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► **To cite this version:**

Rémi Barat, Cédric Chevalier, François Pellegrini. Balance-Enforced Multi-Level Algorithm for Multi-Criteria Graph Partitioning. SIAM CSE 2017, Mar 2017, Atlanta, United States. hal-01671514

**HAL Id: hal-01671514**

**<https://inria.hal.science/hal-01671514v1>**

Submitted on 22 Dec 2017

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# Balance-Enforced Multi-Level Algorithm for Multi-Criteria Graph Partitioning

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28 February 2017

SIAM Conference on Computational Science and Engineering

- 1 Objective
  - Context
  - Example & Model
  - State of the art
- 2 Approach
  - The multi-level framework
  - Scotch multi-criteria algorithm
- 3 Results
- 4 Perspective
  - Improving robustness
- 5 Conclusion

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In the field of numerical simulation, one challenge is to reduce the restitution time.

We consider simulations:

- Running on distributed memory architectures
- Using 2D or 3D meshes
- Coupling several physics (multi-physics simulations)

Running these simulations efficiently needs to:

- 1 Balance the workloads of each processor for each phase of the computation
- 2 Overlap or minimize communications
- 3 Take care of memory accesses
- 4 Exploit full processor characteristics

We focus on the first 2 items.

## Algorithm 1 Example of the temporal loop of a multi-physics simulation

(2 computing phases:  $f$  and  $g$ , coupling the variables  $X$  and  $Y$ )

**for**  $t \in [0, t_{end} - 1]$  **do**

$$X(t+1) \leftarrow f(t+1, X(t), Y(t))$$

# **First** computing phase

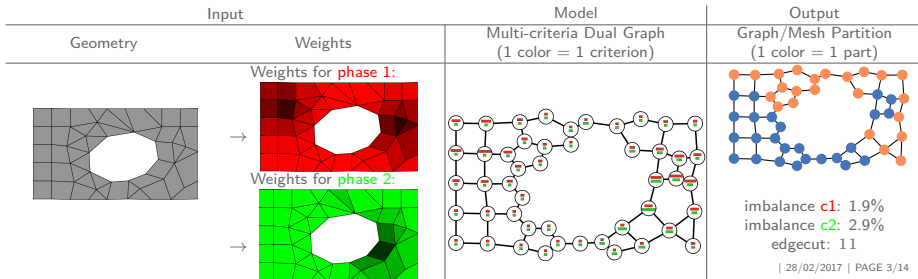
$$Y(t+1) \leftarrow g(t+1, X(t+1), Y(t))$$

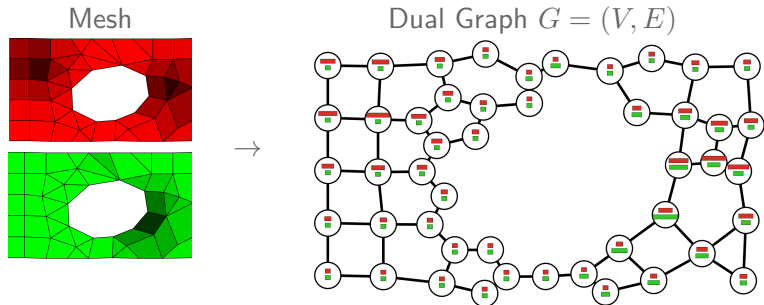
# **Second** computing phase

**end for**

Objectives:

- 1 Balance the workloads of each processor *for each phase*
- 2 Minimize communications between processors





## Problem : Graph partitioning

We search for a partition  $P$  of  $V$  respecting:

- 1 some **constraints**: load-balancing  
(imbalance of  $P$  is for all criteria smaller than a given tolerance)
- 2 an **objective**: minimize the edgecut  
(sum of the weights of the edges cut by the partition).

NP-Hard Problem.

Main existing software:

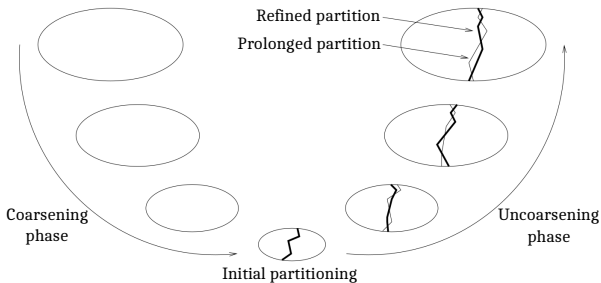
Software	Representations	Multi-criteria	Origin	Limitations for us
Scotch	Topological	No	INRIA	No multi-criteria (yet)
MeTiS	Topological	Yes	University of Minnesota	Does not often meet the balance constraints
PaToH	Topological	Yes	Bilkent University	No parallel version No open-source version Does not always meet the balance constraints
Zoltan	Geometric Topological	Yes No	Sandia National Laboratories	Geometric representations are inefficient

⇒ Lack of efficient multi-criteria partitioning tools.

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A 3-phases algorithm:

- 1 Coarsening
- 2 Initial partitioning of the coarsened hypergraph
- 3 Uncoarsening and refinement



A 3-phases algorithm:

**1 Coarsening**

→ (Ongoing) Selection of a matching policy improving the quality of the returned solutions

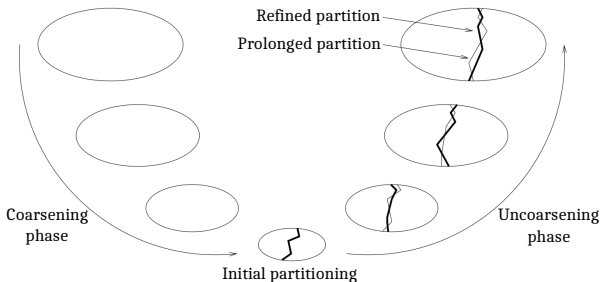
**2 Initial partitioning of the coarsened hypergraph**

→ New algorithm focusing on balance constraints

**3 Uncoarsening and refinement**

→ Adapted Fiduccia-Mattheyses algorithm

Implemented  
in Scotch



- Coarsening:
  - Classical algorithm: Heavy-Edge-Matching
- Initial Partitioning:
  - Local optimization of the balance
  - Enforce balance for all the constraints
  - Presented in SIAM CSC 2016
- Refinement:
  - Fiduccia-Mattheyses algorithm: local optimization of the edgecut with hill-climbing
  - Moves that unbalance the partition more than the tolerance are forbidden

⇒ Finding a balanced partition of the coarsest graph guarantees that the returned partition will respect the balance constraints.



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Comparison between

- MeTiS (5.1.0)
- PaToH (3.2)
- Scotch (6.1.0 - not yet released multi-criteria version)

Instances considered:

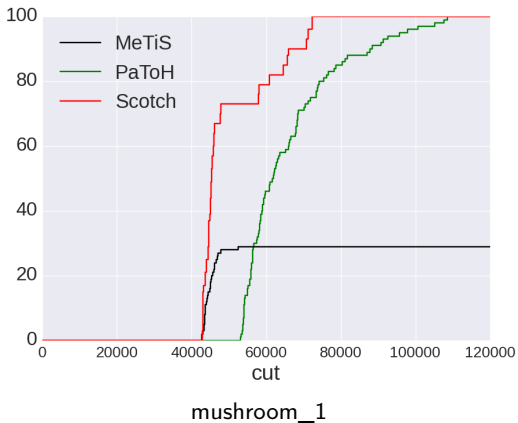
Name	Geometry	Vertices	Edges	Criteria	Edge weights
mushroom_1	2D	22800	45253	3	$\sqrt{w_i + w_j}$
mushroom_2	2D	22800	45253	3	independent
onera	3D	85567	166817	3	$\sqrt{w_i + w_j}$
wave	3D	156317	1059331	3	$\sqrt{w_i + w_j}$

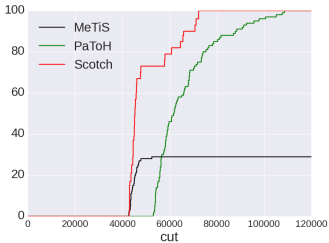
Partitioning parameters:

- Imbalance tolerance 5%
- Number of parts: 2
- Each algorithm is executed 100 times on each instance with random seeds.

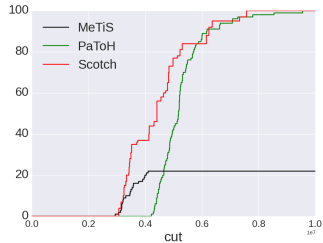
$f_{algo}(x)$  = number of valid solutions of *algo* of cut  $\leq x$ .

- We want to minimize the cut of each returned solution: left is better

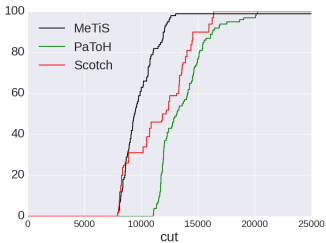




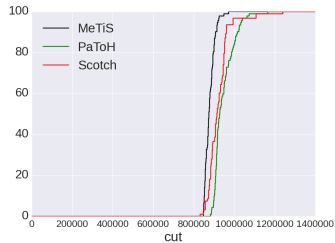
mushroom\_1



mushroom\_2



onera

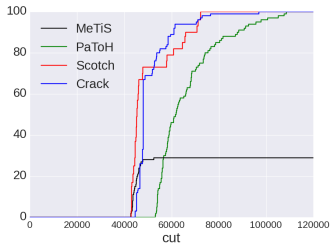


wave

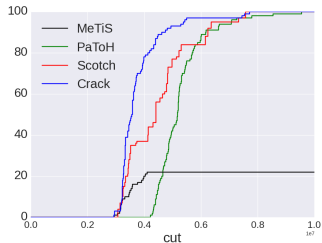
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- Python prototype: Crack
- Allows to test different parameters on the multi-level
- An example: settings of the coarsening parameters
  - Heavy-Edge-Matching
  - Particular order of the vertices:  
sorted by increasing degrees
  - Do not allow to match 2 vertices if the resulting vertex is overweighted

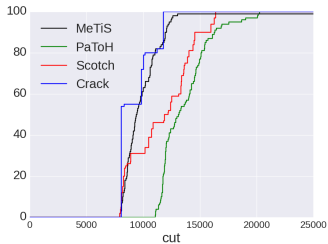
# Comparison of Crack on the previous instances



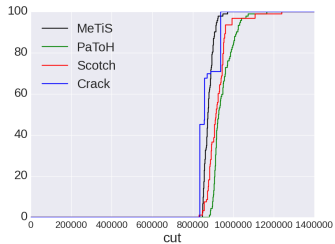
mushroom\_1



mushroom\_2



onera



wave

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## Summary:

- Scotch next release will support multi-criteria graph partitioning
- This new version implements a new initial partitioning and sets the refinement phase to guaranty that the returned solution will respect the balance constraints
- Scotch results are promising

## Perspectives:

- Enforce the algorithm robustness by the study of variations of the algorithms
- Validation on more instances  
Validation on a simulation code
- Set up of a parallel version of the algorithms

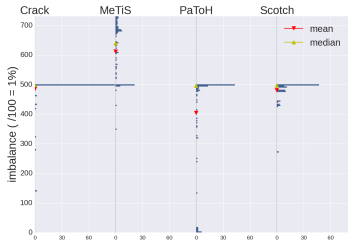
Thank you

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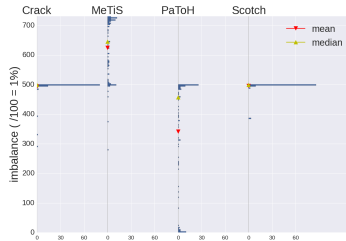
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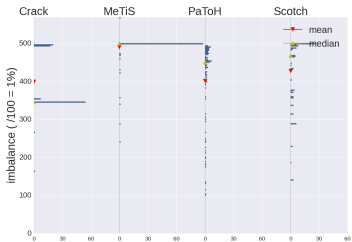
# Imbalance distributions



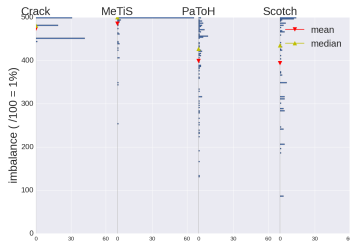
mushroom\_1



mushroom\_2



onera



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