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To cite this version:
Rémi Barat, Cédric Chevalier, François Pellegrini. Balance-Enforced Multi-Level Algorithm for Multi-Criteria Graph Partitioning. SIAM CSE 2017, Mar 2017, Atlanta, United States. hal-01671514

HAL Id: hal-01671514
https://inria.hal.science/hal-01671514
Submitted on 22 Dec 2017

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Balance-Enforced Multi-Level Algorithm for Multi-Criteria Graph Partitioning

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28 February 2017
SIAM Conference on Computational Science and Engineering
Outline

1 Objective
   - Context
   - Example & Model
   - State of the art

2 Approach
   - The multi-level framework
   - Scotch multi-criteria algorithm

3 Results

4 Perspective
   - Improving robustness

5 Conclusion
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In the field of numerical simulation, one challenge is to reduce the restitution time.

We consider simulations:

- Running on distributed memory architectures
- Using 2D or 3D meshes
- Coupling several physics (multi-physics simulations)

Running these simulations efficiently needs to:

1. Balance the workloads of each processor for each phase of the computation
2. Overlap or minimize communications
3. Take care of memory accesses
4. Exploit full processor characteristics

We focus on the first 2 items.
Algorithm 1 Example of the temporal loop of a multi-physics simulation
(2 computing phases: \( f \) and \( g \), coupling the variables \( X \) and \( Y \))

\[
\text{for } t \in [0, t_{\text{end}} - 1] \text{ do}
\]
\[
X(t + 1) \leftarrow f(t + 1, X(t), Y(t)) \quad \# \text{ First computing phase}
\]
\[
Y(t + 1) \leftarrow g(t + 1, X(t + 1), Y(t)) \quad \# \text{ Second computing phase}
\]
\[
\text{end for}
\]

Objectives:
1. Balance the workloads of each processor for each phase
2. Minimize communications between processors

<table>
<thead>
<tr>
<th>Input</th>
<th>Model</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometry</td>
<td>Weights</td>
<td>Multi-criteria Dual Graph (1 color = 1 criterion)</td>
</tr>
<tr>
<td>Graph/Mesh Partition (1 color = 1 part)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Weights for phase 1:

Weights for phase 2:

imbalance c1: 1.9%
imbalance c2: 2.9%
edgecut: 11
Model

Mesh

Dual Graph $G = (V, E)$

Problem : Graph partitioning

We search for a partition $P$ of $V$ respecting:

1. some **constraints**: load-balancing
   (imbalance of $P$ is for all criteria smaller than a given tolerance)

2. an **objective**: minimize the edgecut
   (sum of the weights of the edges cut by the partition).

NP-Hard Problem.
Main existing software:

<table>
<thead>
<tr>
<th>Software</th>
<th>Representations</th>
<th>Multi-criteria</th>
<th>Origin</th>
<th>Limitations for us</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scotch</td>
<td>Topological</td>
<td>No</td>
<td>INRIA</td>
<td>No multi-criteria (yet)</td>
</tr>
<tr>
<td>MeTiS</td>
<td>Topological</td>
<td>Yes</td>
<td>University of Minnesota</td>
<td>Does not often meet the balance constraints</td>
</tr>
<tr>
<td>PaToH</td>
<td>Topological</td>
<td>Yes</td>
<td>Bilkent University</td>
<td>No parallel version&lt;br&gt;No open-source version&lt;br&gt;Does not always meet the balance constraints</td>
</tr>
<tr>
<td>Zoltan</td>
<td>Geometric</td>
<td>Yes</td>
<td>Sandia National Laboratories</td>
<td>Geometric representations are inefficient</td>
</tr>
</tbody>
</table>

⇒ Lack of efficient multi-criteria partitioning tools.
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A 3-phases algorithm:

1. Coarsening

2. Initial partitioning of the coarsened hypergraph

3. Uncoarsening and refinement
Our approach: Multi-level multi-criteria algorithm

A 3-phases algorithm:

1. Coarsening
   → (Ongoing) Selection of a matching policy improving the quality of the returned solutions

2. Initial partitioning of the coarsened hypergraph
   → New algorithm focusing on balance constraints

3. Uncoarsening and refinement
   → Adapted Fiduccia-Mattheyses algorithm

Implemented in Scotch

Coarsening phase

Uncoarsening phase

Initial partitioning

Revised partition

Prolonged partition

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Coarsening:
- Classical algorithm: Heavy-Edge-Matching

Initial Partitioning:
- Local optimization of the balance
- Enforce balance for all the constraints
- Presented in SIAM CSC 2016

Refinement:
- Fiduccia-Mattheyses algorithm: local optimization of the edgecut with hill-climbing
- Moves that unbalance the partition more than the tolerance are forbidden

⇒ Finding a balanced partition of the coarsest graph guarantees that the returned partition will respect the balance constraints.
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Comparison between

- **MeTiS (5.1.0)**
- **PaToH (3.2)**
- **Scotch (6.1.0 - not yet released multi-criteria version)**

Instances considered:

<table>
<thead>
<tr>
<th>Name</th>
<th>Geometry</th>
<th>Vertices</th>
<th>Edges</th>
<th>Criteria</th>
<th>Edge weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>mushroom_1</td>
<td>2D</td>
<td>22800</td>
<td>45253</td>
<td>3</td>
<td>$\sqrt{w_i + w_j}$</td>
</tr>
<tr>
<td>mushroom_2</td>
<td>2D</td>
<td>22800</td>
<td>45253</td>
<td>3</td>
<td>independant</td>
</tr>
<tr>
<td>onera</td>
<td>3D</td>
<td>85567</td>
<td>166817</td>
<td>3</td>
<td>$\sqrt{w_i + w_j}$</td>
</tr>
<tr>
<td>wave</td>
<td>3D</td>
<td>156317</td>
<td>1059331</td>
<td>3</td>
<td>$\sqrt{w_i + w_j}$</td>
</tr>
</tbody>
</table>

Partitioning parameters:

- Imbalance tolerance 5%
- Number of parts: 2
- Each algorithm is executed 100 times on each instance with random seeds.
1 - How to compare the results

\[ f_{algo}(x) = \text{number of valid solutions of } algo \text{ of cut } \leq x. \]

- We want to minimize the cut of each returned solution: left is better.
Results

2 - Comparison on the quality of the returned solutions
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Improving robustness

- Python prototype: Crack
- Allows to test different parameters on the multi-level
- An example: settings of the coarsening parameters
  - Heavy-Edge-Matching
  - Particular order of the vertices: sorted by increasing degrees
  - Do not allow to match 2 vertices if the resulting vertex is overweighted
Comparison of Crack on the previous instances

**mushroom_1**

**mushroom_2**

**onera**

**wave**
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Conclusion

Summary:

- Scotch next release will support multi-criteria graph partitioning
- This new version implements a new initial partitioning and sets the refinement phase to guaranty that the returned solution will respect the balance constraints
- Scotch results are promising

Perspectives:

- Enforce the algorithm robustness by the study of variations of the algorithms
- Validation on more instances
  - Validation on a simulation code
- Set up of a parallel version of the algorithms
Imbalance distributions

mushroom_1

mushroom_2

onera

wave