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Abstract. A popular metaheuristic global optimization technique called Cuckoo Search Algorithm (CSA) is employed to design non-integer order integrators (NOIs) in terms of the Infinite Impulse Response (IIR) templates in this paper. Extensive comparisons on the basis of design quality robustness, error convergence, and optimization time of the CSA-based NOIs are carried out with the Particle Swarm Optimization (PSO) based designs. Results demonstrate the efficient performance of CSA in exploring the multimodal, non-linear, and non-uniform error surface for this optimization problem. The CSA-based designs also outperform the recent literature by 9.67 decibel (dB) and 19.26 dB in terms of mean absolute relative magnitude error (MARME) and maximum absolute magnitude error (MAME) metrics, respectively.

Keywords: Non-integer Order Integrator; Cuckoo Search Algorithm; Particle Swarm Optimization; Metaheuristic Optimization.

1 Introduction

An important application area of fractional calculus [1] is in the realm of control systems and signal processing [2, 3] where the additional degrees-of-freedom provided by the non-integer (fractional) operators have resulted in the design of fractional order controllers (FOCs) [4] which can outperform the traditional controller designs based on the integer order operators. The non-integer order integrators (NOIs) are one of the fundamental building blocks of the FOCs. The frequency response of an ideal NOI is described by $H(\mu) = (1/\mu^r)\angle -(90^\circ \times r)$, where, $r \in (0,1)$ is the fractional order, and μ is the angular frequency. Note that r = 1 results in the conventional (integer order) integrator. Thus, it is easy to realize that NOIs are more generalized and provides superior design flexibility than the conventional integrator. However, since the ideal NOI is an infinite dimensional structure, hence, its practical realization can only be of finite dimensions. Thus, the implementation of NOIs can be regarded as an approximation problem with the objective of designing accurate designs with smaller dimensions. Since the digital implementations of the NOIs based on infinite impulse response (IIR) filter structures as compared with the finite impulse response filters are preferred for real-time applications for various reasons such as power consumption and latency, the primary objective of this paper is to design digital NOIs in terms of IIR filters of third order such that it accurately meets the ideal frequency response. It is worth noting that achieving both stable and minimum phase IIR designs makes this design problem even more challenging.

Design of IIR NOIs based on series expansion methods [5, 6, 7, 8, 9], least square technique [10], Chebyshev polynomials [11], indirect discretization [12], Particle Swarm Optimization (PSO) [13], and Colliding Bodies Optimization algorithm [14] are published in the literature. An excellent survey dealing with the design of NOIs is reported in [15].

Literature survey reveals that metaheuristics have shown promising results in the field of digital filter design [16, 17, 18]. Hence, more research effort should be directed towards the design of NOIs based on the nature-inspired optimization algorithms. This paper employs a popular swarm-intelligence based global optimizer called Cuckoo Search Algorithm (CSA) [19] to design an IIR filter based digital NOI with accurate magnitude response. Further comparisons are also carried out with PSO optimized NOIs with respect to the frequency response performance, robustness, and convergence. Comparison with the latest literature further demonstrates the superiority of the CSA-based designs.

The arrangement of the rest of the paper is as follows. In Section 2, the problem formulation is shown. CSA is briefly presented in Section 3. The MATLAB simulations are carried out on i3 CPU (1.70 GHz) with 2 GB RAM and the results are shown in Section 4. Finally, Section 5 concludes the paper.

2 Optimization Problem Formulation

The IIR filter based representation of the digital NOI is given by (1).

$$H_P(z) = \frac{\sum_{j=0}^{M} q_j z^{-j}}{\sum_{j=0}^{M} p_j z^{-j}} \qquad \textbf{(1)}$$

where, M is the order of $H_P(z)$; q_j and p_j, j= 0, 1, …, M, are the coefficients of $H_P(z)$.

The transfer function in the frequency domain of $H_P(z)$ is given by (2).

$$H_P(\mu) = \frac{\sum_{j=0}^{M} q_j e^{-j\mu}}{\sum_{j=0}^{M} p_j e^{-j\mu}} \qquad \textbf{(2)}$$

The purpose of the optimization procedure based on PSO and CSA is find out the set of coefficient values of $H_P(\mu)$ such that the cost function f as defined by (3) is minimized.

$$f = \sum_{i=1}^{L} [H_P(\mu_i) - H(\mu_i)]^2 \qquad \textbf{(3)}$$

where, L is the total number of points where the frequency responses are determined. Here, $0.1 \le \mu\ (radians/\sec) \le ®$, $M = 3$, L=1000, and $r = 0.5$ are chosen.

Note that this particular optimization problem is a multidimensional (dimension, D=8) and multimodal one.

3 Cuckoo Search Algorithm (CSA)

CSA [19, 20] is a nature-inspired optimization technique which mimics the parasitism behavior of some species of the cuckoo bird. The efficient nature of CSA lies in employing Levy flights as compared with the simple random walks used in PSO. CSA is based on the following rules: (a) each cuckoo bird can lay only one egg which is dumped in a randomly chosen nest, (b) only the best nests containing eggs of superior quality are carried over to the following generation, (c) a fixed number of host nests is only available, (d) the probability of identifying the egg laid by the cuckoo on the host nest by the host bird is given by a probability p_a, where, $p_a \in (0,1)$.

The control parameter p_a allows efficient switching between the global and the local random walks. The local random walk is given by (4).

$$x_i^{t+1} = x_i^{\,t} + \sim s \otimes H(p_a - £) \otimes (x_j^{\,t} - x_k^{\,t}) \qquad \textbf{(4)}$$

where, x_j^t and x_k^t are two solutions chosen in a random manner by permutation, $H(.)$ is the Heaviside function, £ is a random number which is chosen from a uniform distribution, ~ is the step-size scaling factor, and s is the step size. The entry-wise product is denoted by $\otimes$.

The Levy flight based global random walk is defined by (5).

$$x_i^{t+1} = x_i^{\,t} + \sim \otimes L(s, ª) \qquad \textbf{(5)}$$

where, $L(s, ª) = \dfrac{ª\Gamma(ª)\sin(ª®/2)}{®} \dfrac{1}{s^{1+ª}}$.

After performing 50 runs of the algorithm for each set of values of n (= 20, 50, 100, 200) and p_a (=0.10, 0.25, 0.40, 0.50, 0.80), the following parameters of CSA which produced the best design results are: n=50 and p_a=0.25. The random numbers with the distribution L is drawn using the Mantegna algorithm.

The pseudo code of CSA is shown in Fig. 1.

Define fitness function $f(\boldsymbol{x})$, $\boldsymbol{x}=(x_1, \ldots, x_{2M+2})^T$
Randomly generate n number of host nests $\boldsymbol{x}_i$
while (t<1000)
    Randomly select a cuckoo
    Generate a solution as per (5)
    Determine its fitness (f_i)
    Randomly select a nest (say, the jth nest)
        if ($f_i < f_j$)
            Substitute the jth solution with the ith
        end if
    Abandon p_a fraction of host nests
    Generate new nests as per (4)
    Retain the best nests
    Determine the current best nest
    Increment t
end while
Declare the best nest as the optimal solution

Fig. 1. Pseudo code of CSA.

4 Simulation Results and Discussions

The optimal non-integer order ($r = 0.5$) integrators based on CSA and PSO are given by (6) and (7), respectively.

$$H_p(z) = \frac{0.9495 - 0.9436z^{-1} + 0.0646z^{-2} + 0.0563z^{-3}}{1.0270 - 1.6080z^{-1} + 0.6006z^{-2} + 0.0055z^{-3}} \quad \text{(6)}$$

$$H_p(z) = \frac{0.8008 + 0.5178z^{-1} - 0.1260z^{-2} - 0.0594z^{-3}}{0.8818 + 0.0936z^{-1} - 0.4763z^{-2} - 0.0652z^{-3}} \quad \text{(7)}$$

The comparison of the magnitude responses of the designed NOIs with the ideal half integrator is shown in Fig. 2. The MARME and the MAME values attained for CSA are -52.19 dB and -28.94 dB, respectively, which is better than PSO based design by 6.97 dB and 25.62 dB, respectively. The absolute relative magnitude error (ARME) response of PSO and CSA based designs are shown in Fig. 3. The better frequency domain fitting to the continuous-time domain for the CSA-based NOI can be credited to the alternating arrangement of zeros and poles in the real axis of the z-plane as shown in the pole-zero diagram in Fig. 4. Furthermore, this diagram also demonstrates that the CSA based half integrator is stable and exhibits a minimum-phase response which is an essential requirement for control and signal processing applications.

After conducting 100 runs for PSO and CSA, the performance of the designs are analyzed on the basis of best, worst, mean, and standard deviation (SD) for the MARME and MAME metrics, and the results are shown in Table 1. The proposed

CSA based design achieves a small SD, which exemplify the robust performance of CSA as compared with PSO for solving this multimodal optimization problem.

The fitness convergence of PSO and CSA for the design of half integrator is shown in Fig. 5 which shows a faster convergence rate with respect to iterations for CSA. A further study based on 100 trial runs is carried out to determine the mean and SD performances of convergence characteristics and the optimization time (in seconds) of PSO and CSA and the results are shown in Table 1. CSA outperforms PSO in achieving a faster convergence speed and a smaller execution time as highlighted by the mean and the SD values.

Table 1. Performance of CSA based designs based on different performance metrics.

Optimizer	Metric	Best	Worst	Mean	SD
PSO	MARME (dB)	-45.22	-30.88	-38.02	3.13
	MAME (dB)	-3.32	14.75	5.56	3.89
	Iterations to converge	146	222	181	14
	Optimization time (s)	88.652	89.691	89.092	0.205
CSA	MARME (dB)	-52.19	-36.24	-45.24	2.99
	MAME (dB)	-28.94	-16.03	-21.23	2.74
	Iterations to converge	84	123	106	10
	Optimization time (s)	85.789	86.648	86.215	0.188

Table 2 shows the comparison of the CSA based NOI with the literature based on the MARME and MAME metrics. Compared with the designs based on direct discretization [6], interpolation [13], and PSO [13], the proposed designs achieve improvements of 22.74%, 75.66%, and 63.86%, respectively, in terms of MARME, and 220.84%, 327.47%, and 198.96%, respectively, in terms of MAME. Thus, CSA yields the most accurate optimal NOIs. The ARME responses of CSA-based NOI with the literature are shown in Figure 6.

Table 2. Comparison with the literature.

Reference	Approach	MARME (dB)	MAME (dB)
[6]	Direct discretization	-42.52	-9.02
[13]	Linear Interpolation	-29.71	-6.77
[13]	Particle Swarm Optimization	-31.85	-9.68
Proposed	Cuckoo Search Algorithm	-52.19	-28.94

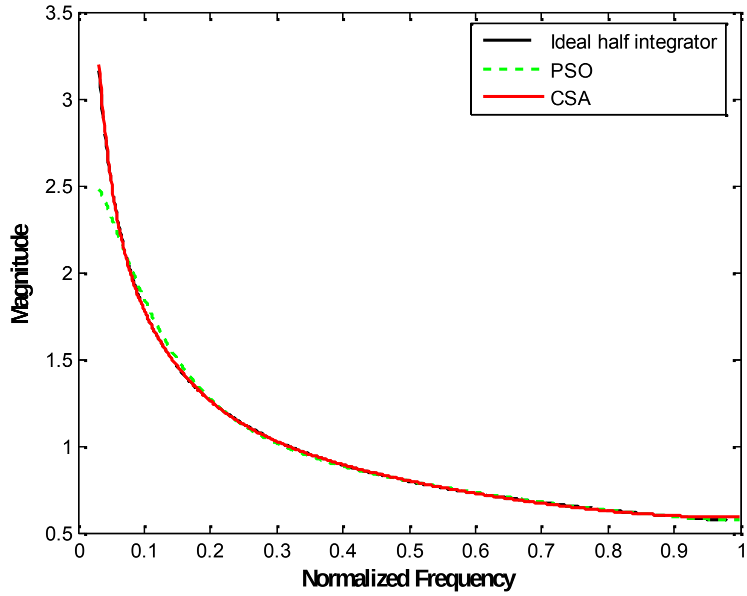

Fig. 2. Magnitude response of the digital half integrators designed using PSO and CSA.

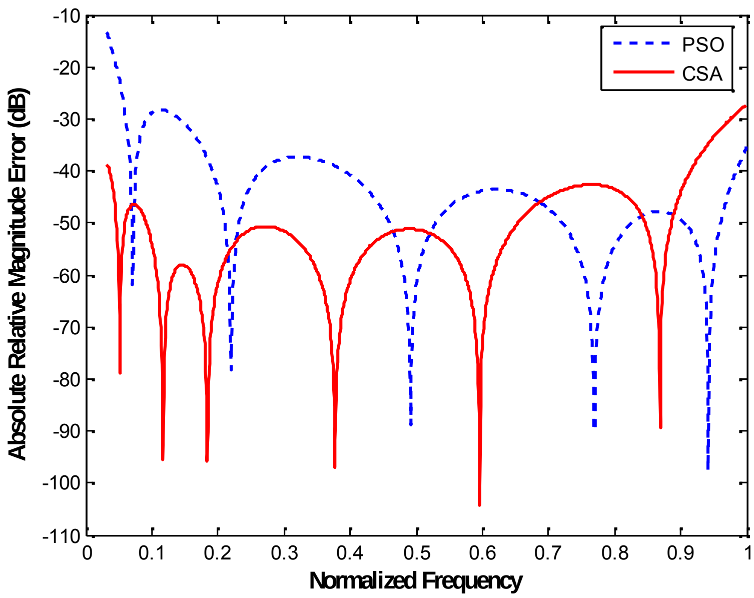

Fig. 3. ARME response comparison of the CSA based half integrator with the literature.

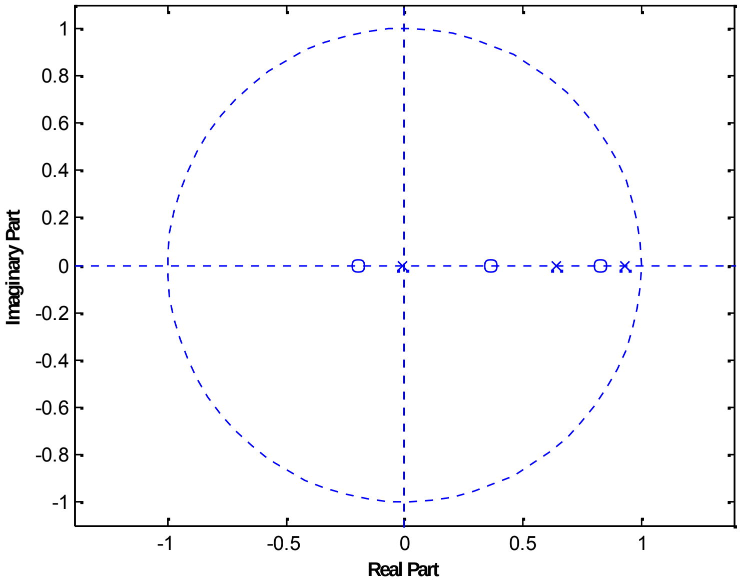

Fig. 4. Pole-zero diagram of the CSA based digital half integrator.

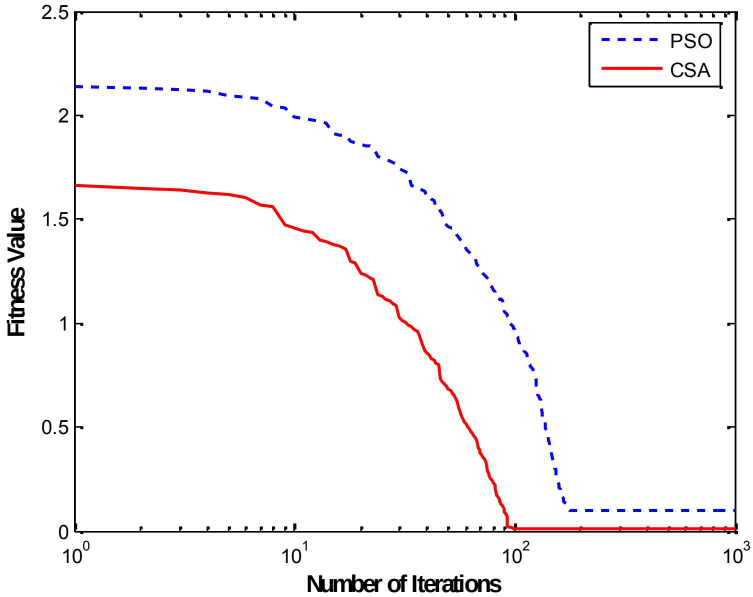

Fig. 5. Convergence curves of PSO and CSA.

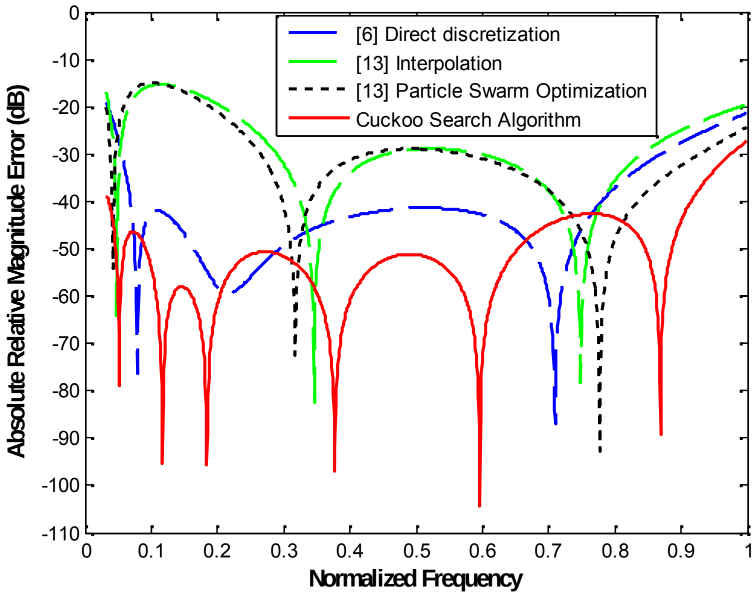

Fig. 6. ARME response comparison of the CSA based half integrator with the literature.

5 Conclusions

Cuckoo Search Algorithm is used to design IIR non-integer order integrator with a wideband frequency response in this work. Magnitude response, speed of convergence, and optimization time are considered to compare among the PSO and CSA based designs. The CSA based model outperforms the PSO based designs for all the afore-mentioned parameters. The proposed CSA based half integrator achieves 22.74% and 198.96% improvement for MARME and MAME metrics, respectively, over the reported literature.

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