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Interoperability between arithmetic proofs using Dedukti

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Motivations & objectives

- There exist many logics and many proof checkers for these logics.
- Some logics are more powerful than others (e.g., quantify over proofs).
- Theorems and proofs are not shared between different proof checkers: Well-known theorems and proofs are proved manually for each new proof checker/logic.
- There is no standard for these logics.
- Our objective is to translate automatically a small part of arithmetic proofs from an expressive logic called The Calculus of Inductive Constructions (CIC), to a less expressive logic, The Higher Order Logic (HOL).

Dedukti is handy for interoperability

- Dedukti implements the λΠ-calcul modulo theory. It is a logical framework.
- Logical frameworks are a kind of proof checker that allows to embed several logics.
- Logical framework are good systems to make interoperability easier.
- λΠ-calcul modulo theory is a simple logic that combines dependent types and rewrite rules.
- The substantial advantage of dedukti for interoperability is that the encoding of a logical system L to dedukti is shallow:
  - Use of Higher-order Abstract syntax
  - Type preservation: \( \Gamma \vdash t : T \Rightarrow \Gamma \vdash \|t\|_L : \|T\|_L \)
  - Computation preservation: \( t_1 \rightarrow_L t_2 \Rightarrow \|t_1\|_L \rightarrow_L \|t_2\|_L \)

Dedukti[CIC] to Dedukti[HOL]

- This translation is not always possible: In CIC, it is possible to quantify over proofs, not in HOL or in CIC, there is an infinite hierarchy of universes that does not exist in HOL.
- But there should be a translation for arithmetic proofs: one does not need universes nor to quantify over proofs.
- Features to remove:
  - Universes
  - Dependent types
  - Inductive definitions and recursive definitions (encoded in λΠ-calcul modulo theory by rewrite rules).

Dedukti as a proof assistant

- Implements tactic in Dedukti (R. Cauderlier).
- Use external provers to prove intermediate results (A. Defourné).
- Prove that the convertibility test is decidable by proving the termination (G. Genestier).
- Implicits, Elaboration and Unification in Dedukti (R. Bocquet).

Implementation

- Implementation in OCaml (5000 lines).
- Compilation time with Ediloh (Fermat little theorem): 20s.
- Several independent tools:
  - Universo (remove universes).
  - Deduktipli (remove dependent types).
  - Ediloh (the compiler from Dedukti[HOL] to OpenTheory).

Future Work

- Automatize the translation.
- Extend that arithmetic library to other provers such as Coq, Matita, PVS...
- Create the W3P (W3C of proofs) in order to create the first standards for proofs.
- OCaml is not really handy to write such compilers especially for binders. A joint work with Prof. Brigitte Pientka is to look at a new logical/programming system that would be handy to express proof transformations.
- Embed other logical systems like CubicalTT (C. Leena Subramaniam).
- Extend Dedukti to rewrite modulo a congruence (G. Ferey).

Dedukti[HOL] to OpenTheory

- Open Theory (OT) is already a tool for interoperability between HOL family provers.
- \( \forall \) and \( \Rightarrow \) are primitives in Dedukti[HOL] but not in OT. This extends to logical rules like the Modus Ponens.
- Terms in Dedukti[HOL] are modulo \( \beta \) but not in OT where this conversion is explicit.
- Other technical problems arise such as that Dedukti uses De Bruijn indices but OT does not, polymorphism in Dedukti[HOL] does not work the same way as in OT...