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# Attitude Estimation with Smartphones

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## Abstract

We investigate the precision of attitude estimation algorithms in the particular context of pedestrian navigation with commodity smartphones and their inertial/magnetic sensors. We report on an extensive comparison and experimental analysis of existing algorithms. We focus on typical motions of smartphones when carried by pedestrians. We use a precise ground truth obtained from a motion capture system. We test state-of-the-art and built-in attitude estimation techniques with several smartphones, in the presence of magnetic perturbations typically found in buildings. We discuss the obtained results, analyze advantages and limits of current technologies for attitude estimation in this context. Furthermore, we propose a new technique for limiting the impact of magnetic perturbations with any attitude estimation algorithm used in this context. We show how our technique compares and improves over previous works. A particular attention was paid to the study of attitude estimation in the context of augmented reality motions.

*Keywords:* Attitude Estimation, Smartphone, Inertial Sensors, Augmented Reality Motions, Magnetic Field, Perturbations, Benchmark.

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## 1. Introduction

Pervasive applications on smartphones increasingly rely on techniques for estimating attitude. Attitude is the orientation of the smartphone with respect to Earth's local frame [1]. Augmented Reality (AR) applications [2, 3, 4], pedestrian dead-reckoning systems for indoor-localization [5], and photo sphere creations and previews [6] constitute examples in which precision and stability of attitude estimation matter.

Modern smartphones embed sensors such as accelerometer, gyroscope, and magnetometer which make it possible to leverage existing attitude estimation algorithms. Such algorithms have been extensively investigated in various domains such as: robotics [7], aerospace [8], unmanned aerial vehicles [9], bio-logging [10], indoor positioning [5]. However, the particular context of smartphones carried by pedestrians brings new challenges due to singular accelerations and magnetic perturbations, which sometimes invalidate the basic hypotheses that underly state-of-the-art attitude estimation algorithms. In particular, the absence of model describing the smartphone motions (preventing control), and the presence and variations of magnetic perturbations during the estimation phase, both introduce additional difficulties.

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*Contribution.* We investigate the precision of attitude estimation algorithms in the context of commodity smartphones carried by pedestrians with a specific focus on AR. We consider eight typical motions (such as texting, phoning, running, etc.) with various impacts on external accelerations, as well as the presence/absence of magnetic perturbations typically found in indoor environments. We systematically analyze, compare and evaluate eight state-of-the-art algorithms (and their variants). We precisely quantify the attitude estimation error obtained with each technique, owing to the use of a precise ground truth obtained with a motion capture system. We make our benchmark available<sup>1</sup> and pay attention to the reproducibility of results. We analyze and discuss the obtained results and report on conclusions. We also present a new technique which helps in improving precision by limiting the effect of magnetic perturbations with all considered algorithms.

*Outline of the paper.* We first introduce required preliminaries in §2. We then review the closest related works in §3. We present the considered algorithms in §4, our new technique in §5, and our experimental protocol in §6. We finally report on obtained results and lessons learned in §7 and §8 before concluding in §9.

## 2. Background for attitude estimation

### 2.1. Sensors measurements and calibration

The sensors configuration of a smartphone is composed of a triad of MEMS (Micro-Electro-Mechanical Systems) sensors consisting of a 3-axis gyroscope, a 3-axis accelerometer and a 3-axis magnetometer. The outputs of these low-cost sensors are imprecise as they suffer from several problems: noise, bias, full scale range, axes misalignment, axes non-orthogonality and temperature variations. Sensors models are described below.

#### 2.1.1. Gyroscope

The 3-axis gyroscope measures the angular velocity of the smartphone in  $rad.s^{-1}$ :  $gyr = [gyr_x \quad gyr_y \quad gyr_z]^T$ . The widely used continuous time model for a gyroscope can be written as:

$$gyr = gyr_r + gyr_b + gyr_n \quad (1)$$

where:

$gyr$  is the angular rate measured by the gyroscope.

$gyr_r$  is the true angular rate.

$gyr_b$  is the gyroscope bias.

$gyr_n$  is the gyroscope noise.

A gyroscope during a static phase should provide an angular velocity of 0 for each axis. Due to the poor quality of sensors, measurements show a small bias. This bias can be detected during static phases and is then subtracted from measurements during online phase.

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<sup>1</sup><http://tyrex.inria.fr/mobile/benchmarks-attitude>

### 2.1.2. Accelerometer

The 3-axis accelerometer measures the acceleration of the smartphone, including the gravity and external acceleration in  $m.s^{-2}$ :  $acc = [acc_x \quad acc_y \quad acc_z]^T$ . The continuous time model for an accelerometer can be written:

$$acc = acc_r + acc_b + acc_n \quad (2)$$

where:

$acc$  is the sum of the gravity and external acceleration of the body measured by the accelerometer (Eq. 3).

$acc_r$  is the true sum of the gravity and external acceleration of the body.

$acc_b$  is the accelerometer bias.

$acc_n$  is the accelerometer noise.

Gravity is the force of attraction by which terrestrial body tends to fall toward the center of the earth and external accelerations are all others accelerations applied on the body:

$$acc = gravity + acc^{ext}. \quad (3)$$

An accelerometer during a static phase provides a magnitude of acceleration close to  $g$ , where  $g$  is the acceleration due to the gravity at the Earth's surface ( $g \approx 9.8 m.s^{-2}$ ). In [11], authors provide an accelerometer calibration algorithm based on a minimum of 9 static phases. This calibration allows to remove the bias and misalignment by normalizing the acceleration vector in multiple smartphone orientations.

### 2.1.3. Magnetometer

The 3-axis magnetometer measures the magnetic field in the smartphone frame in micro-tesla ( $\mu T$ ):  $mag = [mag_x \quad mag_y \quad mag_z]^T$ . The continuous time model for a magnetometer can be written such as:

$$mag = mag_r + mag_b + mag_n, \text{ where:} \quad (4)$$

where:

$mag$  is the sum of the Earth's magnetic field and other magnetic fields measured by the magnetometer (Eq. 5).

$mag_r$  is the true sum of the Earth's magnetic field and other magnetic fields.

$mag_b$  is the magnetometer bias.

$mag_n$  is the magnetometer noise.

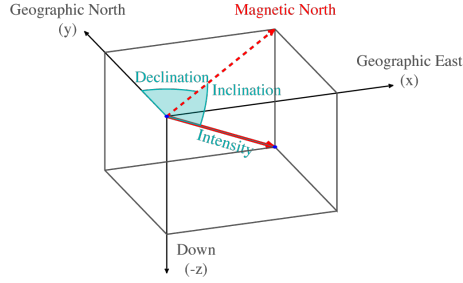


Figure 1: Earth's magnetic field representation

The Earth's magnetic field can be modeled by a dipole and follows basic laws of magnetic fields. At any location, the Earth's magnetic field can be represented by a three-dimensional vector and its intensity varies from  $25\mu T$  to  $65\mu T$ . The National Geospatial-Intelligence Agency (NGA) and the United Kingdoms Defence Geographic Centre (DGC) provide a World Magnetic Model (WMM) [12] every 5-years. Declination is used to know the angle between the Magnetic North and Geographic North, while inclination and intensity are used to build the reference vector.

Unfortunately, the magnetometer does not measure only the Earth's magnetic field. Most of the time in indoor environments, we are in presence of magnetic dipoles which perturb the measure of Earth's magnetic field. These perturbations can be caused by electromagnetic devices (speakers, magnets), manmade structures (walls, floors) or other ferromagnetic objects like belts, keys, etc. For example, a smartphone speaker has a field of about  $200\mu T$  (4 times more than the Earth's magnetic field). The study found in [13] categorizes the environmental characteristics with respect to the magnetic deviations.

Earth's magnetic field is a vector pointing toward magnetic north and its magnitude is noted  $F$ . All other magnetic fields applied on the body are called magnetic perturbations and noted  $\text{mag}^{ext}$ :

$$\text{mag} = \text{Earth's magnetic field} + \text{mag}^{ext}. \quad (5)$$

Magnetic perturbations are categorized in two groups: hard and soft iron distortions. Hard iron distortions are caused by ferromagnetic materials in the same frame than the smartphone (e.g. speaker for a smartphone). Soft iron distortions are caused by objects that produce a magnetic field (buildings walls, machines, heaters...) in Earth's frame. In a context free from magnetic interferences, hard and soft iron distortions can be partially corrected at the same time by normalizing the magnetic field vector in multiple smartphone orientations [14, 15]. In theory, due to soft iron distortions, when the device is moving or when the magnetic context changes, the calibration phase needs to be reprocessed.

## 2.2. Attitude representation

The smartphone attitude is determined when the axis orientation of the Smartphone-Frame SF ( $SF_x$ ,  $SF_y$ ,  $SF_z$ ) is specified with respect to the Earth-Frame EF ( $EF_x$ ,  $EF_y$ ,  $EF_z$ ) (or Local Tangent Plane (LTP)), see Fig. 2.

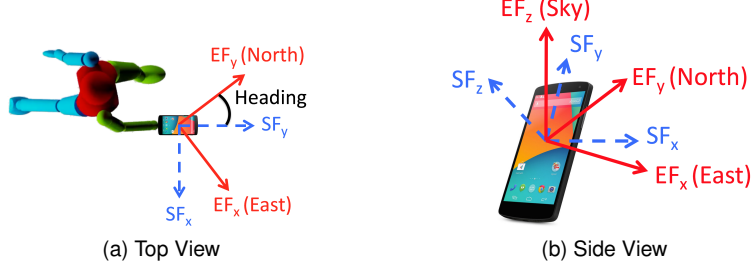


Figure 2: The Smartphone-Frame  $SF$  (dashed line) and Earth-Frame  $EF$  (solid line).

The  $SF_x$ -axis is horizontal and points to the right, the  $SF_y$ -axis is vertical and points up and the  $SF_z$ -axis points towards the outside of the front face of the screen. The  $EF_y$ -axis points to the North. The  $EF_z$ -axis points to the sky perpendicular to the reference ellipsoid and the  $EF_x$ -axis completes the right-handed coordinate system, pointing East (ENU : East, North, Up). Another convention is often used by aerial vehicles called NED for North, East and Down.

Based on the literature, the attitude can be expressed with four different mathematical representations [16]. Euler angles (yaw, pitch, roll), rotation matrices, quaternions or axis/angle.

A unit-norm quaternion, which defines the rotation between  $SF$  and  $EF$ , is defined by:

$$q = {}^E_S q = [q_w \quad q_x \quad q_y \quad q_z]^T \in \mathbb{R}^4, \quad (6)$$

where  $q_w$  and  $[q_x \quad q_y \quad q_z]$  are respectively the scalar and the vector parts of the quaternion.

To express a vector  $v = [v_x \quad v_y \quad v_z]^T$  from  $EF$  to  $SF$ , Hamilton product [17] is used (Eq. (7)). Conversely, from  $EF$  to  $SF$ , Eq. (8) is used.

$${}^S v_q = q^{-1} \otimes {}^E v_q \otimes q, \quad (7)$$

$${}^E v_q = q \otimes {}^S v_q \otimes q^{-1}, \quad (8)$$

where  $v_q$  is the quaternion form of  $v$  (Eq. (9))

$$v_q = [0 \quad v_x \quad v_y \quad v_z]^T. \quad (9)$$

The well-known kinematic equation can be used to describe the variation of the attitude in term of quaternion:

$$\dot{q} = \frac{1}{2} q \otimes \omega_q, \quad (10)$$

where  $\omega_q$  is the quaternion form of angular velocity. More details about quaternion algebra can be found in [17].

Each representation has some drawbacks. In our context, Euler angles cannot be used due to the well-known gimbal-lock problem [18], when the device is in a pocket or held for phoning, the yaw angle can vary widely. Nevertheless, quaternions avoid the singularity problem, they provide basic primitives with cheap computation cost. All the algorithms that we have implemented in Java/Matlab and benchmarked in §7 use the quaternion algebra. A simple mathematical transformation between quaternions and Euler angles can be found in [18].

### 2.3. Attitude Estimation

The problem of finding the optimal attitude estimation solution was formulated for the first time by Wahba in 1965 [1]. Wahba's problem seeks to find a rotation matrix between two coordinate systems from a set of vector observations (minimum two vectors known in a fixed frame and in a body frame). In our case, the two coordinate systems are the Smartphone Frame (SF) and the Earth Frame (EF) as shown in Fig. 2. A typical Inertial Measurement Unit (IMU) in a smartphone can provide two vector observations expressed in two frames:

- acceleration in SF provided by an accelerometer noted  ${}^S\text{acc}$  and its projection in EF noted  ${}^E\text{acc}$ .
- magnetic field in SF provided by a magnetometer noted  ${}^S\text{mag}$  and its projection in EF noted  ${}^E\text{mag}$ .

These 2 observation vectors can be modeled as following:

$${}^S\text{acc}_q = q^{-1} \otimes {}^E\text{acc}_q \otimes q, \quad (11)$$

$${}^S\text{mag}_q = q^{-1} \otimes {}^E\text{mag}_q \otimes q. \quad (12)$$

If the smartphone is in static phase (not translating),  $\text{acc}^{ext} = [0 \ 0 \ 0]^T$  and

$${}^E\text{acc} = [0 \ 0 \ g]^T. \quad (13)$$

In absence of magnetic deviations,  $\text{mag}^{ext} = [0 \ 0 \ 0]^T$  and

$${}^E\text{mag} = [m_x \ m_y \ m_z]^T, \quad (14)$$

where  $m_x$ ,  $m_y$  and  $m_z$  can be obtained using the WMM [12].

Figure 3 shows these two vectors:  ${}^E\text{acc}$  in blue and  ${}^E\text{mag}$  in green.

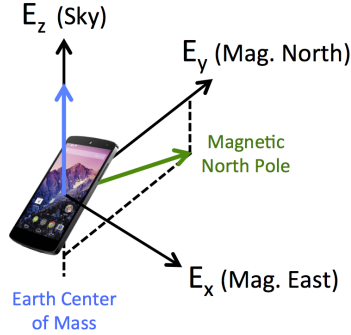


Figure 3: Reference vectors when the smartphone is static and in the absence of magnetic deviations.

In addition to accelerometer and magnetometer, the gyroscope is used to estimate variation of attitude. Unfortunately, the gyroscope bias leads after integration (Eq. (10)) to an angular drift, increasing linearly over time. Since the use of only gyroscope is not enough for attitude estimation, accelerometer and magnetometer are used to get an absolute quaternion and compensate

the drift. The crux in solving an attitude estimation problem then consists in combining inertial and magnetic sensor measurements in a relevant manner. Fig. 4 illustrates the whole approach, where  $K$  is the fusion gain between data merged from accelerometer-magnetometer fusion and gyroscope integration. This gain is adjusted depending on sensors reliability.

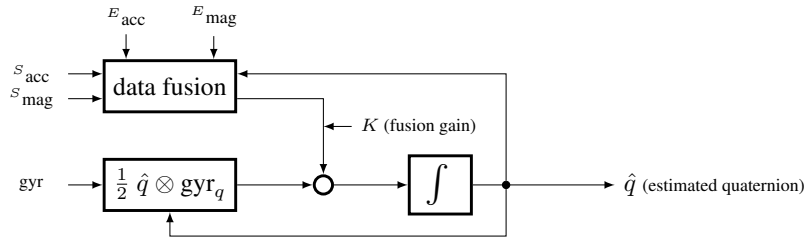


Figure 4: Method for attitude estimation.

#### 2.4. Attitude in Geo Augmented Reality

Augmented Reality is a live view of real world environment where virtual objects are shown over the camera image of an hand-held device. Geo Augmented Reality [19] (or Gravimetric AR) is an AR method which relies exclusively on device position and orientation (Fig. 5). This technique does not use image processing. GPS, Wifi, Bluetooth or any kind of location sensors can be used to determine device position. The orientation of the device is processed by an attitude filter using the onboard MEMS sensors described above. Precision of attitude estimation is crucial in Geo AR, as virtual objects should be seen at the right place. Stability of attitude estimation is crucial as well, since movements of virtual objects should be perceived as following camera movements. This is why we further study feasible balances between precision and attitude in Section 8.

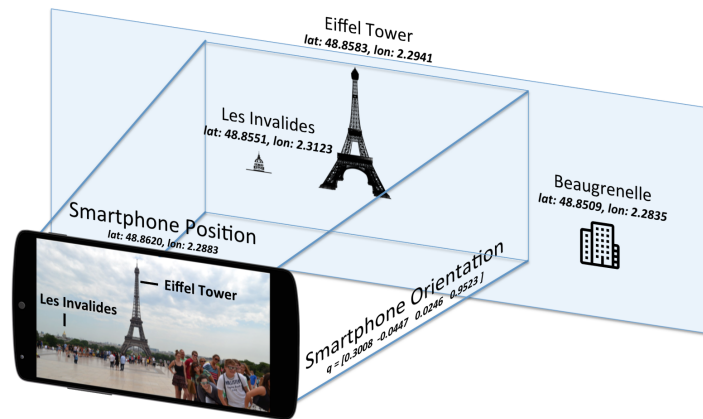


Figure 5: Geo Augmented Reality.



### 3. Related Works

Since 1965, a multitude of solutions have been proposed to resolve attitude estimation problem, such as TRIAD [20], QUaternion ESTimator (QUEST) [21], Singular Value decomposition method (SVD) [22], Kalman Filters (KF) [23, 24, 25, 26, 27], Extended Kalman Filters (EKF) [28, 29, 30, 31, 5], Unscented Kalman Filters (UKF) [32], Adaptive Kalman Filters (AKF) [33, 34], Particle Filters [35] and more recently Observers [10, 36, 37, 38]. A survey and an analysis of these methods can be found in [39]. In 2007, Crassidis et al. provide another survey with a focus on nonlinear attitude estimation methods. In this paper we further focus on algorithms that use measurements from the 3 sensors that are now commonly found on smartphones: gyroscopes, accelerometers and magnetometers, and attempt to leverage on these measurements to provide precise attitude estimation of smartphones carried by pedestrians.

Preliminary versions of this work were presented at the IPIN 2015 [40] and PerCom 2017 [41] conferences. Compared to these earlier results, the present article mainly comprises three additions: (i) more detailed descriptions on considered motions and magnetic perturbations, in particular with a quantitative characterization; (ii) the overall study is extended with a specific focus on augmented reality applications; (iii) novel results concerning stability and precision balances achievable with each filter. We report on a systematic search of parameters adjustment with each filter to shed light on the feasibility envelope in terms of precision and stability.

Most algorithms developed so far rely on a common assumption: the external acceleration is negligible. However, when used in the context of smartphone carried by a pedestrian, this assumption is questionable (we have experimentally observed high external accelerations: see e.g. second column of Table 1). Specifically, the relation between  $^S acc$  and  $^E acc$  given by Eq. (11) is true only if no external acceleration is applied on the smartphone. Assumption of external acceleration is not a new problem, in [23, 24, 29, 26] authors propose to discard accelerometer measurements in the update phase of their KF. They set values of covariance matrix to infinity when:

$$\underbrace{\left| \| ^S acc \| - \| ^E acc \| \right|}_{\mu} > \gamma_{acc}. \quad (15)$$

In [31] and [42], they explain how they adjust the covariance matrix in function of the left term of Eq. (15). In [33] and [34], authors use KF residual errors to detect external acceleration. The technique proposed in [33] needs time to let residual matrix converge in a static phase to identify bias before estimating external accelerations. Finally, in [5], Renaudin et al. only perform the update phase of their KF during periods considered as Quasi Static Field (QSF). During QSF, a low variance is given to measurements and  $^E acc$  is adjusted during these phases. To the best of our knowledge, the use of a detector à la (15) has not been investigated yet with an observer-based filter.

Most algorithms found in the literature do not consider magnetic perturbations. However, in the pedestrian context, the smartphone is often exposed to ferromagnetic objects, and this is known to yield a bad attitude estimation [13, 43]. Few papers are concerned with magnetic perturbations for attitude estimation on a smartphone carried by a pedestrian. In [38], authors consider the impact of magnetic perturbations on the North-East plane, abstracting over other possible impacts. In [23] and [29], authors set the covariance matrix of magnetic measurements to infinity when:

$$\left| \| ^S mag \| - \| ^E mag \| \right| > \gamma_{mag}. \quad (16)$$

In [23], in addition to detector (16), Harada et al. use the following property to detect magnetic perturbations:

$$\theta(^S \text{acc}, ^S \text{mag}) - \theta(^E \text{acc}, ^E \text{mag}) > \gamma_\theta, \quad (17)$$

where:  $\theta(v_1, v_2) = \arccos \frac{v_1 \cdot v_2}{\|v_1\| \cdot \|v_2\|}$ .

Similarly to how external accelerations are treated, Renaudin et al. [5] use a QSF detector based on variance of measurements.

In the present paper, we develop a new technique for limiting the impact of magnetic perturbations on attitude estimation algorithms that are executed on smartphones carried by pedestrians. In addition, we conduct extensive practical experiments with several (and typical) motions of smartphones carried by a pedestrian, and show how our approach compares and improves over previous works in this context. To the best of our knowledge, our systematic comparison of attitude estimation algorithms is the first in this context. Our experiments include 126 datasets with several typical motions, several devices, realistic magnetic perturbations, and a fine-grained analysis.

#### 4. Selected Attitude Estimation Algorithms for Comparison

We now review the state-of-the-art algorithms that we consider in our study. We have selected 8 filters from the literature which are representative of the different techniques developed for solving the attitude estimation based on IMU sensors. Our selection of algorithms can roughly be divided into two categories: those based on observers, and those based on KFs (with their EKF, UKF, and AKF variants). We summarize the main principles and objectives of each algorithm (see [40] for a more formal description of each algorithm using a common notation). For reproducibility purposes, we also indicate parameters that we used with each algorithm – which we set in accordance with authors guidelines found in their papers. We also consider the “black-box algorithms” embedded in Android and iOS. The considered algorithms are the followings:

**Madgwick et al.** [38]. This filter is a Gradient Descent (GD) based algorithm designed for pedestrian navigation. The authors propose to consider magnetic field deviations only on North-East plane using the following technique:  $^E \text{mag} = [0 \quad m_y \quad m_z]^T$ , where  $m_y = \sqrt{h_x^2 + h_y^2}$ ,  $m_z = h_z$  and  $h = \hat{q}^{-1} \otimes ^S \text{mag} \otimes \hat{q}$ . *Madgwick* is the common implementation of the filter, and *MadgwickB* the same with a gyro bias. Parameters:  $\beta = 0.08$ ,  $\zeta = 0.016$ .

**Martin et al.** [37]. This filter is an observer with a new geometrical structure (invariant observer). The authors introduce new measurements based on the cross product of acceleration and magnetic field. *Martin* is the common implementation of the filter. Parameters:  $l_a = 0.7$ ,  $l_c = 0.1$ ,  $l_d = 0.01$ ,  $n = 0.01$ ,  $o = 0.01$ ,  $k = 10$ ,  $\sigma = 0.002$ .

**Mahony et al.** [36]. This filter is a complementary filter designed for aerial vehicles. The main idea is to calculate the error by cross multiplying measured and estimated vectors. *Mahony* is the common implementation of the filter. *MahonyB* is the implementation which takes into account a gyro bias. Parameters:  $\beta = 1$ ,  $\zeta = 0.2$ .

We provided a new variant of this filter (*MahonyMartin*), the observation vector from magnetometer is replaced by the cross product of accelerometer and magnetometer from Martin et al. Parameters:  $\beta = 0.2$ ,  $K_a = 1$ ,  $K_c = 0.5$ .

**Fourati et al.** [10]. This filter is a mix between a complementary filter algorithm and the Marquardt approach designed for bio-logging. *Fourati* is the common implementation of the filter. *FouratiExtAcc* is an extension which takes external accelerations into account using Eq. (15)). Parameters:  $\beta = 0.3$ ,  $K_a = 2$  and  $K_m = 1$ .  $K_a = 0$  when  $\gamma_{acc} = 0.5m.s^{-2}$ .

In the same way we provided *MahonyMartin* variant, we proposed a new filter based on Fourati et al. algorithm, *FouratiMartin* which uses the cross product of accelerometer and magnetometer. Parameters:  $\beta = 0.3$ ,  $K_a = 2$ ,  $K_c = 1$ .

**Choukroun et al.** [25]. This filter provides a linearization of measurement equations. A KF is proposed and guarantees a global convergence. *Choukroun* is the common implementation of the filter.

**Renaudin et al.** [5]. This filter is an EKF designed for Pedestrian Dead Reckoning (PDR). In addition to Eq. (11) and Eq. (12), they use two others properties:

$$acc_{t+1} = q_\omega^{-1} \otimes acc_t \otimes q_\omega, \quad (18)$$

$$mag_{t+1} = q_\omega^{-1} \otimes mag_t \otimes q_\omega, \quad (19)$$

where  $q_\omega$  is interpreted as a rotation between two successive epochs. Eq. (11), (12), (18) and (19) are applied only during QSF periods. The detector for QSF works by analyzing variance of acceleration and magnetic field measurements on a small window ( $\approx 0.2s$ ). This filter has to be initialized ( $\approx 5s$  at the beginning) without external accelerations and magnetic perturbations (mostly outside). *Renaudin* is the common implementation of the filter. In *RenaudinBG*, the gyro bias estimation is added where, gradients update from Eq. (18) and Eq. (19) are considered. *RenaudinExtaccExtmag* takes both QSF detectors into account. Parameters:  $QSF\_Window = 10$ ,  $\gamma_{QSF\_Acc} = 3.92m.s^{-2}$ ,  $\gamma_{QSF\_Mag} = 6\mu T$ ,  $outliers_{QSF\_Acc} = 4.90m.s^{-2}$ ,  $outliers_{QSF\_Mag} = 8\mu T$ .

**Sabatini et al.** [29]. This filter is an EKF which considers external acceleration and magnetic perturbations as explained in §3. *Sabatini* is the common implementation of the filter. *SabatiniExtacc* and *SabatiniExtmag* takes respectively external accelerations and magnetic perturbations into account. We did not implement the gyro bias part of this filter. Parameters:  $\gamma_{acc} = 0.5 m.s^{-2}$ ,  $\gamma_{mag} = 15 \mu T$ ,  $\gamma_\theta = 10^\circ$ ,  $mov\_average_{mag} = 0.1s$

*Ekf* is the common implementation of the Extended KF.

**OS** The Android API of Nexus 5 and iOS API of iPhones also provides quaternions generated by undisclosed “black-box” algorithms. We include them in our comparisons.

## 5. Design of a new algorithm for better limitation of magnetic perturbations impact

The presence of magnetic perturbations in indoor environments is well-known [43]. For example, Figure 6 illustrates variations of the magnetic field observed inside Inria’s research center compared to Earth’s magnetic field. To limit the impact of such magnetic perturbations,

we propose a new approach that further builds on the idea of detectors à la (16). The overall principle is twofold: (1) during periods when we detect magnetic perturbations, we can discard magnetometer measurements for a short period ( $\approx 2 - 3s$ ) so that more importance are given to gyroscope measurements; (2) this period should be reasonably short-enough so that the impact of gyroscope’s bias<sup>2</sup> is limited.

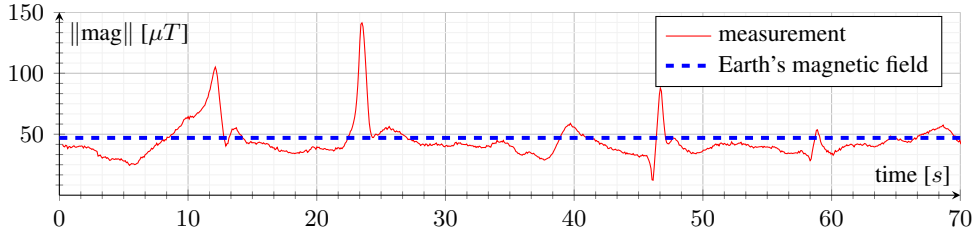


Figure 6: Magnitude of magnetic field measurements and Earth’s magnetic field in the indoor environment of Inria building in Grenoble.

We propose an improvement of the magnetic perturbation detector (Eq. (16)) adapted to the pedestrian context. When a person is moving with a normal speed (walk) in a building, we have observed huge variations of magnitude of magnetic field  $\|S_{\text{mag}}\| > 100 \mu T$  (see for example Fig. 6 at  $t = 24s$ ). The main problem with the detector (16) is to find a proper  $\gamma_{\text{mag}}$  which should be: (i) high enough not to discard magnetometer measurements due to low magnetic perturbations omnipresent in an indoor environment and (ii) small enough to reject high perturbations which affect attitude estimation (such as those coming from the proximity of e.g. heaters, see: §6.3).

When the threshold of (Eq. (16)) is reached, generally the filter is already diverging. This means that when this detection occurs, and therefore when gyroscope integration starts, magnetometer measurements involving perturbations below the threshold have already impacted attitude estimation.

Figure 7 presents our new technique to limit the impact of magnetic perturbations. The principle is that we reprocess the filter for the  $t_{\text{mag, rep}}$  last seconds without magnetometer measurements (Eq. (12)). When the detection occurs, attitude estimation is immediately replaced by these values. This technique avoids the attitude divergence during the  $t_{\text{mag, rep}}$  last seconds before the detection (Eq. (16)). This technique can be used for real-time attitude estimation (time for reprocessing being negligible when compared to  $t_{\text{mag, rep}}$ ), in which case a discontinuity of some degrees can be observed when the detection occurs (see Fig. 12).

During periods of magnetic perturbation, Eq. (16) can be true for a small duration. This is because magnitude of magnetometer measurement can be similar to Earth’s magnetic field magnitude during a perturbation phase, it depends on the direction of the perturbation. For this purpose a last condition is added: Eq. (12) can be used only if there is no detection (Eq. (16)) during the last  $t_{\text{mag, nopert}}$  seconds.

This technique works with all filters where updates (Eq. (11)) from magnetometer can be temporarily removed (which is the case of all algorithms considered here). An important prerequisite is magnetometer calibration. In a context without magnetic perturbations, magnitude of magnetometer measurements should be equal to the magnitude of Earth’s magnetic field.

<sup>2</sup>We experimentally measured the drift due to gyroscope’s bias integration as approximately  $5^\circ/min$ .

```

Data:
f (gyr, acc, mag, dT, mag_update) is a basic filter (KF or observer) where mag_update is a boolean indicating whether magnetometer measurements have to be used.
vec_states_and_values is a moving vector keeping track of filter state and measurements from sensors over a sliding window.
last_mag_pert is the elapsed time since the last magnetic perturbation detected. Initially it is set to 0.

// Detecting magnetic perturbations
mag_updatek = abs( $\|S\text{mag}\| - \|E\text{mag}\|$ ) <  $\gamma_{\text{mag}}$ 

// Enforcing minimal durations
if mag_updatek then
  last_mag_pert = last_mag_pert + dT
  if last_mag_pert <  $t_{\text{mag, nopert}}$  then
    mag_updatek = false
  end
else
  last_mag_pert = 0
end

// Reprocessing last values without mag data
if !mag_updatek-1 and mag_updatek then
  f.setState(vec_states_and_values.first)
  foreach element e of vec_states_and_values do
    f(e.gyr, e.acc, e.mag, e.dT, false)
  end
end

attitude, state = f(gyr, acc, mag, dT, mag_updatek)

// Store state and measurements
vec_states_and_valuesk = state, gyr, acc, mag, dT
remove lines of vec_states_and_values where elapsed time >  $t_{\text{mag, rep}}$ 

```

Figure 7: Pseudo-code for limiting the impact of magnetic perturbations.

In addition to the algorithms presented before, we also consider 2 new algorithms based on the aforementioned technique. The first one, *MichelObsF*, is an implementation of the technique where **f** is the observer function from Fourati et al. [10]. The second algorithm, *MichelEkfF*, is designed with **f** set to the well known EKF filter from the literature. For both algorithms we use the following common parameters:  $\gamma_{\text{mag}} = 15\mu T$ ,  $t_{\text{mag, nopert}} = 2s$  and  $t_{\text{mag, rep}} = 3s$ .

## 6. Experimental Protocol

In this section, we explain our experimental methodology. A total of 126 trials have been conducted by 3 peoples with 3 different smartphones, following several typical motions in an environment with low and high magnetic disturbances.

### 6.1. Ground Truth

Reference measurements have been made by a Qualisys system. This technology provides quaternions with a precision of  $0.5^\circ$  of rotation. Our room is equipped with 20 Oqus cameras connected to a server and a Qualisys Tracker software with a sampling rate at 150Hz. For the

purpose of aligning timestamps of our ground truth data with the one of smartphone’s sensors, we used a slerp interpolation [44]. The motion tracker reference frame has been aligned with  $EF$  using room orientation provided by architects. The room is a  $10m \times 10m$  square motion lab<sup>3</sup> (see Fig. 8). In this room, we observed that the magnetic field is almost homogeneous from a sub-place to another (variations are less than  $3\mu T$ ), and with negligible variations over time.



Figure 8: Kinovis room at Inria, Grenoble, France.

A smartphone handler with infrared markers has been created with a 3D printer for this study and its markers have been aligned with  $SF$  (see Fig. 9).

## 6.2. Typical Smartphone Motions

We identify 8 typical smartphone motions, inspired from [45]:

- Querying the context in **augmented reality** (see Fig. 9a).
- Walking while user is **texting** a message (see Fig. 9b).
- Walking while the user is **phoning** (see Fig. 9c).
- Walking with a **swinging** hand (see Fig. 9d).
- Walking with the device in the **front pocket** (see Fig. 9e).
- Walking with the device in the **back pocket** (see Fig. 9f).
- **Running** with the device in the **hand** (see Fig. 9g).
- **Running** with the device in the **pocket** (see Fig. 9h).

AR motion is a slow motion typically found during AR experiences. Other motions happen when pedestrians move and are relevant for navigation applications. Each motion is characterized by a particular external accelerations. The Table 1 shows some statistics on external acceleration magnitude grouped by motion, for the 126 tests. The second column of Table 1 shows the average (AVG) of external acceleration magnitude grouped by motion where the third column shows the estimated one from Eq.15.

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<sup>3</sup>See: <http://kinovis.inrialpes.fr>

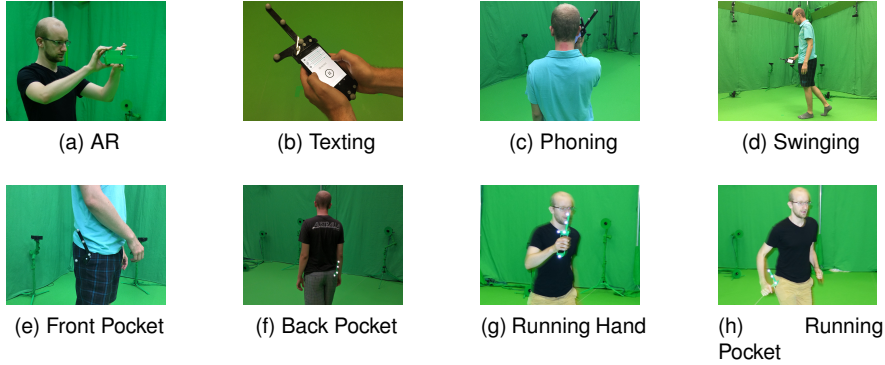


Figure 9: The eight typical motions for a smartphone.

Table 1: Statistics on Magnitude of External Accelerations for each motion

	Ext. Acc. AVG ( $m.s^{-2}$ )	Ext. Acc. AVG est. ( $m.s^{-2}$ )	Ratio	$> 0.5m.s^{-2}$	$> 1.5m.s^{-2}$	$> 5m.s^{-2}$
AR	0.56	0.24	2.39	46.4%	2.4%	0.0%
Texting	1.08	0.61	1.81	83.5%	20.7%	0.1%
Phoning	1.08	0.57	1.96	83.1%	21.0%	0.1%
Front Pocket	2.48	1.40	1.81	97.1%	68.2%	7.5%
Back Pocket	2.53	1.23	2.10	97.5%	72.0%	7.7%
Swinging	5.28	2.30	2.42	99.7%	96.8%	52.5%
Running Pocket	9.56	5.93	1.61	99.6%	98.2%	84.4%
Running Hand	16.34	8.44	2.02	99.9%	99.7%	98.6%

During tests, we observed that external accelerations almost never reach zero because the device is always moving, and constant speed is very unlikely when the device is held or carried in a pocket. However, we noticed a high variety of external accelerations: some motions involve external accelerations that are 20 times lower than gravity while others (like running hand) are closer to twice the value of gravity. We also noticed that the maximum swing of accelerometer ( $\pm 2g$ ) is often reached during our running experiments.

### 6.3. Introducing Magnetic Perturbations

During tests, we noticed that magnetic disturbances are always present in indoor-environments, and they vary between different buildings. This is mainly due to building structure. We also observed in some cases, if user is close to heaters, electrical cabinets or simply close to a wall, magnitude of magnetic field can grow up to  $150 \mu T$  during few seconds, that is 3 times more than Earth's magnetic field (see e.g. Fig. 6).

The motion capture system used is located in a room with low and constant magnetic perturbations. In order to reproduce typical magnetic perturbations of indoor environments inside the motion lab, we used several magnetic boards (see Fig. 10). This allowed us to introduce magnetic perturbations similar to the ones described above in Fig. 6. Specifically, during the 2

minutes tests, we brought the device to a few centimeters away from magnetic boards; and we repeated this 3 or 4 times (see Fig. 11).



Figure 10: Magnetic boards for building structure and heaters simulation.

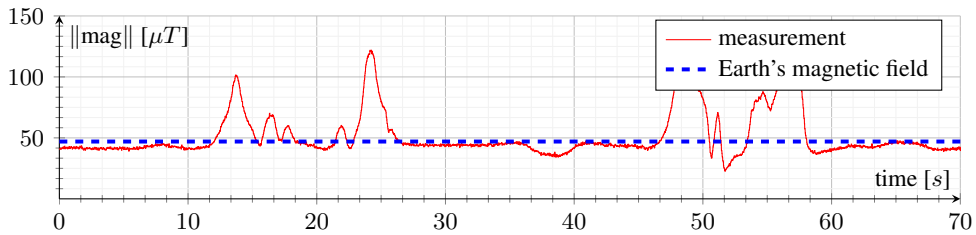


Figure 11: Magnitude of magnetic field measurements and Earth's magnetic field during our simulation with magnetic boards.

The Table 2 shows some statistics on External Magnetic Field Magnitude (EMFM). When we do not consider white magnetic boards, magnitude of magnetic field is not totally equal to the magnitude of Earth's magnetic field, so perturbations cannot be entirely omitted. If we add magnetic boards, a difference between the two magnitudes can be clearly observed (column 2). In average, 26.5% of the time, magnetic perturbations have a magnitude higher than  $> 5\mu T$  and they not exist if we remove magnetic boards.

Table 2: Statistics on Magnitude of Magnetic Field with low and high magnetic perturbations

	EMFM ( $\mu T$ )	EMFM Estimated ( $\mu T$ )	Ratio	STD ( $\mu T$ )	$> 0.5\mu T$	$> 1.5\mu T$	$> 5\mu T$
High	29.57	18.61	1.65	43.09	46.7%	31.2%	26.5%
Low	7.12	5.18	1.40	1.99	13.0%	0.2%	0.0%

#### 6.4. Different Devices

Measurements have been recorded with 3 different smartphones from 2 manufacturers. The 3 smartphones used are a LG Nexus 5, an iPhone 5 and an iPhone 4S. We implemented a log



application<sup>4</sup> for Android and iOS. Table 3 summarizes sensors specifications for the 3 devices.

Table 3: Sensors specifications with the max. sampling rate

	Accelerometer	Gyroscope	Magnetometer
iPhone 4S	STMicro STM33DH 100Hz	STIMicro AGDI 100Hz	AKM 8975 40Hz
iPhone 5	STMicro LIS331DLH 100Hz	STIMicro L3G4200D 100Hz	AKM 8963 25Hz
Nexus 5	InvenSense MPU6515 200Hz	InvenSense MPU6515 200Hz	AKM 8963 60Hz

For the purpose of aligning timestamps of magnetic field and gyroscope data with data obtained from accelerometer, we used a linear extrapolation. In order to keep the focus on a real-time process, interpolation is not allowable here. We choose to align data at 100Hz. Moreover, for each trial, we chose to process 31 algorithms at 4 sampling rates and with 7 different calibrations, that is a total of more than 110 000 tests and 804 millions quaternions compared.

#### 6.5. Common Basis of Comparison and Reproducibility

To ensure a reasonably fast convergence of algorithms, we initialize the first quaternion (for estimation algorithms) using the first measurement of accelerometer and the first measurement of magnetometer. In addition, we discard the first five seconds from our results, to allow time for filter to converge.

Most smartphone APIs (including Nexus 5 and iPhones) provide both calibrated and uncalibrated data from magnetometer and gyroscope<sup>5</sup>, and only uncalibrated data from accelerometer. Calibration phases can be triggered by the Android operating system at anytime. However, we notice that the gyroscope bias is removed during static phases and the magnetometer is calibrated during the drawing of an infinity symbol. For iOS devices, magnetometer calibration must be explicitly triggered by the user. The exact calibration algorithms embedded in both iOS and Android are not disclosed so we consider them as “black-boxes”.

To investigate the impact of calibration, we also developed a custom calibration procedure: every morning, we applied our own implementation of the calibration based on Bartz et al. [15] to remove soft and hard iron distortions from magnetometer and based on Frosio et al. [11] for the accelerometer. In addition, for all calibrations we applied a scale to adjust magnitude to  $9.8m.s^{-2}$  for accelerometer and Earth’s magnetic field magnitude for magnetometer. For the gyroscope, we simply remove the bias by subtracting measured values in each axis during static phases.

The precision error is reported using the Mean Absolute Error (MAE) on the Quaternion Angle Difference (QAD) [46]. It allows to avoid the use of Euler angles with the gimbal-lock problem. The formula of QAD is defined by:

$$\theta = \cos^{-1}(2\langle q1, q2 \rangle^2 - 1). \quad (20)$$

Since the accuracy of the system that provides the ground truth is  $\pm 0.5^\circ$ , we consider that two algorithms exhibiting differences in precision lower than  $0.5^\circ$  rank similarly.

<sup>4</sup><https://github.com/tyrex-team/senslogs>

<sup>5</sup>not from iOS API

## 7. Results and Discussions

We made available the whole benchmark including the 110000+ of 2-minute results and the 126 datasets at: <http://tyrex.inria.fr/mobile/benchmarks-attitude>. Tests can thus be reproduced. This benchmark makes it possible to evaluate new filters over a common ground truth, and to compute additional analytics like e.g. precision errors using Euler angles. In this Section we report on a few discussions, backed by aggregated views on a fraction of the obtained results.

### 7.1. Importance of Calibration

We tested attitude estimation algorithms in 6 different situations where magnetometer, gyroscope and accelerometer are either calibrated or not. Table 4 presents the results, showing that precision is impacted in the same way with all algorithms.

Table 4: Precision of attitude estimation according to calibration with all motions

	Mag: No Gyr: No Acc: No	Mag: Yes Gyr: No Acc: No	Mag: Yes Gyr: No Acc: Yes	Mag: Yes Gyr: Yes Acc: No	<b>Mag: Yes Gyr: Yes Acc: Yes</b>	Mag: OS Gyr: OS* Acc: No
<i>Choukroun</i>	95.1°	16.5°	16.5°	9.9°	<b>10.0°</b>	17.3°
<i>Fourati</i>	83.7°	15.6°	15.5°	10.3°	<b>10.4°</b>	16.3°
<i>Madgwick</i>	77.5°	18.2°	18.2°	8.1°	<b>8.1°</b>	17.7°
<i>Renaudin</i>	82.2°	19.5°	19.5°	8.0°	<b>8.1°</b>	18.1°
<i>Ekf</i>	79.8°	19.4°	19.4°	7.9°	<b>8.0°</b>	18.2°
<i>MichelEkfF</i>	82.0°	20.1°	20.1°	6.9°	<b>7.0°</b>	18.2°
<i>MichelObsF</i>	82.1°	13.6°	13.5°	5.9°	<b>5.9°</b>	15.1°

\* Not available for iOS devices

In a context free from magnetic perturbations, the magnitude of uncalibrated magnetic field is about  $350\mu T$ . This is why it is impossible to estimate attitude if calibration of hard iron distortions has not been done before. The gyroscope calibration phase is mostly important during periods with no update from accelerometer and magnetometer values. If gyroscope is not calibrated, integration drift will grow from  $5^\circ.\text{min}^{-1}$  to  $20^\circ.\text{min}^{-1}$ . We observe that accelerometer calibration does not significantly improve the precision of attitude estimation for the considered datasets. The way we performed calibration (see §6.5) provides a significantly better precision in attitude estimation than the calibration performed by device-embedded algorithms.

### 7.2. The Difficulty with Noise for Kalman Filters

KFs are often used in the general domain of attitude estimation where white noises naturally model physical sensors noise. We know from theory that KF converge when the smartphone is static and magnetometer values correspond to Earth's magnetic field. However, this is not the case in the context that we consider. The magnitude of external accelerations and magnetic perturbations experienced by the smartphone is much higher than its physical sensors noise.

With values for sensors noise experimentally extracted (as commonly found in the literature), filters yield high precision errors and diverge quickly. This is shown in Table 5 where

*ChoukrounSn*, *RenaudinSn* and *EkfSn* respectively denote the algorithms initialized with values for noise measured from physical sensors.

Table 5: Precision of attitude estimation according to sensor noises without magnetic perturbations.

	AR	Texting	Phoning	Front Pocket	Back Pocket	Swinging	Running Pocket	Running Hand
<i>Choukroun</i>	5.1°	4.3°	4.4°	4.8°	4.6°	6.3°	7.9°	21.1°
<i>ChoukrounSn</i>	15.6°	20.6°	15.9°	17.8°	16.9°	11.5°	17.6°	35.2°
<i>Ekf</i>	4.5°	4.0°	3.7°	4.6°	4.6°	5.9°	8.2°	16.8°
<i>EkfSn</i>	44.0°	57.8°	36.1°	20.6°	30.8°	29.1°	23.3°	54.1°
<i>Renaudin</i>	4.5°	3.8°	3.7°	4.7°	4.6°	6.1°	8.5°	17.9°
<i>RenaudinSn</i>	20.8°	18.5°	17.8°	17.3°	18.4°	11.4°	17.4°	36.5°

KFs can still give better results in this context, provided we adapt the “noise values” in a way that does not reflect anymore physical sensors noise, but that instead takes into account the relative importance of sensor measurements in this context. Gyroscope measurements are not impacted by external accelerations nor magnetic perturbations. In our context, we observed that giving more importance to gyroscope measurements (compared to magnetometer and accelerometer measurements) yields better results (despite convergence being a bit longer). Experimentally we obtained the best results (See *Choukroun*, *Renaudin* and *Ekf* in Table 5) by using the following “noise values”:  $\sigma_{\text{acc}} = 0.5$ ,  $\sigma_{\text{mag}} = 0.8$ ,  $\sigma_{\text{gyr}} = 0.3$  for all KFs<sup>6</sup>.

Applying KFs remains non trivial, because the notion of noise to model in this context goes much beyond the setting in which initial KFs were designed.

Observers and KFs exhibit similar results for low to moderate external accelerations. For higher accelerations (typically found when swinging and running), observers were found to improve precision. This is especially the case for *MichelObsF* that outperforms *MichelEkfF*, as shown in Table 7.

### 7.3. Bias Consideration

Many existing filters try to estimate sensors bias and in particular gyroscope bias. For example, in observers, typical procedures use residuals between reference and estimation to estimate bias (e.g. [36, 38]). In our setting however, residuals do not only originate from gyroscope bias but also from magnetic perturbations and external accelerations. Furthermore, a calibration phase is performed in a previous stage.

We can thus wonder how useful classical bias estimation techniques are in our setting. Table 6 compares the results obtained with two variants of each filter: one with bias estimation and one without. We observe that bias estimation seems unnecessary in our context of study. We remark however that bias estimation can still be useful for situations where the gyroscope is not calibrated. In this particular case, precision of attitude estimation is improved with bias estimation, provided external accelerations remain small.

<sup>6</sup>except for the Linear KF from Choukroun where we adapt these values for the linearized model:  $\sigma_{\text{acc}} = 0.3$ ,  $\sigma_{\text{mag}} = 0.3$ ,  $\sigma_{\text{gyr}} = 0.5$

Table 6: Precision of attitude according to bias estimation without magnetic perturbations.

	AR	Texting	Phoning	Front Pocket	Back Pocket	Swinging	Running Pocket	Running Hand
<i>Madgwick</i>	4.8°	4.1°	4.6°	4.9°	5.0°	5.8°	7.1°	16.5°
<i>MadgwickB</i>	5.2°	4.8°	5.4°	5.8°	6.2°	11.5°	10.5°	19.8°
<i>Mahony</i>	5.0°	4.6°	4.2°	5.1°	5.2°	7.5°	7.9°	11.2°
<i>MahonyB</i>	5.6°	4.9°	4.7°	6.1°	5.7°	9.9°	13.1°	26.4°
<i>Renaudin</i>	4.5°	3.8°	3.7°	4.7°	4.6°	6.1°	8.5°	17.9°
<i>RenaudinBG</i>	4.5°	3.7°	3.8°	4.5°	4.6°	6.9°	12.8°	19.3°

#### 7.4. Behaviors during Typical Smartphone Motions

Table 7 compares the precision of attitude estimation for each motion. We observe a negative correlation between magnitude of external accelerations and precision of attitude estimation. This is verified for all algorithms.

Table 7: Precision of Attitude estimation according to typical motions without magnetic perturbations.

	AR	Texting	Phoning	Front Pocket	Back Pocket	Swinging	Running Pocket	Running Hand
<i>OS</i>	7.1°	5.9°	5.8°	12.7°	13.2°	20.3°	24.4°	62.0°
<i>Choukroun</i>	5.1°	4.3°	4.4°	4.8°	4.6°	6.3°	7.9°	21.1°
<i>Madgwick</i>	4.8°	4.1°	4.6°	4.9°	5.0°	5.8°	7.1°	16.5°
<i>Mahony</i>	5.0°	4.6°	4.2°	5.1°	5.2°	7.5°	7.9°	11.2°
<i>Fourati</i>	4.8°	4.0°	4.4°	4.6°	4.8°	5.3°	6.3°	6.6°
<i>FouratiExtacc</i>	4.9°	5.4°	4.7°	6.0°	5.7°	8.4°	12.2°	29.1°
<i>Sabatini</i>	4.5°	4.0°	3.7°	4.6°	4.6°	5.9°	8.2°	16.8°
<i>SabatiniExtacc</i>	4.5°	4.5°	4.0°	5.5°	5.0°	9.7°	15.0°	33.5°
<i>Renaudin</i>	4.5°	3.8°	3.7°	4.7°	4.6°	6.1°	8.5°	17.9°
<i>RenaudinExtacc</i>	4.5°	3.8°	3.7°	4.8°	4.8°	6.0°	8.0°	30.3°
<b><i>MichelObsF</i></b>	<b>4.8°</b>	<b>3.9°</b>	<b>4.4°</b>	<b>4.6°</b>	<b>4.8°</b>	<b>5.3°</b>	<b>6.3°</b>	<b>6.6°</b>
<b><i>MichelEkfF</i></b>	<b>4.5°</b>	<b>4.0°</b>	<b>3.7°</b>	<b>4.6°</b>	<b>4.6°</b>	<b>6.0°</b>	<b>8.2°</b>	<b>16.8°</b>

We observe that two algorithms stand out in terms of precision: *Fourati* and *MichelObsF*.

Table 1 presents the left term  $\mu$  of detector (Eq. (15)) and the magnitude of external accelerations (extracted from the ground truth). We observe that the two series are highly correlated ( $\rho > 99\%$ ). This suggests that it is possible to reasonably distinguish periods with high external accelerations.

We also observe that filters which take external accelerations into account do not yield better precision than others. This can be explained by long periods of perturbations without the smartphone becoming completely static. Moreover, filters are very sensitive to false detections which make them quickly diverge. An interesting perspective for the further development of filters in

this context would be to investigate how to better leverage the detection of periods with high external accelerations in order to improve precision of attitude estimation during those periods (Table 7).

### 7.5. Impact of Magnetic Perturbations

We tested different motions in the presence/absence of magnetic perturbations (§6.3). Results are shown in Table 8.

Table 8: Precision of attitude estimation according to typical motions with magnetic perturbations.

	AR	Texting	Phoning	Front Pocket	Back Pocket	Swinging
<i>OS</i>	29.0°	24.4°	21.1°	19.8°	37.9°	19.2°
<i>Madgwick</i>	18.2°	7.5°	7.8°	8.1°	9.4°	10.0°
<i>Mahony</i>	31.8°	26.1°	30.0°	19.9°	13.9°	26.6°
<i>Renaudin</i>	17.1°	7.0°	7.6°	8.9°	8.7°	9.5°
<i>RenaudinExtmag</i>	16.8°	6.4°	7.3°	8.4°	8.4°	8.9°
<i>Sabatini</i>	16.6°	7.0°	8.0°	8.9°	8.6°	10.1°
<i>SabatiniExtmag</i>	14.6°	8.7°	8.9°	6.4°	8.4°	9.0°
<i>MichelObs</i>	32.1°	14.0°	16.4°	14.6°	8.8°	19.1°
<i>MichelObsExtmag</i>	18.0°	11.9°	11.4°	7.4°	8.8°	10.3°
<i>MichelObsExtmagWt</i>	15.5°	9.2°	9.7°	7.1°	7.3°	10.1°
<b><i>MichelObsF</i></b>	<b>10.6°</b>	<b>5.4°</b>	<b>6.0°</b>	<b>5.8°</b>	<b>7.1°</b>	<b>7.7°</b>
<i>MichelEkf</i>	16.6°	7.0°	8.0°	8.9°	8.6°	10.1°
<i>MichelEkfExtmag</i>	14.2°	8.9°	9.0°	5.5°	8.6°	9.2°
<i>MichelEkfExtmagWt</i>	12.3°	6.3°	7.2°	5.3°	8.5°	8.7°
<b><i>MichelEkfF</i></b>	<b>10.8°</b>	<b>5.3°</b>	<b>5.5°</b>	<b>5.7°</b>	<b>10.3°</b>	<b>7.5°</b>

We observe that filters which implement a magnetic perturbations detector do not systematically exhibit a better behavior when compared to their native variant. However, if we extend them with our technique for enforcing minimal durations (See Fig. 7), precision is systematically improved when compared to their native variant.

*RenaudinExtmag* implements a different detector for magnetic perturbations based on variances which improves *Renaudin*. However, *RenaudinExtmag* is very sensitive to false detections because Earth’s magnetic field is known only during the initial phase.

We observe that the two variants of our technique (*MichelEkfF* and *MichelObsF*) gives better precisions for all motions except for the back pocket motion in the case of *MichelEkfF*. *MichelObsF* thus stands out: it provides a significantly better precision during periods of magnetic perturbations even with high accelerations. We also notice that precision is improved regardless of the motion.

Figure 12 illustrates the relative improvements in precision brought by the respective components of our new technique presented in §5, in the case of yaw.

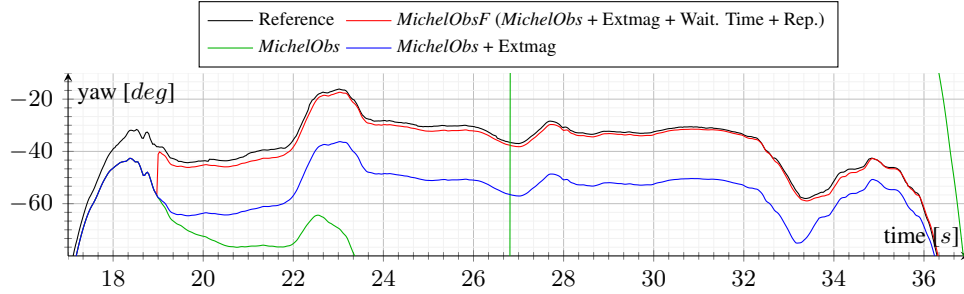


Figure 12: Sample run of the reprocessing technique (red) when a magnetic perturbation occurs, in comparison to ground truth (black) and earlier techniques.

### 7.6. Pitch and Roll in Augmented Reality mode

In many applications which use attitude, the goal is to provide the most accurate estimation. In Augmented Reality (AR), for example, it is interesting to use algorithms which provide good estimations of only two of the three Euler angles in order to enhance rendering. We defined pitch and roll as the rotations around y-axis and z-axis. As Euler angles suffer from singularity and this singularity is a problem when the smartphone is held in AR mode we apply a rotation of  $90^\circ$  around x-axis then another rotation of  $90^\circ$  around z-axis. The smartphone is now considered in “Camera landscape” frame, as shown in Figure 13.

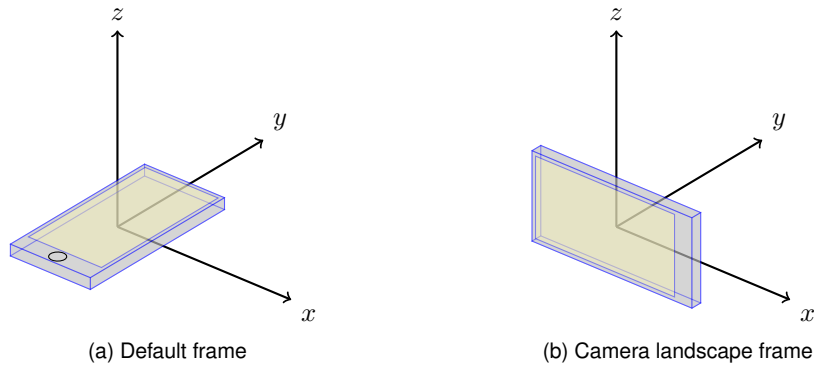


Figure 13: From default frame to camera landscape frame (rotation of  $90^\circ$  around x-axis then another rotation of  $90^\circ$  around z-axis)

Table 9 shows algorithms precision during AR motions in a highly perturbed magnetic environment. During motions with low external accelerations, which this is especially the case for AR motions, we can use a specific technique for limiting the impact of magnetic perturbations. We use a cross product between the magnetometer and the accelerometer as yielding our observation vector. This allows algorithms to be more robust to errors from magnetometer measurements on pitch and roll angles [37]. Algorithms using this technique yield better precision than others. For example, *FouratiMartin* is twice accurate than its classical version. The same behavior is observed for *MahonyMartin* algorithm which is 5 times more accurate than the

Table 9: Precision of attitude estimation according to Augmented Reality motions with magnetic perturbations.

	QAD	Yaw	Pitch	Roll
<i>Choukroun</i>	24.8°	22.1°	<b>6.0°</b>	<b>6.6°</b>
<i>Fourati</i>	32.1°	31.5°	<b>2.3°</b>	<b>3.0°</b>
<i>FouratiMartin</i>	21.7°	21.3°	<b>1.4°</b>	<b>1.6°</b>
<i>FouratiMartinF</i>	10.2°	9.8°	<b>1.4°</b>	<b>1.6°</b>
<i>Madgwick</i>	18.2°	17.1°	<b>3.2°</b>	<b>3.1°</b>
<i>Mahony</i>	31.8°	28.9°	<b>6.9°</b>	<b>7.9°</b>
<i>MahonyMartin</i>	14.4°	14.1°	<b>1.1°</b>	<b>1.4°</b>
<i>MahonyMartinF</i>	10.1°	9.8°	<b>1.2°</b>	<b>1.5°</b>
<i>Martin</i>	34.4°	34.1°	<b>0.9°</b>	<b>1.2°</b>
<i>MichelEkfF</i>	10.8°	10.5°	<b>1.1°</b>	<b>1.4°</b>
<i>MichelObsF</i>	10.7°	10.3°	<b>1.3°</b>	<b>1.6°</b>
<i>OS</i>	29.0°	28.9°	<b>1.1°</b>	<b>1.2°</b>
<i>RenaudinExtmag</i>	16.8°	16.0°	<b>2.6°</b>	<b>2.9°</b>
<i>SabatiniExtmag</i>	14.6°	14.3°	<b>1.7°</b>	<b>1.9°</b>

*Mahony* version. It is also possible to equip our filter with this technique in order to enhance overall results (*MahonyMartinF* and *FouratiMartinF*).

It should also be noticed that embedded algorithms have a good behavior in this specific context. It is likely that they use a similar technique.

### 7.7. Comparison with Device-Embedded Algorithms

Table 10 shows algorithms precision depending on the smartphone used. For each algorithm, we observe rather similar results across the different smartphones.

Table 10: Precision according to device with all motions and with/without magnetic perturbations.

	iPhone 4S	iPhone 5	LG Nexus 5
<i>OS</i>	23.6°	28.6°	12.7°
<i>Choukroun</i>	8.6°	10.4°	10.9°
<i>Mahony</i>	10.8°	15.2°	16.6°
<i>Madgwick</i>	7.1°	8.7°	8.6°
<i>Ekf</i>	6.7°	8.7°	8.5°
<b><i>MichelObsF</i></b>	<b>5.4°</b>	<b>6.5°</b>	<b>5.9°</b>
<b><i>MichelEkfF</i></b>	<b>5.6°</b>	<b>8.3°</b>	<b>7.0°</b>

We also observe that all algorithms exhibit a similar or better precision compared to OS-embedded algorithms. We know that this is at least partially due to a bad calibration (especially for iPhones). Finally, we notice that *MichelEkfF* and *MichelObsF* provide much better precision with all smartphones. Specifically, on 126 tests, we noticed that they improve the precision of OS-embedded algorithms on iPhone 4S by 300%, iPhone 5 by 250% and Nexus 5 by 100%.

### 7.8. Empirical Computational Complexity

Because of smartphone’s limited resources (e.g. battery), we study to which extent improvements in precision of attitude estimation have an impact in terms of empirical computational complexity. Figure 14 summarizes the relative times spent with each algorithm, where unit time corresponds to the running time of *Mahony*. Ratios have been obtained using the offline implementations executed across all 126 datasets.

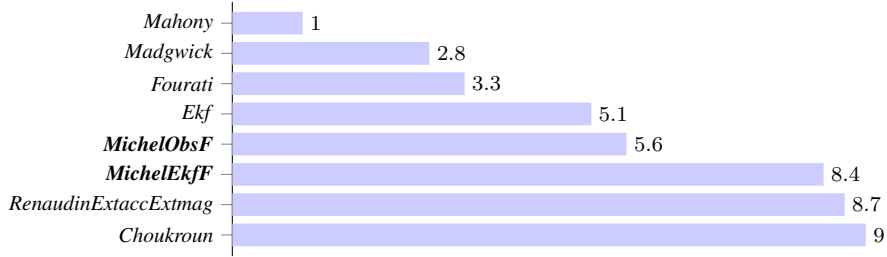


Figure 14: Relative performance in terms of CPU cost (lower is better).

We observe that all algorithms can be executed on smartphones even at much higher frequencies than current sensors capabilities (see Table 3). For example, our implementation of *Mahony* running on the Nexus 5 can output up to 45000 quaternions per second.

### 7.9. Relevant Sampling Rates

In all aforementioned results, sensors sampling rate was set to 100Hz. We studied the behavior of algorithms whenever the sampling rate varies. Table 11 presents precision according to sampling rate. We observe that results with a sampling at 100Hz and 40Hz are relatively similar, and much more precise than with lower frequencies. This suggests to implement filters with a sampling rate of 40Hz to save smartphone’s battery life, for a negligible loss in precision.

Table 11: Precision according to sampling with all motions and with/without magnetic perturbations.

	100Hz	40Hz	10Hz	2Hz
<i>Choukroun</i>	10.0°	10.1°	15.6°	34.7°
<i>Mahony</i>	14.2°	14.3°	19.7°	48.9°
<i>Madgwick</i>	8.1°	8.1°	17.3°	62.8°
<i>Ekf</i>	8.0°	8.1°	15.3°	49.5°
<b><i>MichelObsF</i></b>	<b>5.9°</b>	<b>6.0°</b>	<b>14.8°</b>	<b>52.5°</b>
<b><i>MichelEkfF</i></b>	<b>7.0°</b>	<b>7.1°</b>	<b>14.8°</b>	<b>51.3°</b>

In our specific context, we obtain a mean error of 6° using our best algorithm (*MichelObsF*). When used in an AR application with geolocation and close tracked objects, this might be enough to avoid huge offsets during rendering. This might also be suitable for a navigation application with short trips. For longer trips, the additional use of a map-matching algorithm might be considered.



## 8. Parameter Adjustment for a Balance Between Stability and Precision

In the previous section, we evaluated algorithms using parameter values as recommended by their authors (§ 4). If authors did not provide instructions on setting parameter values, we chose them empirically. In the present section, we evaluate several sets of parameters for each filter in order to observe their feasibility envelope. For the rest of the study, precision error of filters is shown in function of the stability.

### 8.1. Stability of a filter

In some specific contexts, eg. AR, rendering is very important. When the device is static, augmented point of interests should be static and not moving nor blinking. For this purpose, we added to our benchmark the stability component. The stability is also strongly related to the noise of the sensors and especially the noise of the magnetometer and accelerometer [40]. Precision error's STD cannot be used directly to know the stability of the filter. We used a moving STD with a window of  $0.1s$  which corresponds to the *moving picture rate* [47] observable by a user. Obviously, the stability measurement makes sense only when filters assumptions are met (few magnetic perturbations and few external accelerations).

### 8.2. Parameters & Algorithms

Tests have been conducted with different sets of parameter values for each algorithm on a systematic basis. Parameter values have been chosen empirically to cover a spectrum of possibilities and show the trade-off between the stability and the precision error. We recall below the set of parameters of each filter, and for each parameter we give the set of parameter values tested. We consider the cartesian product of all sets of parameter values. We indicate the size of the cartesian product (i.e. the number of configurations tested) next to each filter name.

$$\begin{aligned} \sigma_{acc} &= [0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5] \\ \text{Choukroun (125)} \quad \sigma_{mag} &= [0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5] \\ \sigma_{gyr} &= [0.3 \quad 0.4 \quad 0.5 \quad 0.6 \quad 0.7] \end{aligned}$$

$$\begin{aligned} \sigma_{acc} &= [0.3 \quad 0.4 \quad 0.5 \quad 0.6 \quad 0.7] \\ \text{EKF (125)} \quad \sigma_{mag} &= [0.6 \quad 0.7 \quad 0.8 \quad 0.9 \quad 1.0] \\ \sigma_{gyr} &= [0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5] \end{aligned}$$

$$\begin{aligned} \beta &= [0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5] \\ \text{Fourati (125)} \quad K_a &= [1 \quad 1.5 \quad 2 \quad 2.5 \quad 3] \\ K_m &= [0.5 \quad 1 \quad 1.5 \quad 2 \quad 2.5] \end{aligned}$$

$$\begin{aligned} \beta &= [0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5] \\ \text{FouratiExtAcc (625)} \quad K_a &= [1 \quad 1.5 \quad 2 \quad 2.5 \quad 3] \\ K_m &= [0.5 \quad 1 \quad 1.5 \quad 2 \quad 2.5] \\ \gamma_{acc} &= [0.1 \quad 0.3 \quad 0.5 \quad 1 \quad 3 \quad 5] \end{aligned}$$

$$\beta = [0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5]$$

**FouratiMartin (125)**  $K_a = [1 \quad 1.5 \quad 2 \quad 2.5 \quad 3]$   
 $K_c = [0.5 \quad 1 \quad 1.5 \quad 2 \quad 2.5]$

**Madgwick (28)**  $\beta = [0 \quad 0.05 \quad 0.01 \quad 0.02 \quad \dots \quad 0.24 \quad 0.25]$

$$\beta = [0.5 \quad 0.75 \quad 1 \quad 1.25 \quad 1.5]$$

**Mahony (125)**  $K_a = [0.5 \quad 0.75 \quad 1 \quad 1.25 \quad 1.5]$   
 $K_m = [0.5 \quad 0.75 \quad 1 \quad 1.25 \quad 1.5]$

$$\beta = [0.1 \quad 0.15 \quad 0.2 \quad 0.25 \quad 0.3]$$

**MahonyMartin (125)**  $K_a = [0.25 \quad 0.5 \quad 0.75 \quad 1 \quad 1.25]$   
 $K_c = [0.5 \quad 0.75 \quad 1 \quad 1.25 \quad 1.5]$

$$\beta = [0.2 \quad 0.3 \quad 0.4]$$

$$K_a = [1.5 \quad 2 \quad 2.5]$$

$$K_m = [0.5 \quad 1 \quad 1.5 \quad 2]$$

**MichelObsF (1944)**  $\gamma_{mag} = [12 \quad 13 \quad 14 \quad 15 \quad 16 \quad 17]$   
 $t_{mag, nopert} = [1 \quad 2 \quad 3]$   
 $t_{mag, rep} = [2 \quad 3 \quad 4]$

$$\sigma_{acc} = [0.4 \quad 0.5 \quad 0.6]$$

$$\sigma_{mag} = [0.7 \quad 0.8 \quad 0.9]$$

**SabatiniExtMag (486)**  $\sigma_{gyr} = [0.2 \quad 0.3 \quad 0.4]$   
 $\gamma_{mag} = [12 \quad 13 \quad 14 \quad 15 \quad 16 \quad 17]$   
 $\gamma_{\theta} = [8 \quad 10 \quad 12]$

For example, for *MichelObsF* we tested 1944 ways of setting initial parameter values, given by all the possible combinations of the values described above for each parameter.

### 8.3. Augmented Reality: Parameter Adjustment for a Balance Between Stability and Precision

We have set up an online tool<sup>7</sup> to visualize the spectrum of possibilities for each algorithm. Figures 15 and 16 show the range of possibilities in terms of stability and precision error for a selection of algorithms during AR trials. Each dot of the graph corresponds to the couple (precision error, stability) for one set of parameter values.

In the case of low magnetic perturbations (Fig. 15), we observed a common behavior for kalman filters, whose best results are obtained when  $\sigma_{mag} \approx 2 \sigma_{acc}$  and  $\sigma_{acc} \approx 2 \sigma_{gyr}$ . A similar observation holds with the weights of observers (instead of variances – thus with inverted ratios). Ratios found here between the sensors are directly related to sensor noises from the Allan variance [40].

<sup>7</sup><http://tyrex.inria.fr/mobile/benchmarks-attitude/#comparison-parameters>

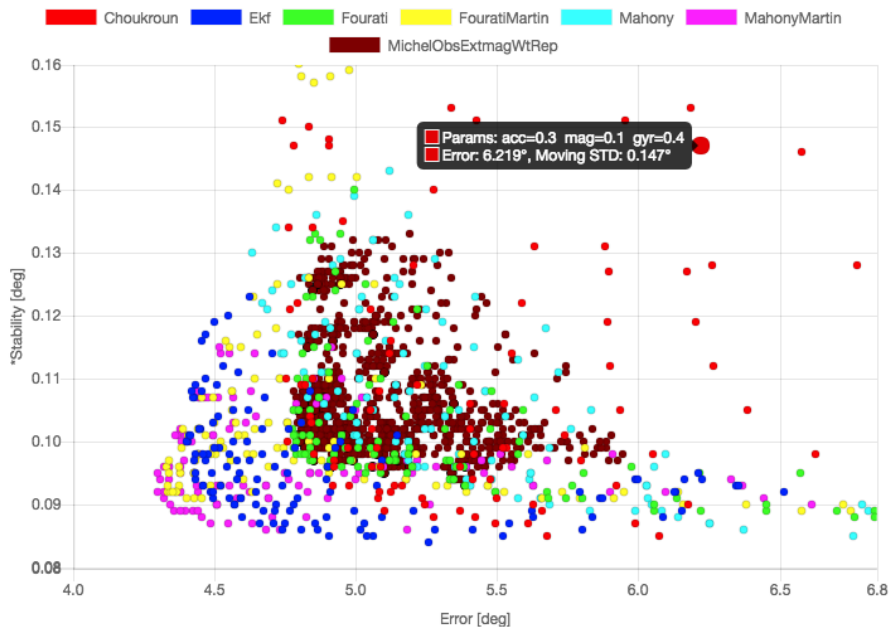


Figure 15: Spectrum of possibilities in terms of stability/precision in AR with few magnetic perturbations



Figure 16: Spectrum of possibilities in terms of stability/precision in AR with high magnetic perturbations

In the case of high magnetic perturbations (Fig. 16), algorithms without detector exhibit a common behavior: their best results are obtained when  $\sigma_{acc} < \sigma_{mag}$  and  $\sigma_{gyr} \ll \sigma_{mag}$ . That behavior shows the impact of magnetic field measurements on the overall results. For algorithms with a magnetic perturbations detector,  $\sigma_{gyr} \ll \sigma_{mag}$  is also true, but  $\sigma_{acc} \approx 0.75 \sigma_{mag}$ .

To conclude, we observed that some filters provide a better feasibility envelope, especially in the presence of magnetic perturbations (*MichelObsExtmagWtRep*). Also, it is preferable to use a filter which deals with magnetic perturbations, this avoids to create a filter with adaptative parameters in function of the magnetic context.

Moreover, this tool allows us to confirm that parameter values chosen empirically in § 4 are among those that yield the best results in this study.

## 9. Conclusions

We investigate the use of attitude estimation algorithms in the particular context of pedestrians using commodity smartphones. We propose a benchmark for evaluating and comparing the precision of attitude estimations during typical smartphone motions with and without magnetic perturbations. For the first time, our experiments shed light on the relative impacts of calibrations, parameters, noises, bias, motions, magnetic perturbations, and sampling rates when estimating attitude on smartphones. We go further in the study in the particular context of attitude estimation during augmented reality motions. An online tool based on the benchmarks has been released in order to help developers in choosing the right filter and appropriate parameter values in function of the expected motions, device, and magnetic perturbations. We also comment on lessons learned from our experiments for further research on the topic. In all cases, we recommend developers to use custom calibration and algorithms in replacement of those provided by smartphone's OS. Our algorithm "*MichelObsF*" provides significant gains in precision when estimating attitude in the presence of magnetic perturbations. In the absence of magnetic perturbations, it offers the same precision than the most precise algorithms.

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