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► **To cite this version:**

S Akshay, Loïc Hélouët, R Phawade. Combining Free choice and Time in Petri Nets . 6th IFIP Working group on trends in Concurrency, Sep 2017, Berlin, Germany. hal-01650751

HAL Id: hal-01650751

<https://inria.hal.science/hal-01650751v1>

Submitted on 28 Nov 2017

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Combining Free choice and Time in Petri Nets

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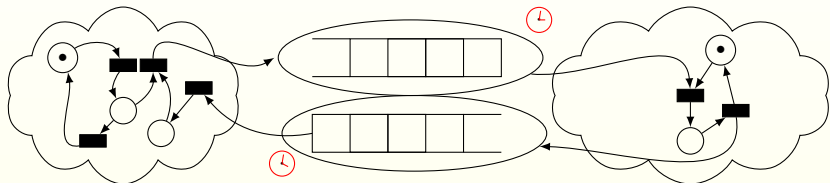
^b) INRIA Rennes, France

TRENDS Sept. 9th 2017

[TIME'16]

Motivation 1 : Modeling issues

Model Time constrained **unbounded** concurrent systems



Desired features

- Latency : messages take **at least 10 ms** to reach their destination
- Timeout/urgency : a message not consumed **after 200ms** is lost
- Rates : a message is received **every γ t.u.,...**

Motivation 2 : Verification

Standard questions

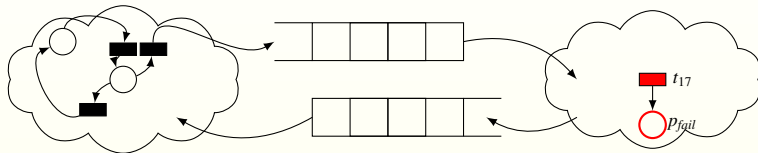
- **Reachability** : is marking M reachable from initial marking M_0 ?
- **Coverability** : Given a marking M , is there a marking M' reachable from M_0 such that $M'(p) > M(p)$ for every place p ?
- **Boundedness** : is there a bound K such that for every reachable marking, every place p , $M(p) \leq K$?
- **Firability** : is there an execution in which transition t is fired ?

Objectives

- Decidability for these questions
- Efficient algorithms

Motivation 3 : Robustness

φ_{17} : Transition t_{17} (a major failure) is not firable.



What if :

- time is measured with some imprecision
- clocks tend to have some drift/jitter/delay, ...

Robustness : reasoning with idealized time representation

Assume a class of properties Φ , a model for time imprecision $\llbracket \cdot \rrbracket_\delta$

Given a model \mathcal{M} , and a value $\delta \in \mathbb{R}$, check that :

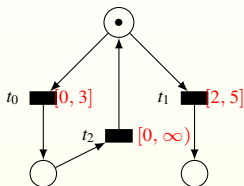
$$\forall \varphi \in \Phi, \mathcal{M} \models \varphi \iff \llbracket \mathcal{M} \rrbracket_\delta \models \varphi$$

Given a model \mathcal{M} , check if :

$$\exists \Delta, \forall \delta \leq \Delta, \forall \varphi \in \Phi, \mathcal{M} \models \varphi \iff \llbracket \mathcal{M} \rrbracket_\delta \models \varphi$$

Time vs Timed Petri nets

Time Petri nets [Merlin74]

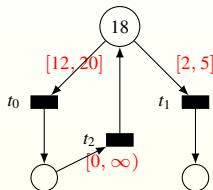


Pros :

- time
- Urgency
- unbounded places
- Expressive power

Cons : Undecidability

Timed Petri nets [Walter83]



Pros :

- time
- unbounded places
- Ages
- WSTS \equiv decidability of coverability, boundedness,...

Cons : no urgency

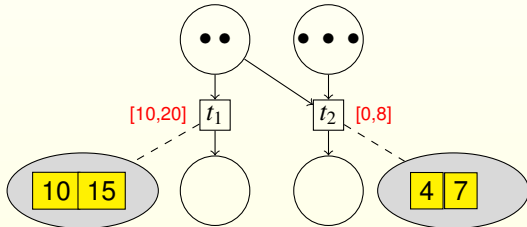
- Free-choice Multiserver Time Petri nets
- Processes and their relation to untimed nets
- Firability
- Robustness

TPN Multiserver Semantics : Configuration

Configuration (Threshold Semantics)

$$C = (M, mem)$$

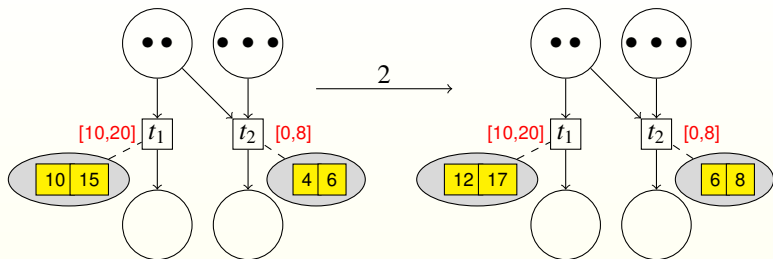
- M : marking, enables transitions **several** times
- mem : remembers for each **enabling instance** of a transition for how long it has been enabled



$$M(p_0) = 2; M(p_1) = 3; M(p_3) = 0; M(p_4) = 0$$

$$mem(t_1) = \{10; 15\} \quad mem(t_2) = \{4; 7\}$$

TPN Multiserver Semantics : timed move



Timed Move : $C \xrightarrow{\delta} C'$

Let a duration δ elapse = update memorized durations

Urgency of a transition t in $C = (M, mem)$

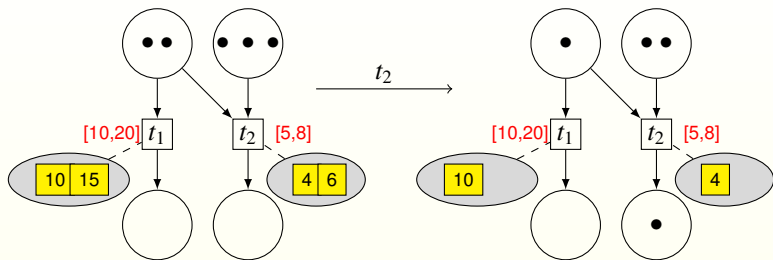
Let t be a transition such that :

- $I(t) = [l, u]$
- t has been enabled for u t.u, i.e., $\max mem(t) = u$

then, t is **urgent**.

Time **cannot** progress, a **discrete** move must occur.

TPN Multiserver Semantics : discrete move



Discrete firing : $C \xrightarrow{t_i} C'$

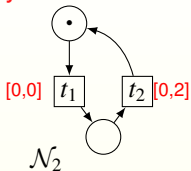
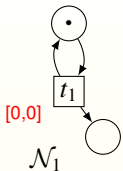
- $I(t_i) = [l, u], \max(\text{mem}(t_i)) > l$ (t_i need not be urgent)
- modification of memory for transitions in competition with t_i
- Other enabling instances remain untouched

Behaviors of a net

- Labeled Transition System : $(\mathcal{C}, \longrightarrow, C_0)$
- $\text{Lang}(\mathcal{N}) \subseteq T \times \mathbb{R}$:
set of timed words of \mathcal{N} : $w = (t_1, d_1)(t_2, d_2) \dots$

Restrictions (to obtain decidable classes)

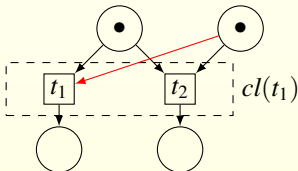
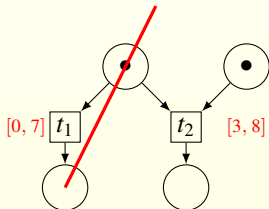
Forbid nets that can **force zero-delay behaviors**.



Free choice PN and free choice TPN

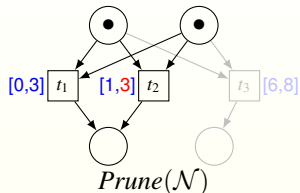
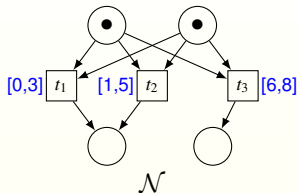
(FC-PN) $\mathcal{U} = (P, T, F)$ is **free choice** if $\forall t, t' \in T, \bullet t \cap \bullet t' \neq \emptyset \implies \bullet t = \bullet t'$.
(FC-TPN) $\mathcal{N} = (\mathcal{U}, M_0, I)$ is a **free choice TPN** if $Untime(\mathcal{N}) = \mathcal{U}$ is free choice.

The **cluster** of transition t is $Cl(t) = \{t' \in T \mid \bullet t \cap \bullet t' \neq \emptyset\}$



Pruning FC-TPN (a.k.a normalization [Chatain13])

Some transitions in FC-TPNs will obviously never fire !



To obtain a Pruned FC-TPN

- remove unfirable transitions :

$$t : I(t) = [a, b] \wedge \exists t', I(t') = [c, d] \text{ with } d < a$$

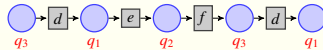
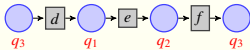
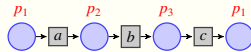
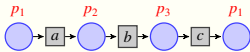
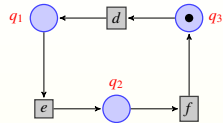
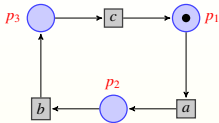
- Associate to remaining transition possible values for intervals

$$I(t) = [a, b] \Rightarrow I'(t) = [a, \max_{t' \in Cl(t)} (lft(t'))]$$

Pruning Lemma : Let \mathcal{N} be a **FC-TPN**, then

the transition systems associated with \mathcal{N} and $Prune(\mathcal{N})$ are isomorphic.
(not true outside FC-TPNs)

Causal processes



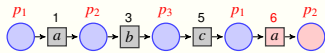
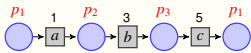
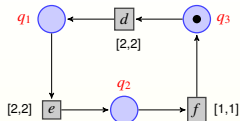
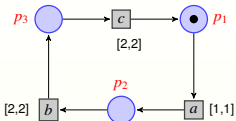
Untimed causal processes ON_1 and ON_2

Causal processes

- A partial order representation of executions
- **Principle** : Unfold the net by glueing transitions/places occurrences one after another starting from the initially marked places

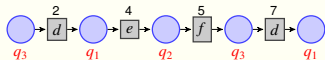
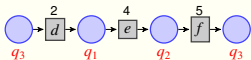
Still an untimed setting !

Timed causal processes



TON_3

TON_4



Advantages of Timed Causal processes

- Partial order timed representation of timed executions
- **URGENCY** is considered : if d occurs at date 7 in TON , all urgent transitions before d also occur in TON .

Relation between Timed causal Processes and timed languages

Let \mathcal{P} be the set of timed causal processes of \mathcal{N} . Then,

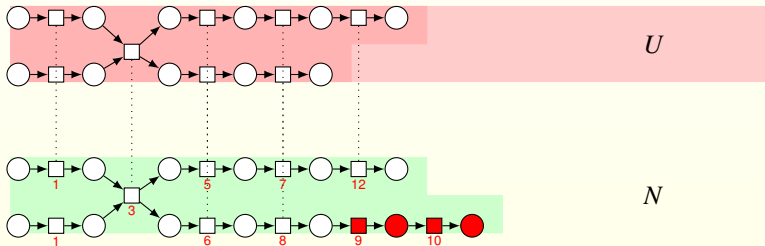
$$Lang(\mathcal{N}) = \bigcup_{N \in \mathcal{P}} Lang(N)$$

Properties of Free Choice TPN

Theorem 1 : Inclusion of untimed prefixes

Let $\mathcal{N} = (\mathcal{U}, M_0, I)$ be a **pruned FC-TPN** (w.o. forced 0-delay sequences).
 U be an (untimed) causal process of $\mathcal{U} = \text{Untime}(\mathcal{N})$.

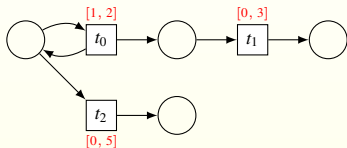
Then there exists a timed causal process N of \mathcal{N} such that $U \leq \text{Untime}(N)$.



Theorem 2 : Fireability

Let $\mathcal{N} = (\mathcal{U}, M_0, I)$ be an FC-TPN (w.o. forced 0-delay).
Checking fireability of a transition $t \in T$ in \mathcal{N} is **decidable**.

Proof idea : Firability \sim coverability in untimed nets [Rack78]



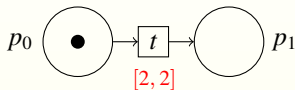
Theorem 3 : Termination

Let $\mathcal{N} = (\mathcal{U}, M_0, I)$ be an FC-TPN (w.o. forced 0-delay). It is decidable if \mathcal{N} terminates.

Proof Idea :

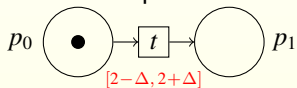
an infinite run of $\mathcal{U} = \text{Untime}(\text{Prune}(\mathcal{N}))$ has a timed counterpart in \mathcal{N} and conversely. (and termination is decidable in untimed Petri nets)

A major drawback of timed models :

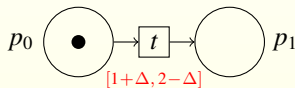


```
public class MyProgram {  
    throws InterruptedException {  
        Myprogram prog=new MyProgram();  
        //Pause for 2 seconds  
        Thread.sleep(2000);  
        prog.t();  
    }  
}
```

What the implementation might really do :



Guard enlargement



Guard Shrinking

Form now : $\mathcal{N}_\delta = \mathcal{N}$ with enlarged guards

Definition (robustness problems for TPNs)

Given a TPN \mathcal{N} , does there exist $\Delta \in \mathbb{Q}_{>0}$ such that $\forall \delta \leq \Delta$

- $\text{Fireable}(\mathcal{N}) = \text{Fireable}(\mathcal{N}_\delta)$?
- \mathcal{N} terminates iff \mathcal{N}_δ terminates
- \mathcal{N} is bounded iff \mathcal{N}_δ is bounded...

Note : Firability, termination, boundedness are not a priori robust/non robust properties of the whole class of FC-TPNs.

Theorem 4

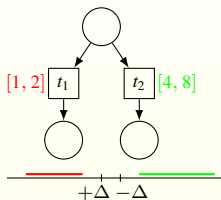
Let \mathcal{N} be a FC-TPN without forced 0-delay time firing sequences. Then robustness of fireability is decidable. If \mathcal{N} has robust fireability, bound Δ can be effectively computed.

Theorem 5

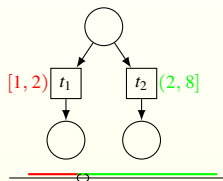
Let \mathcal{N} be a FC-TPN without forced 0-delay time firing sequences. Then it is decidable whether termination is a robust property of \mathcal{N} .

Proof idea

If $\text{Prune}(\mathcal{N})$ and $\text{Prune}(\mathcal{N}_\Delta)$ have the same clusters, then Δ -enlargement of \mathcal{N} does not modify firable transitions.



t_2 not firable



t_2 firable $\forall \Delta$

Check for each accessible cluster C whether intervals can be enlarged by some Δ_C without changing firable transitions

Contributions so far :

- A FC-Multiserver PN variant with its process semantics
- Decidable firability, termination
- Decidability of robustness of firability, termination, wrt enlargement (and shrinking)

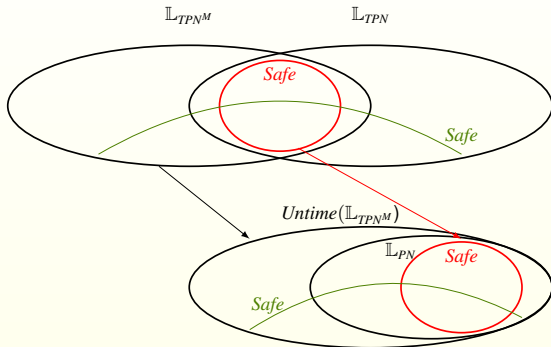
Open questions :

- decidability of Coverability, reachability and boundedness for FC-TPNs ?
- Robustness of more properties ?
- same issues without multi-enabledness ?
- Expressiveness of FC-TPNs ?

\mathbb{L}_{TPNM} : Timed languages expressible with Time Petri nets (Multiserver)

\mathbb{L}_{TPN} : Timed languages expressible with Time Petri nets

\mathbb{L}_{PN} : Untimed languages expressible with Petri nets



(Thm. 1) In free choice nets, timed / untimed processes tightly related
What about **Languages** ?

$$\mathbb{L}_{PN} \subseteq Untime(\mathbb{L}_{TPNM}) \quad \mathbb{L}_{PN \cap Safe} = Untime(\mathbb{L}_{TPNM \cap Safe}) \quad \mathbb{L}_{PN} \neq Untime(\mathbb{L}_{TPNM} ??)$$

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