G_1'(n, M_n)$ is:*

$$D_n = O(n^{1-\alpha/(d_m-1)})$$

Remark 3. *It is important to note that although the orders of magnitude of path length D_n are the same in both Theorem 3.3 and Theorem 3.5, the results consider two different giant components: (cumulated) $G_1(n, E_n)$ and (maximum) $G_1'(n, M_n)$.*

Remark 4. *We have the identity*

$$(M_n/P_{\max})^{1/\delta} D_n^{d_m-1} = O(n^{d_m-1-\delta}). \tag{20}$$

Theorem 3.6. *Let the maximum path energy between two points belonging to the giant component, $G_1'(n, M_n)$ be $M_n = P_{\max}$. The number of hops D_n on the shortest path is $O(n^{1-2/(d_r(1+1/d_m))})$.*

This theorem gives, in fact, the path length when no constraint on energy exists (or that the maximum energy allowed is the highest energy for a transmission between two neighbors in the hyperfractal map). Therefore, we obtain here the same results of [5], where an infinite radio range is considered.

3.3 Remarks on the network throughput capacity

Let us consider the scaling of the network throughput capacity when a constraint on the energy is imposed. In [29], the authors express the throughput capacity of random wireless networks as:

$$\zeta(n) = \Theta\left(\frac{n^2 \sum_{i\in G} \omega_i(n)}{\sum_{i,j\in G} r_{ij}}\right). \tag{21}$$

where $\zeta(n)$ is the throughput capacity, defined as the expected number of packets delivered to their destinations per slot, $\omega_i(n)$ is the expected transmission rate of each node i among all the nodes n and G is the giant component. In the following, denote by C the transmission rate of each node.

Using our results of Theorems 3.3 and 3.5 and substituting them in the expression 21, we obtain the following corollary on a lower bound of the network throughput capacity with constraints on path energy.

Corollary 2. *In a hyperfractal with n nodes, fractal dimension of nodes d_m, and $\alpha < 1$ and C the transmission rate of each node: when $E_n = O(n^{(1-\delta)(1-\alpha)} P_{\max})$ is the maximum cumulated energy of the minimal path between any pair of nodes in the giant component $G_1(n, E_n)$ or when $M_n = O(n^{-\delta(1-\alpha)} P_{\max})$ is the maximum path energy of the minimal path between any pair of nodes in the giant component $G_1'(n, M_n)$, a lower bound on the network throughput is:*

$$\zeta(n) = \Omega\left(C n^{\frac{\alpha}{d_m-1}}\right) \tag{22}$$

Remark 5. *We notice that with $\alpha < 1$ and $d_m > 3$ we have $\zeta(n)$ of order which can be smaller than $n^{1/2}$ which is less than the capacity in a random uniform network with no Canyon effect as described in [30].*

Remark 6. *When $\alpha = 1$, i.e. with no energy constraint $E_n = c_E P_{\max}$ the path length can drop down to $D_n = O\left(n^{1-2/((1+1/d_m)d_r)}\right)$ and, in this case, we have $\zeta(n) = \Omega(n^{2/((1+1/d_m)d_r)})$ which tends to be in $O(n)$ when $d_m \to \infty$ and $d_r \to 2$. In this situation the capacity is of optimal order since D_n tends to be $O(1)$.*

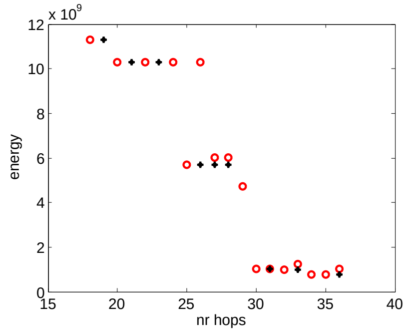


Figure 6: Minimum cumulated end-to-end energy versus hops for a transmitter-receiver pair.

4 NUMERICAL EVALUATION

This section presents an evaluation of the accuracy of our theoretical findings in different scenarios by comparing them to results obtained by simulating the events in a two-dimensional network. We use our own discrete time event-based simulator (developed in Matlab) which follows the model presented in Section 2.

We consider the length of the map to be 1000 and, therefore, $P_{\max}$ is just 1000^{δ}, where δ is the pathloss coefficient that will be chosen to be either 3 or 4, as it will be mentioned.

Let us look at Figure 6 to get a grasp of the trade-offs between cumulated end-to-end energy and hop count for a transmitter-receiver pair. The pair has been selected randomly from the simulations performed in a hyperfractal map with $n = 800$, pathloss coefficient $\delta = 4$, fractal dimension of nodes $d_m = 4.33$ and fractal dimension of relays $d_r = 3$. The plot shows the minimum cumulated energy for the end-to-end transmission for a fixed and allowed number of hops, k, in red circle markers. Note that the energy does not decrease monotonically as forcing to take a longer path may not allow to take the best path. However when considering the minimum cumulated energy of all paths *up to a number of hops*, the black star markers in Figure 6, the energy decrease and exhibits clearly the behavior analyzed in Theorem 3.3. That is, the minimum cumulated energy is indeed decreasing when the number of hops is allowed to grow (and the end-to-end communication is allowed to choose longer, yet cheaper, paths).

Let us further validate Theorem 3.3 through simulations performed for 100 randomly chosen transmitter-receiver pairs in hyperfractal maps with various configurations. We run simulations for different values of the number of nodes, $n = 800$ nodes and 1000 nodes respectively, different values of pathloss, $\delta = 3$ and $\delta = 4$ and different configurations of the hyperfractal map. The setups of the hyperfractal maps are: node fractal dimension $d_m = 4.33$ and relay fractal dimension $d_r = 3.3$ for the first setup and $d_m = 3.3$ and $d_r = 2.3$ for the second setup.

The results exhibited in Figure 7 are obtained by computing, for each of the transmitter-receiver pair, the minimum cumulated end-to-end energy for a path smaller than k, then averaging over the 100 results.

The left-hand sides of the Figures 7 (a) and 7 (b) show the variation of the minimum path cumulated energy for the path with the increase of the number of hops in a hyperfractal setup of $d_m = 4.33$ and $d_r = 3$ for $n = 800$ in 7 (a) and $n = 1000$ in 7 (b). The figures illustrate that, indeed, allowing the hop count to grow decreases the energy considerably. The decay of the maximum cumulated energy with the allowed number of hops is even more visible in logarithmic scale in the right side of the same figures.

The decays remains substantial when changing the hyperfractal setup to $d_m = 3.2$, $d_r = 2.3$. Figures 7 (c) and 7 (d) shows the results obtained for $n = 800$ and $n = 1000$ respectively in the new setup. The decay is shown to be very dramatic when looking in logarithmic scale. Even though there can be oscillations around the linearly decreasing characteristic, as one can notice in Figure 7 (d), left hand side, the global behavior stays the same, decreasing, as one can better notice in logarithmic scale in Figure 7 (d), right hand side.

When changing the pathloss coefficient to $\delta = 3$, the effect of Theorem 3.3 remains, as illustrated in Figure 8 for a hyperfractal setup of $d_m = 4.33$, $d_r = 3$, $n = 800$ nodes.

Next, we validate the claims of Theorem 3.5 on the variation of path length with the imposed constraint on maximum energy per node. For that, we choose randomly 100 transmitter-receiver pairs belonging to the central cross and we compute the shortest path by applying a constraint on the maximum transmission energy of nodes belonging to the path. For the configurations chosen, the hyperfractal setups are: nodes fractal dimension $d_m = 3.3$, relays fractal dimension $d_r = 2.3$, pathloss coefficient $\delta = 3.3$ and we vary the number of nodes, n to be either $n = 500$ or $n = 800$.

For both values of n, Figure 9, shows that indeed, decreasing the constraint of path maximum energy leads to an increase in the path length.

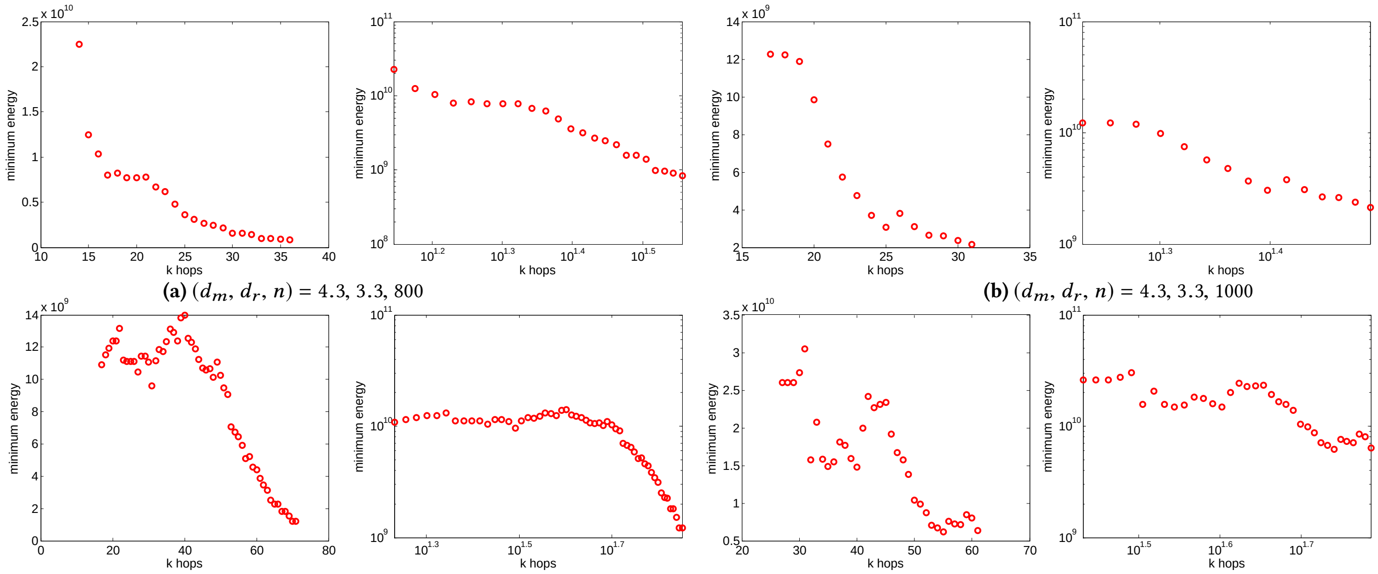
Changing the fractal dimensions (both streets and relays) does not change the behavior, as illustrated in Figure 10. In this case, the hyperfractal setups have the following base configuration: nodes fractal dimension $d_m = 4.33$, relays fractal dimension $d_r = 3$, pathloss coefficient $\delta = 4$ and we vary the number of nodes, n to be either $n = 500$ or $n = 800$. Again, making a tougher constraint on the path maximum energy leads to the increase of the path length, showing that achievable trade-offs in hyperfractal maps of nodes with road-side infrastructure.

5 CONCLUSION

This paper presented results on the trade-offs between the end-to-end communication delay and energy spent for completing a transmission in vehicular communication in urban settings by exploiting the “hyperfractal” model. This model captures self-similarity as an environment characteristic. The self-similar characteristic of the road-side infrastructure has also been incorporated.

Analytical bounds have been derived for the end-to-end communication hop count under the constraints of total accumulated energy, and maximum energy per node, exhibiting the achievable trade-offs in a hyperfractal network. A lower bound on the network throughput capacity with constraints on path energy is given. Further examples of model fitting with data have been given.

Analytical results have been validated using a discrete time event-based simulator developed in Matlab.



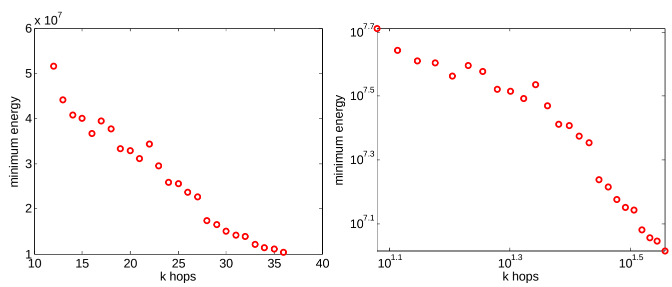
(a) $(d_m, d_r, n) = 4.3, 3.3, 800$

(b) $(d_m, d_r, n) = 4.3, 3.3, 1000$

(c) $(d_m, d_r, n) = 3.3, 2.3, 800$

(d) $(d_m, d_r, n) = 3.3, 2.3, 1000$

Figure 7: Minimum cumulated end-to-end energy versus hops, averaging over 100 transmitter-receiver pairs, $\delta = 4$, linear scale left side of sub-figures, logarithmic scale right side of sub-figures



(a) Linear **(b) Logarithmic scale**

Figure 8: Minimum cumulated end-to-end energy versus hops, averaging over 100 transmitter-receiver pairs, $\delta = 3$

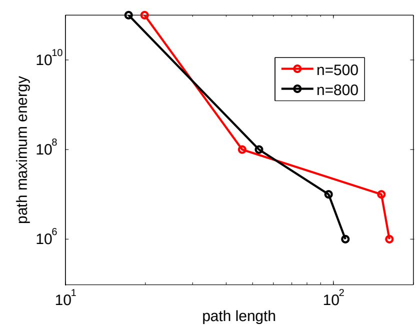


Figure 10: Path maximum energy versus hops $d_m = 4.33, d_r = 3$

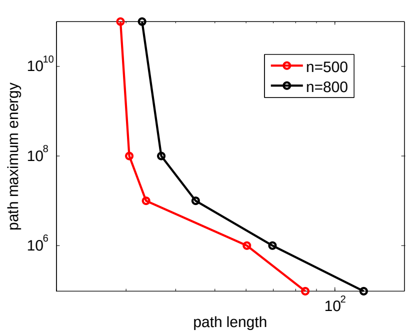


Figure 9: Path maximum energy versus hops $d_m = 3.3, d_r = 2.3$

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APPENDIX

A second relevant example of data fitting tot he hyperfractal model is showcased here. Figure 11 shows the annual average traffic estimates in a neighborhood of Adelaide, Australia. The data is provided by [31].

By making use of the same fitting procedure described earlier, the estimated fractal dimension of this neighborhood of Adelaide is $d_m = 2.8$. The fitting is further illustrated in Figure 12.

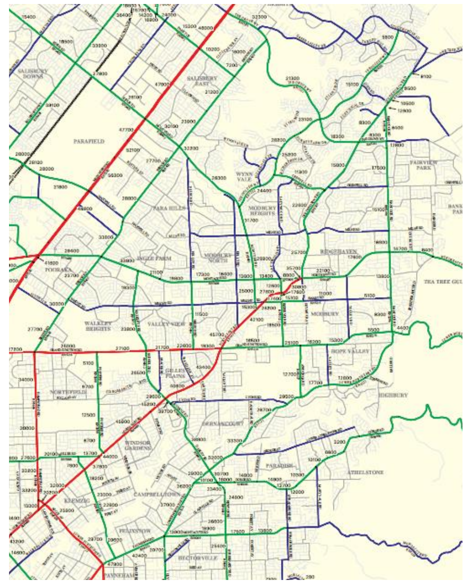

Figure 11: Annual Average Traffic Estimates in Adelaide

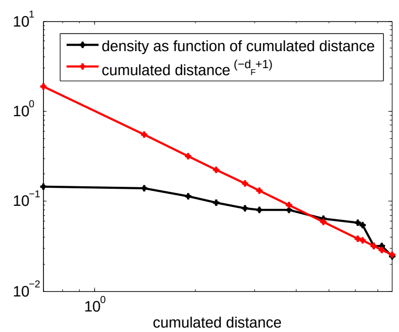

Figure 12: Hypefractal model fitting for the city of Adelaide