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Interoperability between arithmetic proofs using Dedukti

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Motivations & objectives
- There exist many logics and many proof checkers for these logics
- Some logics are more powerful than others (e.g. quantify over proofs)
- Theorems and proofs are not shared between different proof checkers: Well-known theorems and proofs are proved manually for each new proof checker/logic
- There is no standard for these logics
- Our objective is to translate automatically a small library of arithmetic proofs from an expressive logic called The Calculus of Inductive Constructions (CIC), to a less expressive logic, The Higher Order Logic (HOL).

Dedukti is handy for interoperability
- Dedukti implements the λΠ-calcul modulo theory. It is a logical framework
- Logical frameworks are a kind of proof checker that allows to embed several logics
- Logical framework is good systems to make interoperability easier
- λΠ-calcul modulo theory is a simple logic that combines dependent types and rewrite rules
- The substantial advantage of dedukti for interoperability is that the encoding of a logical system $L$ to dedukti is shallow:
  - Use of Higher-order Abstract syntax
  - Type preservation : $Γ ⊢ t : T$ ⇒ $Γ' ⊢ t : T'$
  - Computation preservation : $t_1 →_L t_2$ ⇒ $||t_1|| →_{βΠ} ||t_2||$

Dedukti[CIC] to Dedukti[HOL]
- This translation is not always possible: In CIC, it is possible to quantify over proofs, not in HOL or in CIC, there is an infinite hierarchy of universes that does not exist in HOL.
- But there should be a translation for arithmetic proofs: one does not need universes nor to quantify over proofs
- Features to remove:
  - Universes
  - Dependent types
  - Inductive definitions and recursive definitions (encoded in λΠ-calcul modulo theory by rewrite rules)

Dedukti as a proof assistant
- Implements tactic in Dedukti (R. Cauderlier)
- Use external provers to prove intermediate results (A. Defourné)
- Prove that the convertibility test is decidable by proving the termination (G. Genestier)
- Implicits, Elaboration and Unification in Dedukti (R. Bocquet)

Implementation
- Implementation in OCaml (5000 lines)
- Compilation time with Ediloh (Fermat little theorem): 20s
- Several independent tools:
  - Universo (remove universes)
  - Deduktipli (remove dependent types)
  - Ediloh (the compiler from Dedukti[HOL] to OpenTheory)

Future Work
- Automatize the translation
- Extend that arithmetic library to other provers such as Coq, Matita, PVS...
- Create the W3P (W3C of proofs) in order to create the first standards for proofs
- OCaml is not really handy to write such compilers especially for binders. A joint work with Prof. Brigitte Pientka is to look at a new logical/programming system that would be handy to express proof transformations
- Embed other logical systems like CubicalTT (C. Leena Subramaniam)
- Extend Dedukti to rewrite modulo a congruence (G. Ferey)