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Engineering Notes

Ultimate Boundedness Results for Noise-Corrupted Quaternion Output Feedback Attitude Tracking Controllers

1 Sungpil Yang*

University of Texas at Austin, Austin, Texas 78712

Frédéric Mazenc[†]

2 Centre National de la Recherche Scientifique, 91192

Gif-sur-Yvette, France

and

Maruthi R. Akella[‡]

University of Texas at Austin, Austin, Texas 78712

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I. Introduction

THE importance of the rigid-body attitude control problem is well established with a wide range of applications from robot manipulators to satellites, and it continues to receive significant attention in recent literature. In spite of the technical difficulties of this problem that stem from the mathematical description of the attitude and the nonlinearity of the dynamics, this subject has been extensively studied and numerous solutions proposed over the past two decades [1]: Three-parameter representations like Euler angles, Rodrigues parameters, and modified Rodrigues parameters suffer from the kinematic singularity, and singularity-free quaternions are limited by the nonlinear unit norm constraint. In the Euler rotational dynamics, the formulation is more complex due to the Coriolis term.

Both stabilizing and tracking solutions, using various attitude representations, are now well understood: especially for full-state feedback control systems [1–5]. However, full-state feedback is not always possible due to cost limitations or design constraints. In the cases where rate-gyroscope measurements do not directly appear in the control law or gyroscope failure occurs during the mission, passivity-based approaches have been suggested for alternate solutions. High-gain observers or numerical differentiation schemes are other options, but the closed-loop system may be subject to peaking phenomena or noise inversely proportional to a sampling time.

Attitude stabilization without using the rate state or its estimate was originally proposed by Lizarralde and Wen via the passivity framework [6]. A stable filter driven by attitude signals is considered to achieve a passive system interconnection, and both the input and output of the filter are employed to design a velocity-free control law. Many further extensions including different parameterizations and generalization to tracking control have appeared in the literature [7–12].

Because passivity-based velocity-free controllers are model independent for the setpoint regulation problem, robustness properties to inertia parameter uncertainties automatically follow. For the tracking case, however, the controller is dependent on the inertia matrix, so it can be subject to model mismatch. In the literature, output feedback adaptive attitude tracking control was studied in [13], which was automatically robust to the inertia uncertainty. In addition to system identification error, these controllers are, in general, subject to other types of uncertainty. In reality, the body orientation cannot be perfectly measured and external disturbance torques always exist.

The attitude states are determined directly from line-of-sight measurements or are estimated using filters [14]. Accelerometers, magnetometers, and star sensors can provide vector measurements that determine the body orientation through deterministic methods like TRIAD, QUEST, or FOAM in a way that measurement errors are minimized [15–17]. Sometimes, rate gyroscopes are used to filter out measurement noise [14]. Recently, the velocity-free stabilization problem using vector measurements instead of converting them to the attitude variables was addressed, but measurement errors were excluded in the analyses [18,19].

Environmental disturbances within a feedback loop are another source of uncertainty. Solar radiation pressure, atmospheric drag, Earth-oblateness, or gravity gradient effects can all generate nonnegligible external torques that perturb the body orientation. To deal with this robustness question, the attitude control problem is typically formulated by including an additive bounded disturbance term in the Euler rotational dynamics. In the literature, on one hand, several disturbance-rejection controllers are proposed using modified filters: Subbarao and Akella [9] designed a control algorithm where integral terms appeared in both the controller and the filter, whereas Bin et al. [11] proposed a nonsmooth control, achieving semiglobal stability in the presence of disturbances. On the other hand, in [20,21], full-state and observer-based output feedback controllers with exponential gains were proposed, establishing that closed-loop systems are uniformly practically asymptotically stable (UPAS) when disturbance torques exist (roughly speaking, UPAS means the asymptotic stability of a ball for which the radius becomes smaller by tuning controller gains [20–22]). More recently, Xian et al. [23] proposed a velocity-free output feedback tracking controller that was robust to bounded external disturbance torques.

When full states were available, Bandyopadhyay et al. [24] proposed nonlinear tracking controllers with various attitude representations for a spacecraft carrying an unknown large object that significantly influences inertia parameters. In the presence of modeling uncertainties, measurement errors, and disturbance torques, the controllers guaranteed global exponential convergence of tracking errors with finite-gain \mathcal{L}_p stability. In this Note, we propose a passivity-based output feedback tracking controller with quaternion kinematics while considering measurement errors, disturbance torques, and parameter uncertainties. The proposed controller achieves asymptotic stability of the tracking errors in the ideal case and guarantees ultimate boundedness of the closed-loop trajectories through proper gains that can be selected from a stability analysis.

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*Ph.D. Candidate, Department of Aerospace Engineering and Engineering Mechanics, 1 University Station; yang.sungpil@utexas.edu. Student Member AIAA.

[†]Chargé de Recherche, French Institute for Research in Computer Science and Automation Project Team, Dynamical Interconnected Systems in Complex Environments, Laboratoire des Signaux et Systèmes (Unités Mixtes de Recherche, CNRS 8506), CentraleSupélec, Université Paris-Sud, 3 Rue Joliot Curie; frederic.mazenc@l2s.centralesupelec.fr.

[‡]Professor, Department of Aerospace Engineering and Engineering Mechanics, 1 University Station. Myron L. Begeman Fellow in Engineering; makella@mail.utexas.edu. Associate Fellow AIAA.

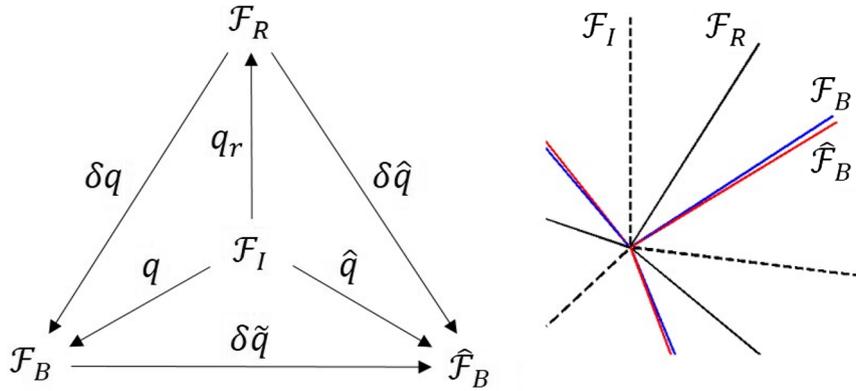


Fig. 1 Frames and corresponding quaternions.

Because the problem is formulated with the unit-quaternion representation, we can avoid geometric singularity normally caused by minimal three-dimensional parameterizations. Compared to the global representation of attitude kinematics described by direction cosine matrices, it does not require high-dimensional formulation, and thus higher-order numerical renormalization to satisfy the orthogonality constraint. Unlike the control laws with the quaternion kinematics in [24] and the ones in [13,23], the proposed controller does not include control singularity that occurs when the scalar part of the error quaternion becomes zero. Although those controllers are nonsingular if they are initially, quaternion tracking errors in the presence of measurement uncertainties around the singular point may lead to undesirable behavior. Specifically, to achieve disturbance rejection, one of the gains of the controller in [23] became arbitrarily large, even for arbitrarily small disturbances if the initial conditions were close to the singular point; therefore, appropriate reference designs must precede the gain selection to avoid any potential singularity issues.

This work builds on the preliminary results from [25,26] by generalizing to the tracking case and some further analysis of the filter-free case. Extending the strictification technique (see [27–29] for details), a partially strict Lyapunov function is constructed toward establishment of stability and ultimate boundedness properties for the closed-loop system. With known upper bounds of the magnitude of measurement errors, disturbance torques, and parameter uncertainties, a feasible range for the feedback gains is derived in terms of bounds on the initial conditions in such a way to ensure asymptotic convergence of all closed-loop signals to within a residual set. Because the system is not input-to-state stable, boundedness properties do not come for free when all the uncertainties and disturbance torques are considered. In spite of the nonlinear structure of the kinematics and dynamics of the problem, however, the closed-loop system is rigorously analyzed through the standard Lyapunov analysis methods. This is achieved, owing to the fact that the strictified Lyapunov function allows us to deal with this nontrivial problem in a standard way. As the passivity-based controller is not new for the attitude control problem, the key contribution of this Note would be a theoretical proof how an ideal-case design is robust to uncertainties through a Lyapunov stability analysis.

The Note is organized as follows. The Euler rotational dynamics with the quaternion kinematics are described in Sec. II, and the problem is formulated in Sec. III. Next, a velocity-free tracking controller is proposed, and then the closed-loop system is analyzed in the presence of the uncertainties in Sec. IV. Numerical results for stabilization and tracking cases are provided in Sec. V. Finally, Sec. VI presents some concluding remarks.

II. Equations of Motion

A. Quaternion

Let \mathcal{F}_I be the inertial frame and \mathcal{F}_B be a body-fixed frame. Let \mathcal{F}_R be a reference frame with which the body-fixed frame is required to be aligned by a controller. The body orientation is assumed to be measured in such a way that it is different from the true orientation

due to measurement errors or estimation errors. So, we consider an estimated body-fixed frame $\hat{\mathcal{F}}_B$. Let q , \hat{q} , and q_r be the quaternions representing the orientations of \mathcal{F}_B , $\hat{\mathcal{F}}_B$, and \mathcal{F}_R with respect to the inertial frame \mathcal{F}_I . The rotations from \mathcal{F}_R to \mathcal{F}_B and $\hat{\mathcal{F}}_B$ are represented by the quaternions δq and $\delta \hat{q}$. Also, the error between true and estimated body orientations is denoted as $\delta \tilde{q}$. The aforementioned quaternions and their corresponding frame rotations are described in Fig. 1.

Quaternions are converted to rotation matrices. Let $C: \mathbb{Q} \rightarrow SO(3)$ be a map defined as [30]

$$C(q) = I_3 - 2q_0 q_v^* + 2q_v^* q_v^* \quad (1)$$

where the domain

$$\mathbb{Q} = \{q = (q_0, q_v) \in \mathbb{R}^4 | q_0 \in \mathbb{R}, q_v \in \mathbb{R}^3, q^T q = 1\}$$

is the set of all quaternions and the codomain

$$SO(3) = \{R \in \mathbb{R}^{3 \times 3} | R^T R = R R^T = I_3, \det(R) = 1\}$$

is the three-dimensional rotation group, I_3 is the three-dimensional identity matrix, and the superscript $*$ is the skew-symmetric matrix operator such that $v^* w = v \times w$ for any $v, w \in \mathbb{R}^3$. Note that because the quaternion parameterization is defined over non-Euclidean space, it is free from the mathematical singularity that arises from all minimal three-parameter representations.

Because successive rotations can be represented as one rotation, the rotation from \mathcal{F}_I to \mathcal{F}_B is represented as

$$C(q) = C(\delta q)C(q_r) \quad (2)$$

Equivalently, the quaternion q is expressed as

$$q = G(q_r)\delta q \quad (3)$$

where $G: \mathbb{Q} \rightarrow \mathbb{R}^{4 \times 4}$ is defined as

$$G(q) = \begin{bmatrix} q_0 & -q_v^T \\ q_v & q_v^* + q_0 I_3 \end{bmatrix} \quad (4)$$

Note that one can easily derive G is an orthogonal matrix from the definition. The true quaternion tracking error is then given by

$$\delta q = G^T(q_r)q \quad (5)$$

which corresponds to

$$C(\delta q) = C(q)C^T(q_r) \quad (6)$$

The role of tracking controllers is to make $C(\delta q)$ converge to I_3 . In quaternion representation, that means δq_v becomes zero, whatever its scalar part is.

B. Kinematics

The kinematic equation for the body attitude described in terms of the quaternion q is given by

$$\dot{q}(t) = \frac{1}{2}E(q(t))\omega(t) \quad (7)$$

where ω is the angular rate of the body with respect to the body-fixed frame \mathcal{F}_B , and the map $E: \mathbb{Q} \rightarrow \mathbb{R}^{4 \times 3}$ is given by

$$E(q) = \begin{bmatrix} -q_v^T \\ F(q) \end{bmatrix} \quad (8)$$

with

$$F(q) = q_v^* + q_0 I_3 \quad (9)$$

Let the reference angular rate in the frame \mathcal{F}_R be denoted as ω_r , which is assumed to be smooth and bounded so that we have

$$\sup_{t \geq 0} \|\omega_r(t)\|_2 = \nu_\omega \quad (10)$$

$$\sup_{t \geq 0} \|\dot{\omega}_r(t)\|_2 = \nu_\alpha, \quad (11)$$

for some finite positive numbers ν_ω and ν_α . It is also assumed that the reference states are governed by

$$\dot{q}_r(t) = \frac{1}{2}E(q_r(t))\omega_r(t) \quad (12)$$

Let $\delta\omega$ be the difference between the actual and desired rates in the body-fixed frame \mathcal{F}_B . Then, the rate error is defined as

$$\delta\omega(t) = \omega(t) - \omega_r^B(t) \quad (13)$$

where

$$\omega_r^B(t) = C(\delta q(t))\omega_r(t) \quad (14)$$

By differentiating Eq. (6) with respect to time, we obtain [31]

$$\delta\dot{q}(t) = \frac{1}{2}E(\delta q(t))\delta\omega(t) \quad (15)$$

C. Dynamics

The mathematical model for the fully actuated rigid-body rotational motion is described by the Euler equations given by

$$J\dot{\omega}(t) = -\omega^*(t)J\omega(t) + u(t) \quad (16)$$

where $J = J^T \in \mathbb{R}^{3 \times 3}$ is the moment-of-inertia matrix, and $u(t) \in \mathbb{R}^3$ is the input torque. Note that, when we let J_M and J_m be the largest and the smallest eigenvalues of J , respectively, J satisfies

$$J_m \|v\|_2^2 \leq v^T J v \leq J_M \|v\|_2^2 \quad (17)$$

for all $v \in \mathbb{R}^3$.

From Eq. (13), the error dynamics are derived as

$$J\delta\dot{\omega}(t) = -\omega^*(t)J\omega(t) + u(t) + J\delta\omega^*(t)\omega_r^B(t) - JC(\delta q(t))\dot{\omega}_r(t) \quad (18)$$

Alternatively, from the identity

$$(J\delta\omega)^* + (J\delta\omega^*) + (\delta\omega^*J) = \text{tr}(J)\delta\omega^* \quad (19)$$

where $\text{tr}(J)$ is the trace of J , we have

$$J\delta\dot{\omega}(t) = -\delta\omega^*(t)(J\delta\omega(t) + \bar{J}\omega_r^B(t)) + u(t) - \eta(t) \quad (20)$$

with

$$\bar{J} = 2J - \text{tr}(J)I_3 \quad (21)$$

and

$$\eta(t) = \omega_r^{B*}(t)J\omega_r^B(t) + JC(\delta q(t))\dot{\omega}_r(t) \quad (22)$$

III. Problem Description

We assume that the system is equipped with only attitude sensors so that full-state feedback control is not feasible. In spite of the lack of gyroscope sensors, it is well known that the system can be stabilized by using passivity-based control strategies for which a low-pass filter driven by the quaternion signal is added to the original dynamics [6,7]. Now, we consider, in the presence of uncertainties, the stabilization of the system

$$\begin{aligned} \delta\dot{q}(t) &= \frac{1}{2}E(\delta q(t))\delta\omega(t) \\ J\delta\dot{\omega}(t) &= -\delta\omega^*(t)(J\delta\omega(t) + \bar{J}\omega_r^B(t)) + u(t) - \eta(t) + d(t) \\ \dot{\sigma}(t) &= -\sigma(t) + \delta\hat{q}(t) \end{aligned} \quad (23)$$

where $\delta\hat{q}$ is the estimated quaternion tracking error, and $d(t)$ is a bounded disturbance: there is a constant $\mu_d > 0$ such that, for all $t \geq 0$, the inequality

$$\|d(t)\|_2 \leq \mu_d \quad (24)$$

is satisfied. Also, it is assumed that the exact value of J is not available, so its estimate \hat{J} is used for the control. Our objective is to show a passivity-based PD-type control law stabilizing the system in the ideal situations guarantees the boundedness of the closed-loop trajectories in the presence of measurement errors, disturbance torques and inertia parameter uncertainties. 10

From Fig. 1, the attitude estimate error $\delta\tilde{q} \in \mathbb{Q}$ is obtained as

$$\delta\tilde{q}(t) = G^T(\delta q(t))\delta\hat{q}(t) \quad (25)$$

We assume that it is governed by the following dynamics:

$$\delta\dot{\tilde{q}}(t) = \frac{1}{2}E(\delta\tilde{q}(t))\tilde{\omega}(t) \quad (26)$$

where the forcing term $\tilde{\omega}$ is a random variable [32]. Observe that this assumption allows the quaternion estimate $\hat{q}(t)$ to remain in the set \mathbb{Q} for all $t \geq 0$. We also assume, for the analysis, that $\tilde{\omega}$ is bounded by a known quantity and restricts the magnitude of $\delta\tilde{q}_v$ within a small number:

$$\|\delta\tilde{q}_v(t)\|_2 \leq \mu_v \ll 1 \quad (27)$$

$$\|\tilde{\omega}(t)\|_2 \leq 2\mu_\omega \ll 1 \quad (28)$$

for all $t \geq 0$. Typically, μ_v and μ_ω are very small quantities that are available based on the sensor characteristics. Due to the unit-norm constraint of the error quaternion $\delta\tilde{q}(t)$, its scalar part $\delta\tilde{q}_0(t)$ is bounded by

$$\mu_0 \leq \delta\tilde{q}_0(t) \leq 1 \quad (29)$$

where $\mu_0 = \sqrt{1 - \mu_v^2}$ with the assumption $\delta\tilde{q}_0(0) > 0$. In other words, $\delta\tilde{q}$ is very close to the identity quaternion $(1, 0, 0, 0)$.

Throughout the Note, $x = (\delta q, \delta\omega, \dot{\sigma}) \in \mathbb{Q} \times \mathbb{R}^7$ represents the entire states and $\bar{x} = (\delta q_v, \delta\omega, \dot{\sigma}) \in \mathbb{R}^{10}$ represents the states of interest to be stabilized. We also introduce the notations $z = (\|\delta q_v\|_2, \|\delta\omega\|_2, \|\dot{\sigma}\|_2) \in \mathbb{R}^3$ and $\bar{z} = (\|\delta\omega\|_2, \|\dot{\sigma}\|_2) \in \mathbb{R}^2$ for later

use. Arguments of functions are often omitted for notational simplicity whenever no confusion arises within a context.

IV. Main Result

In this section, boundedness of the solution of Eq. (23) is established through a partially strictified Lyapunov function in the presence of quaternion estimate errors, disturbance torques, and parameter uncertainties.

Let us consider the attitude dynamics [Eq. (23)] with $\delta\hat{q}(t) = \delta q(t)$ and $d(t) = 0$, and let us assume that the initial conditions are such that $\|\delta q(0)\|_2 = 1$, $\delta\omega(0) \in \mathbb{R}^3$, and $\dot{\sigma}(0) = 0$. Then, by analyzing

$$V_0(t) = \alpha[\delta q_v^T(t)\delta q_v(t) + (\delta q_0(t) - 1)^2] + \frac{1}{2}\delta\omega^T(t)J\delta\omega(t) + \frac{b}{2}\dot{\sigma}^T(t)\dot{\sigma}(t) \quad (30)$$

where α and b are positive constants, one can show that the control

$$u(t) = -\alpha\delta q_v(t) - \frac{b}{2}E^T(\delta q(t))\dot{\sigma}(t) + \eta(t) \quad (31)$$

stabilizes system (23) [25].

When the uncertainties are considered, we employ $\delta\hat{q}(t)$ and \hat{J} instead of $\delta q(t)$ and J , respectively, for the controller and the filter. Then, the proposed control law is rewritten as

$$\begin{aligned} u(t) &= -\alpha\delta\hat{q}_v(t) - \frac{b}{2}E^T(\delta\hat{q}(t))\dot{\sigma}(t) + \hat{\eta}(t) \\ &= -\alpha[\delta\tilde{q}_0(t)\delta q_v(t) + F(\delta q(t))\delta\tilde{q}_v(t)] \\ &\quad + \frac{b}{2}E^T(\dot{\sigma}(t))G(\delta q(t))\delta\tilde{q}(t) + \hat{\eta}(t) \end{aligned} \quad (32)$$

where

$$\hat{\eta}(t) = [C(\delta\hat{q}(t))\omega_r(t)]^* \hat{J}C(\delta\hat{q}(t))\omega_r(t) + \hat{J}C(\delta\hat{q}(t))\dot{\omega}_r(t)$$

The filter becomes

$$\begin{aligned} \dot{\sigma}(t) &= -\sigma(t) + \delta\hat{q}(t) \\ &= -\sigma(t) + G(\delta q(t))\delta\tilde{q}(t) \end{aligned} \quad (33)$$

with its second time derivative

$$\ddot{\sigma}(t) = -\dot{\sigma}(t) + \delta\tilde{q}_0(t)\delta\dot{q}(t) + E(\delta\dot{q}(t))\delta\tilde{q}_v(t) + G(\delta q(t))\delta\dot{\tilde{q}}(t) \quad (34)$$

Once the proposed control law is implemented, the rate dynamics are obtained as

$$\begin{aligned} J\delta\dot{\omega}(t) &= -\delta\omega^*(t)(J\delta\omega(t) + \bar{J}\omega_r^B(t)) + \tilde{\eta}(t) + d(t) \\ &\quad - \alpha[\delta\tilde{q}_0(t)\delta q_v(t) + F(\delta q(t))\delta\tilde{q}_v(t)] + \frac{b}{2}E^T(\dot{\sigma}(t))G(\delta q(t))\delta\tilde{q}(t) \end{aligned} \quad (35)$$

where $\tilde{\eta} = \hat{\eta} - \eta$. With the uncertainties, we now analyze

$$V(t) = V_1(t) + \frac{c_3}{J_m b} V_0^2(t) \quad (36)$$

where

$$V_1(t) = c_1 V_0(t) + \delta q_v^T(t)J\delta\omega(t) - 2c_2\delta\dot{q}^T(t)\dot{\sigma}(t) \quad (37)$$

and c_1 , c_2 , and c_3 are positive constants to be determined.

First, by differentiating Eq. (30), we have

$$\begin{aligned} \dot{V}_0 &= \delta\omega^T \left[\alpha(1 - \delta\tilde{q}_0)\delta q_v - \alpha F(\delta q)\delta\tilde{q}_v + \frac{b}{2}E^T(\dot{\sigma})G(\delta q)\delta\tilde{q} + \tilde{\eta} + d \right] \\ &\quad + b\dot{\sigma}^T \left[-\dot{\sigma} + \frac{\delta\tilde{q}_0}{2}E(\delta q)\delta\omega + E(\delta\dot{q})\delta\tilde{q}_v + G(\delta q)\delta\dot{\tilde{q}} \right] \\ &= \delta\omega^T [\alpha(1 - \delta\tilde{q}_0)\delta q_v - \alpha F(\delta q)\delta\tilde{q}_v + \tilde{\eta} + d] \\ &\quad + b\dot{\sigma}^T [-\dot{\sigma} + G(\delta q)\delta\dot{\tilde{q}}] + \frac{b}{2}\delta\omega^T [E^T(\dot{\sigma})E(\delta q) - E^T(\delta q)E(\dot{\sigma})]\delta\tilde{q}_v \end{aligned} \quad (38)$$

After arranging terms and applying the known or derived upper bounds for the measurement errors, external disturbances, and parameter uncertainties, we obtain

$$\begin{aligned} \dot{V}_0 &\leq -b\|\dot{\sigma}\|_2^2 + \alpha(1 - \mu_0)\|\delta q_v\|_2\|\delta\omega\|_2 + b\mu_v\|\delta\omega\|_2\|\dot{\sigma}\|_2 \\ &\quad + (\alpha\mu_v + \mu_\eta + \mu_d)\|\delta\omega\|_2 + b\mu_\omega\|\dot{\sigma}\|_2 \end{aligned} \quad (39)$$

where

$$\mu_\eta = 4\sqrt{J_M J_\Delta} \nu_\omega^2 \mu_v^2 + 2J_M(2\nu_\omega^2 + \nu_\alpha)\mu_v + (\nu_\omega^2 + \nu_\alpha)\mu_J \quad (40)$$

with $J_\Delta = J_M - J_m$, and μ_J such that $\|\hat{J} - J\|_2 \leq \mu_J$. Note the following inequality:

$$\begin{aligned} \|C(\delta\hat{q}) - C(\delta q)\|_2 &= \|(C(\delta\tilde{q}) - I_3)C(\delta q)\|_2 \\ &= \|-2F^T(\delta\tilde{q})\delta\tilde{q}_v^* C(\delta q)\|_2 \\ &\leq 2\|\delta\tilde{q}_v\|_2 \end{aligned} \quad (41)$$

is used to obtain

$$\|\tilde{\eta}\|_2 \leq \mu_\eta \quad (42)$$

To have, in the right-hand side of Eq. (39), a negative definite quadratic function of $z(t)$, we consider $V_1(t)$. The cross terms in $V_1(t)$ allow us to have nonpositive quadratic terms in $\delta q_v(t)$ and $\delta\omega(t)$ within $\dot{V}_1(t)$. Because the upper bound of $\dot{V}_1(t)$ does not include a nonnegative term in $(1 - \delta q)$, one cannot say $V_0(t)$ is strictified. Instead, the term ‘‘partial’’ is introduced to describe this procedure.

To ease the calculation, let us introduce

$$N_1(t) = \delta q_v^T(t)J\delta\omega(t) \quad (43)$$

$$N_2(t) = -\delta\dot{q}^T(t)\dot{\sigma}(t) \quad (44)$$

for which the time derivatives are computed as

$$\begin{aligned} \dot{N}_1 &= \frac{1}{2}\delta\omega^T F(\delta q)J\delta\omega - \alpha\delta\tilde{q}_0\|\delta q_v\|_2^2 + \frac{b}{2}\delta q_v^T E^T(\dot{\sigma})\delta\hat{q} \\ &\quad - \alpha\delta q_v^T F(\delta q)\delta\tilde{q}_v + \delta q_v^T(\tilde{\eta} + d) - \delta q_v^T \delta\omega^* \bar{J}\omega_r^B \end{aligned} \quad (45)$$

$$\begin{aligned} \dot{N}_2 &= \delta\dot{q}^T \dot{\sigma} - \delta\tilde{q}_0\|\delta\dot{q}\|_2^2 - \delta\dot{q}^T G(\delta q)\delta\dot{\tilde{q}} + \frac{1}{4}\|\delta\omega\|_2^2 \dot{\sigma}^T \delta q \\ &\quad - \frac{1}{2}\dot{\sigma}^T E(\delta q)\delta\dot{\omega} \end{aligned} \quad (46)$$

Because $\|\delta q(t)\|_2 = \|E(\delta q(t))\|_2 = 1$ and $\|F(\delta q(t))\|_2 \leq 1$ for all $t \geq 0$, the upper bounds of Eqs. (45) and (46) are obtained as

$$\begin{aligned} \dot{N}_1 \leq & -\alpha\mu_0\|\delta q_v\|_2^2 + \frac{J_M}{2}\|\delta\omega\|_2^2 + \bar{J}_M\nu_\omega\|\delta q_v\|_2\|\delta\omega\|_2 \\ & + \frac{\mathfrak{b}}{2}\|\delta q_v\|_2\|\dot{\sigma}\|_2 + (\alpha\mu_v + \mu_\eta + \mu_d)\|\delta q_v\|_2 \end{aligned} \quad (47)$$

$$\begin{aligned} \dot{N}_2 \leq & -\frac{\mu_0}{4}\|\delta\omega\|_2^2 + \frac{\mathfrak{b}}{4J_m}\|\dot{\sigma}\|_2^2 + \frac{\alpha}{2J_m}\|\delta q_v\|_2\|\dot{\sigma}\|_2 \\ & + \left(\frac{1}{2} + \frac{\bar{J}_M\nu_\omega}{2J_m}\right)\|\delta\omega\|_2\|\dot{\sigma}\|_2 \\ & + \left(\frac{1}{4} + \frac{\sqrt{J_M J_\Delta}}{2J_m}\right)\|\delta\omega\|_2^2\|\dot{\sigma}\|_2 + \frac{\mu_\omega}{2}\|\delta\omega\|_2 \\ & + \frac{(\alpha\mu_v + \mu_\eta + \mu_d)}{2J_m}\|\dot{\sigma}\|_2 \end{aligned} \quad (48)$$

Then, using Eq. (47) and (48), we have

$$\begin{aligned} \dot{V}_1 \leq & -z^T Q_1 z + \gamma_1\|\delta q_v\|_2 + \gamma_2\|\delta\omega\|_2 + \gamma_3\|\dot{\sigma}\|_2 \\ & + \left(\frac{1}{2} + \frac{\sqrt{J_M J_\Delta}}{J_m}\right)c_2\|\delta\omega\|_2^2\|\dot{\sigma}\|_2 - \frac{1}{2}(c_2\mu_0 - J_M)(1 - \xi_1)\|\delta\omega\|_2^2 \end{aligned} \quad (49)$$

where

$$\gamma_1 = \alpha\mu_v + \mu_\eta + \mu_d \quad (50)$$

$$\gamma_2 = c_1\gamma_1 + c_2\mu_\omega \quad (51)$$

$$\gamma_3 = c_1\mathfrak{b}\mu_\omega + \frac{c_2\gamma_1}{J_m} \quad (52)$$

and

$$Q_1 = \begin{bmatrix} \alpha\mu_0 & -\frac{1}{2}(c_1\alpha(1 - \mu_0) + \bar{J}_M\nu_\omega) & -\frac{1}{2}\left(\frac{\mathfrak{b}}{2} + \frac{c_2\alpha}{J_m}\right) \\ -\frac{1}{2}(c_1\alpha(1 - \mu_0) + \bar{J}_M\nu_\omega) & \frac{1}{2}(c_2\mu_0 - J_M)\xi_1 & -\frac{1}{2}\left(c_1\mathfrak{b}\mu_v + c_2 + \frac{c_2\bar{J}_M\nu_\omega}{J_m}\right) \\ -\frac{1}{2}\left(\frac{\mathfrak{b}}{2} + \frac{c_2\alpha}{J_m}\right) & -\frac{1}{2}\left(c_1\mathfrak{b}\mu_v + c_2 + \frac{c_2\bar{J}_M\nu_\omega}{J_m}\right) & \left(c_1 - \frac{c_2}{2J_m}\right)\mathfrak{b} \end{bmatrix} \quad (53)$$

with $\xi_1 \in (0, 1)$.

Remark 1 For the ideal setpoint regulation problem with $\nu_\omega = \nu_\alpha = 0$, the explicit inequalities

$$c_2 > J_M \quad (54)$$

and

$$\begin{aligned} c_1 > \max \left\{ \frac{c_2}{2J_m} + \frac{1}{4\alpha\mathfrak{b}} \left(\frac{\mathfrak{b}}{2} + \frac{c_2\alpha}{J_m} \right)^2 \right. \\ \left. + \frac{c_2^2}{2(c_2 - J_M)\mathfrak{b}\xi_1}, \sqrt{\frac{c_2^2}{J_m\mathfrak{b}} + \frac{J_M}{2\alpha}} \right\} \end{aligned} \quad (55)$$

guarantee V_1 and Q_1 are positive definite in the absence of uncertainties. Thus, the preceding conditions help to select proper values of c_1 and c_2 initially. Once reference tracking and uncertainties are considered, those lower bounds will be increased.

Next, we analyze $V(t)$ defined in Eq. (36) as the final step. We assume that the positive constants c_1 and c_2 in V_1 are chosen such that Q_1 is positive definite and V_1 is nonnegative. Let

$$\begin{aligned} N_3(\|\dot{\sigma}\|_2) = & c_3\|\dot{\sigma}\|_2^2 - \left(\frac{1}{2} + \frac{\sqrt{J_M J_\Delta}}{J_m}\right)c_2\|\dot{\sigma}\|_2 \\ & + \frac{1}{2}(c_2\mu_0 - J_M)(1 - \xi_1) \end{aligned} \quad (56)$$

Then, using the inequalities

$$V_0(t) \geq \frac{J_m}{2}\|\delta\omega(t)\|_2^2 \quad (57)$$

$$\|\sigma(t)\|_2 \leq 1 \quad (58)$$

$$\|\dot{\sigma}(t)\|_2 \leq 2 \quad (59)$$

together with Eq. (39), it is deduced that

$$\begin{aligned} \dot{V} \leq & -z^T Q_1 z + \gamma_1\|\delta q_v\|_2 + (\gamma_2 + \gamma_4 V_0)\|\delta\omega\|_2 \\ & + (\gamma_3 + \gamma_5 V_0)\|\dot{\sigma}\|_2 - \|\delta\omega\|_2^2 N_3(\|\dot{\sigma}\|_2) \end{aligned} \quad (60)$$

where

$$\gamma_4 = \frac{2c_3}{J_m\mathfrak{b}}[\alpha(1 - \mu_0) + \mathfrak{b}\mu_v + \gamma_1] \quad (61)$$

$$\gamma_5 = \frac{2c_3\mu_\omega}{J_m} \quad (62)$$

Moreover, because the choice

$$c_3 = \frac{c_2^2}{2(c_2\mu_0 - J_M)(1 - \xi_1)} \left(\frac{1}{2} + \frac{\sqrt{J_M J_\Delta}}{J_m} \right)^2 \quad (63)$$

leads to $N_3(\|\dot{\sigma}\|_2) \geq 0$ for any $\|\dot{\sigma}\|_2 \in [0, 2]$, we finally have

$$\begin{aligned} \dot{V} \leq & -z^T Q_2 z + \gamma_1\|\delta q_v\|_2 + (\gamma_2 + \gamma_4 V_0)\|\delta\omega\|_2 \\ & + (\gamma_3 + \gamma_5 V_0)\|\dot{\sigma}\|_2 \end{aligned} \quad (64)$$

To handle the nonnegative terms in Eq. (64), let us rewrite the inequality as

$$\begin{aligned} \dot{V} \leq & -z^T Q_2 z - k_1\|\delta q_v\|_2^2 - k_2\|\delta\omega\|_2^2 - k_3\|\dot{\sigma}\|_2^2 + \gamma_1\|\delta q_v\|_2 \\ & + (\gamma_2 + \gamma_4 V_0)\|\delta\omega\|_2 + (\gamma_3 + \gamma_5 V_0)\|\dot{\sigma}\|_2 \end{aligned} \quad (65)$$

where

$$k_1 = \alpha\mu_0(1 - \xi_2) \quad (66)$$

$$k_2 = \frac{(c_2\mu_0 - J_M)}{2}\xi_1(1 - \xi_3) \quad (67)$$

$$k_3 = \left(c_1 - \frac{c_2}{2J_m} \right) \bar{b}(1 - \xi_4) \quad (68)$$

with $\xi_2, \xi_3, \xi_4 \in (0, 1)$ and $Q_2 = Q_1 - \text{diag}(k_1, k_2, k_3)$. For a vector $v \in \mathbb{R}^n$, $\text{diag}(v) \in \mathbb{R}^{n \times n}$ denotes a square diagonal matrix with the elements of v on the main diagonal. Because the inequality

$$\pm x^T P y \leq c x^T P x + \frac{1}{4c} y^T P y \quad (69)$$

holds for any $P = P^T \in \mathbb{R}^{n \times n} > 0$, $x, y \in \mathbb{R}^n$, and $c > 0$, we further obtain

$$\begin{aligned} \dot{V} \leq & -z^T Q_2 z + \left(\frac{\gamma_4^2}{4k_2} + \frac{\gamma_5^2}{4k_3} \right) V_0^2 + \left(\frac{\gamma_2 \gamma_4}{2k_2} + \frac{\gamma_3 \gamma_5}{2k_3} \right) V_0 \\ & + \frac{1}{4} \left(\frac{\gamma_1^2}{k_1} + \frac{\gamma_2^2}{k_2} + \frac{\gamma_3^2}{k_3} \right) \end{aligned} \quad (70)$$

Let us assume that there exist c_1 and c_2 such that Q_2 is positive definite. Then, by lemma 3 in [25], there exists a positive definite matrix $\bar{Q}_2 \in \mathbb{R}^{2 \times 2}$ such that

$$z^T(t) Q_2 z(t) \geq \bar{z}^T(t) \bar{Q}_2 \bar{z}(t) \quad (71)$$

Also, by Eq. (71), lemma 4 in [25], and the fact that

$$V_0(t) \leq 4\alpha + \frac{J_M}{2} \|\delta\omega(t)\|_2^2 + \frac{\bar{b}}{2} \|\dot{\sigma}(t)\|_2^2 \quad (72)$$

one can show that there exists a positive constant k_4 such that

$$\begin{aligned} \dot{V} \leq & -\frac{1}{k_4} V_0 + \left(\frac{\gamma_4^2}{4k_2} + \frac{\gamma_5^2}{4k_3} \right) V_0^2 + \left(\frac{\gamma_2 \gamma_4}{2k_2} + \frac{\gamma_3 \gamma_5}{2k_3} \right) V_0 \\ & + \left[\frac{1}{4} \left(\frac{\gamma_1^2}{k_1} + \frac{\gamma_2^2}{k_2} + \frac{\gamma_3^2}{k_3} \right) + \frac{4\alpha}{k_4} \right] \end{aligned} \quad (73)$$

Once a parameter $\xi_5 \in (0, 1)$ is introduced, we obtain

$$\dot{V}(t) \leq -\frac{\xi_5}{k_4} V_0(t) + N_4(V_0(t)) \quad (74)$$

where

$$N_4(V_0) = \varphi_1 V_0^2 - \varphi_2 V_0 + \varphi_3 \quad (75)$$

with

$$\varphi_1 = \frac{\gamma_4^2}{4k_2} + \frac{\gamma_5^2}{4k_3} \quad (76)$$

$$\varphi_2 = \frac{(1 - \xi_5)}{k_4} - \left(\frac{\gamma_2 \gamma_4}{2k_2} + \frac{\gamma_3 \gamma_5}{2k_3} \right) \quad (77)$$

$$\varphi_3 = \frac{1}{4} \left(\frac{\gamma_1^2}{k_1} + \frac{\gamma_2^2}{k_2} + \frac{\gamma_3^2}{k_3} \right) + \frac{4\alpha}{k_4} \quad (78)$$

To ensure the existence of $V_0 > 0$ such that $N_4(V_0) \leq 0$, let us assume the control gains α and \bar{b} satisfy

$$\varphi_2 > 2\sqrt{\varphi_1 \varphi_3} \quad (79)$$

Let r_1 and r_2 be two positive distinct roots of the equation $N_4(V_0) = 0$, and let $r_1 < r_2$. Then, $r_1 \leq V_0(t) \leq r_2$ implies

$$\dot{V}(t) \leq -\frac{\xi_5}{k_4} V_0(t) \quad (80)$$

From the inequality $V_1(t) \leq 2c_1 V_0(t)$, it is deduced that

$$\frac{c_3}{J_m \bar{b}} V_0^2(t) \leq V(t) \leq 2c_1 V_0(t) + \frac{c_3}{J_m \bar{b}} V_0^2(t) \quad (81)$$

After some algebra, we have

$$\frac{\sqrt{\varphi_5^2 + 4\varphi_4 V(t)} - \varphi_5}{2\varphi_4} \leq V_0(t) \leq \frac{\sqrt{4\varphi_4 V(t)}}{2\varphi_4} \quad (82)$$

where

$$\varphi_4 = \frac{c_3}{J_m \bar{b}} \quad (83)$$

$$\varphi_5 = 2c_1 \quad (84)$$

Therefore, if

$$\varphi_4 r_1^2 + \varphi_5 r_1 \leq V(t) \leq \varphi_4 r_2^2 \quad (85)$$

then Eq. (80) follows. Again, for $V(t)$ to exist, a stricter condition for α and \bar{b} ,

$$\varphi_2 \geq \sqrt{\frac{\varphi_1^4 \varphi_5^2}{\varphi_2^2 \varphi_4^2} r_1^2 + 4\varphi_1 \varphi_3} \quad (86)$$

is necessary. Now, we can conclude that the set $\{x \in \mathbb{Q} \times \mathbb{R}^7 | V(x) \leq \varphi_4 r_2^2\}$ is positively invariant and trajectories starting in this set will enter the residual set $\{x \in \mathbb{Q} \times \mathbb{R}^7 | V(x) \leq \varphi_4 r_1^2 + \varphi_5 r_1\}$ within a finite amount of time. Note that this result is only local because $\varphi_4 r_2^2$ is finite [33].

Suppose $\delta q_0(t)$ is initially chosen such that $\delta q_0(0) \geq 0$. Then, at $t = 0$, $V(0)$ is bounded by

$$V(0) \leq c_1 \bar{V}_0 + J_M \|\delta q_v(0)\|_2 \|\delta\omega(0)\|_2 + \frac{c_3}{J_m \bar{b}} \bar{V}_0^2 \quad (87)$$

where

$$\bar{V}_0 = 2\alpha(1 - \sqrt{1 - \|\delta q_v(0)\|_2^2}) + \frac{J_M}{2} \|\delta\omega(0)\|_2^2 \quad (88)$$

Hence, so long as $\|\delta q_v(0)\|_2$ and $\|\delta\omega(0)\|_2$ satisfy

$$c_1 \bar{V}_0 + J_M \|\delta q_v(0)\|_2 \|\delta\omega(0)\|_2 + \frac{c_3}{J_m \bar{b}} \bar{V}_0^2 \leq \varphi_4 r_2^2 \quad (89)$$

solutions are bounded for all $t \geq 0$.

In summary, the proposed controller

$$\begin{aligned} u(t) = & -\alpha \delta \hat{q}_v(t) - \frac{\bar{b}}{2} E^T (\delta \hat{q}(t)) \dot{\sigma}(t) \\ & + (C(\delta \hat{q}(t)) \omega_r(t))^* \hat{J} C(\delta \hat{q}(t)) \omega_r(t) + \hat{J} C(\delta \hat{q}(t)) \dot{\omega}_r(t) \end{aligned} \quad (90)$$

regulates the tracking errors in the absence of uncertainties and makes them bounded in the presence of uncertainties if the control gains α and \bar{b} are selected such that all the conditions described in this section are satisfied.

V. Numerical Results

We consider a CubeSat-class satellite for which the dynamics are governed by Eq. (23) closed with control law (32). It is assumed that the inertia matrix is given by [34]

$$J = \begin{bmatrix} 0.0465 & -0.0007 & 0.0004 \\ -0.0007 & 0.0486 & -0.0021 \\ 0.0004 & -0.0021 & 0.0482 \end{bmatrix} \quad (91)$$

and the measurement error bounds $\mu_v = \mu_\omega = 0.01$ guarantee approximately 0.5 deg accuracy. Because existing nanostar trackers have an accuracy less than 0.03 deg [34–36], this specification gives enough margin in the measurement errors. We also assume that the magnitude of the external disturbance is bounded by $\mu_d = 10^{-4}$, which is a few orders greater than the typical magnitude for satellites with a similar size [34,37]. By letting

$$\|\delta q_v(0)\|_2 = 0, 1, \|\delta\omega(0)\|_2 = 0.01$$

and using Eq. (89), one can find a feasible gain region in which the control gains a and b can be selected to ensure the boundedness of the closed-loop trajectories. In this section, we study both the stabilization and tracking problems. Because the stabilizing controller is independent of the inertia parameters, it is expected that the feasible gain region is not smaller than one obtained from general tracking cases.

First, consider the stabilization case by letting $q_r(t) = (1, 0, 0, 0)$ and $\omega_r(t) = 0$, which implies $\nu_\omega = \nu_\alpha = 0$. To maximize the region, multiple values for each ξ_i are considered. That is, the region in Fig. 2a is the union of the regions obtained using different ξ values. Here, we restrict our gain range to a compact set $[0.01, 0.3] \times [0.01, 1.5] \subset \mathbb{R}^2$ for the computation. Note that it would be

recommended to start with sufficiently bigger sets in order to search gain regions efficiently

To demonstrate the boundedness properties, we choose $(a, b) = (0.2, 0.8)$ and numerically integrate the closed-loop system using the following initial conditions:

$$q(0) = \hat{q}(0) = \sigma(0) = (\sqrt{0.9925}, 0.05, 0.05, 0.05)$$

and $\omega(0) = (-0.09, 0.03, -0.03)$. The disturbance torque is assumed to be generated by the gravity gradient, atmospheric drag, and J_2 effect. The model is given by [20]

$$d = \frac{3\mu}{\|r\|_2^5} C(q)r^*Jr - \frac{1}{2}\rho\|v\|_2 C_d A r_c^* C(q)v + \frac{3\mu J_2 R_e^2}{2\|r\|_2^7} r_c^* C(q) \begin{bmatrix} 5r_x r_z^2 - 3r_x \|r\|_2^2 \\ 5r_y r_z^2 - 3r_y \|r\|_2^2 \\ 5r_z^3 - 3r_z \|r\|_2^2 \end{bmatrix} \quad (92)$$

where $r = (r_x, r_y, r_z) \in \mathbb{R}^3$ is the vector from the center of the Earth to the spacecraft represented in the inertial frame \mathcal{F}_I , $v \in \mathbb{R}^3$ is the spacecraft velocity vector in the inertial frame, and $r_c \in \mathbb{R}^3$ is the vector from the spacecraft center of mass to the line of action of the force caused by the J_2 effect in the body-fixed frame \mathcal{F}_B . The other parameters in Eq. (92) are given by $\mu = 398.6 \times 10^{12} \text{ m}^3/\text{s}^2$, $\rho = 1.99 \times 10^{-14} \text{ kg/m}^3$, $C_d = 2.5$, $A = 0.04 \text{ m}^2$, $R_e = 6371.0 \times 10^3 \text{ m}$, and $J_2 = 1082.6 \times 10^{-6}$. The Earth-centered inertial frame \mathcal{F}_I is defined such that the x axis points to the sun and the z axis is

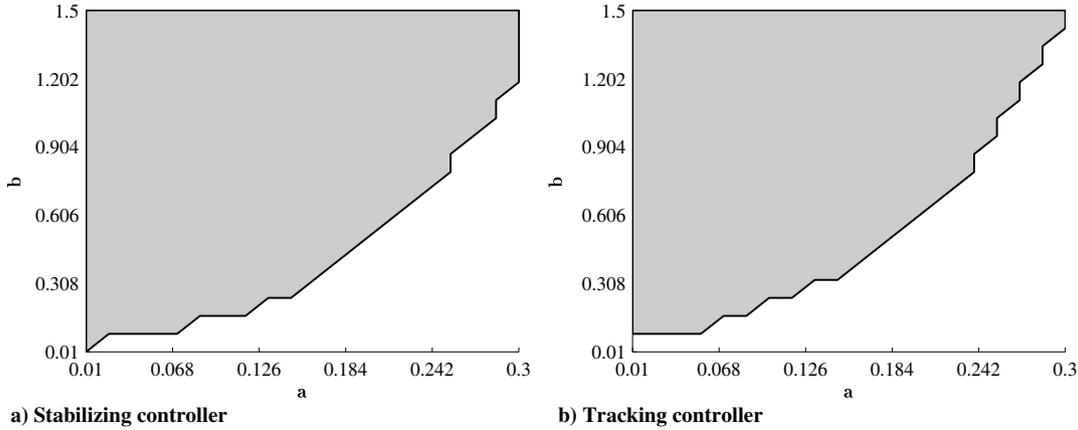


Fig. 2 Feasible gain regions for stabilizing and tracking controllers.

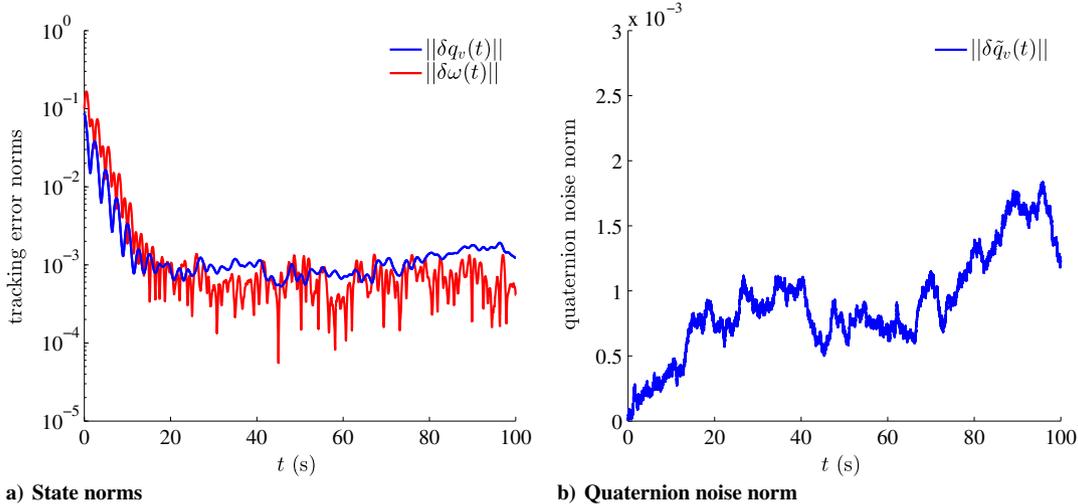


Fig. 3 Time histories of $\|q_v(t)\|_2$, $\|\omega(t)\|_2$, and $\|\delta\tilde{q}_v(t)\|_2$ for stabilization.

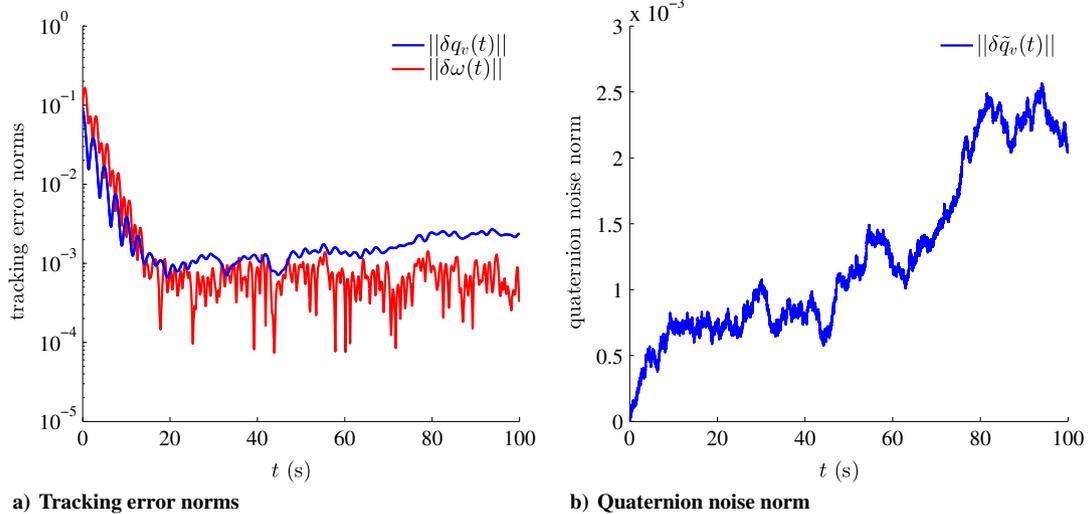


Fig. 4 Time histories of $\|\delta q_v(t)\|_2$, $\|\delta \omega(t)\|_2$, and $\|\delta \tilde{q}_v(t)\|_2$ for tracking.

aligned with the axis of rotation of the Earth. The spacecraft is assumed to be in a circular orbit with an altitude 765 km and an inclination of 70 deg. For the numerical implementation, we choose $r_c = (0.01, 0, 0)$ so that the magnitude of the disturbance torque does not exceed the designed upper limit. As seen in Fig. 3a, the state norms do not converge to zero but are bounded. Figure 3b illustrates the norm of the quaternion estimate error $\|\delta \tilde{q}_v(t)\|_2$, which is strictly less than μ_v during the simulation time. It is assumed that, at each simulation time t , the quaternion estimate error kinematics are driven by a three-dimensional random vector $\tilde{\omega}$ with a uniform distribution of $\|\tilde{\omega}(t)\|_2$ over the closed interval $[0, 2\nu_s]$.

Next, we consider the tracking problem with the same uncertainty specifications. Suppose the reference angular velocity profile is given by

$$\omega_r(t) = 0.05(\sin(0.5t), \sin(0.5t), \sin(0.5t))$$

with the initial reference quaternion $q_r(0) = (1, 0, 0, 0)$. We obtain from the profile that $\nu_\omega = 0.05\sqrt{3}$ and $\nu_\alpha = 0.5\nu_\omega$. It is also assumed that the proposed tracking controller employs the estimated inertia matrix given by $J = \text{diag}(0.046, 0.049, 0.048)$. Using the fact that $\mu_J = 0.005$, a feasible gain region is computed as shown in Fig. 2b. We use the same gains and initial conditions described in the stabilization problem for the simulation. As illustrated in Fig. 4a, the tracking error norms are bounded.

VI. Conclusions

This Note addresses the attitude tracking problem with the quaternion kinematics. It is assumed that only attitude vector measurements are available and are used to estimate the quaternion state. By employing the quaternion state and the output from a low-pass filter driven by the quaternion, a passivity-based tracking controller is proposed. Aided by the strictification technique, it is proved that one of the equilibrium points is asymptotically stable in the ideal case where there are no uncertainties. Even when measurement errors and external disturbances introduce perturbations along with model mismatch in the dynamics, a positive invariant set and a residual set are established in the use of known bounds for the uncertainties. Then, sufficient conditions for the initial states and the feedback gains that guarantee boundedness of the closed-loop trajectories through a Lyapunov analysis are provided.

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