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Interoperability between arithmetic proofs using Dedukti

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Motivations & objectives

- There exist many logics and many proof checkers for these logics
- Some logics are more powerful than others (e.g. quantify over proofs)
- Theorems and proofs are not shared between different proof checkers: Well-known theorems and proofs are proved manually for each new proof checker/logic
- There is no standard for these logics
- Our objective is to translate automatically a small amount of arithmetic proofs from an expressive logic called The Calculus of Inductive Constructions (CIC), to a less expressive logic, The Higher Order Logic (HOL).

Dedukti is handy for interoperability

- Dedukti implements the \( \lambda \Pi \)-calculus modulo theory. It is a logical framework
- Logical frameworks are a kind of proof checker that allows to embed several logics
- Logical framework are good systems to make interoperability easier
- \( \lambda \Pi \)-calculus modulo theory is a simple logic that combines dependent types and rewrite rules
- The substantial advantage of dedukti for interoperability is that the encoding of a logical system \( \mathcal{L} \) to dedukti is shallow:
  - Use of Higher-order Abstract syntax
  - Type preservation: \( \Gamma \vdash T : T \Rightarrow |\Gamma| \vdash |T| \Rightarrow |T| \)
  - Computation preservation: \( t_1 \rightarrow_{\mathcal{L}} t_2 \Rightarrow |t_1| \rightarrow_{\mathcal{H}} |t_2| \)

Dedukti[CIC] to Dedukti[HOL]

- This translation is not always possible: In CIC, it is possible to quantify over proofs, not in HOL or in CIC, there is an infinite hierarchy of universes that does not exist in HOL
- But there should be a translation for arithmetic proofs: one does not need universes nor to quantify over proofs
- Features to remove:
  - Universes
  - Dependent types
  - Inductive definitions and recursive definitions (encoded in \( \lambda \Pi \)-calculus modulo theory by rewrite rules)

Dedukti as a proof assistant

- Implements tactic in Dedukti (R. Cauderlier)
- Use external provers to prove intermediate results (A. Defourné)
- Prove that the convertibility test is decidable by proving the termination (G. Genestier)
- Implicits, Elaboration and Unification in Dedukti (R. Bocquet)

Implementation

- Open Theory (OT) is already a tool for interoperability between HOL family provers.
- \( \forall \) and \( \Rightarrow \) are primitives in Dedukti[HOL] but not in OT. This extends to logical rules like the Modus Ponens
- Terms in Dedukti[HOL] are modulo \( \beta \) but not in OT where this conversion is explicit.
- Other technical problems arise such as that Dedukti uses De Bruijn indices but OT does not, polymorphism in Dedukti[HOL] does not work the same way as in OT...

Future Work

- Automatize the translation
- Extend that arithmetic library to other provers such as Coq, Matita, PVS...
- Create the W3P (W3C of proofs) in order to create the first standards for proofs
- OCaml is not really handy to write such compilers especially for binders. A joint work with Prof. Brigitte Pientka is to look at a new logical/programming system that would be handy to express proof transformations
- Embed other logical systems like CubicalTT (C. Leena Subramaniam)
- Extend Dedukti to rewrite modulo a congruence (G. Ferey)