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Semiconductor laser Markov models in the micro-canonical, canonical and grand-canonical ensembles

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Semiconductor laser Markov models in the micro-canonical, canonical and grand-canonical ensembles

A semiconductor laser Markov picture

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IES, Montpellier University

11/07/17





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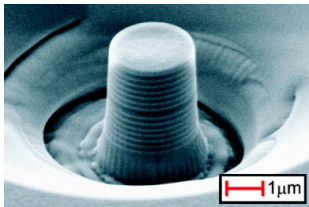
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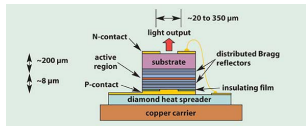
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Micropillar with embedded colloidal CdSe/ZnS quantum dots

- $\approx 1 \mu m^2$
- $\approx 10 \mu W$



Vertical-Cavity Surface-Emitting Laser

- $\approx 0.01 mm^2$
- $\approx 1 mW$



High power VECSEL

- $\approx 1 cm^2$
- $\approx 1.5 kW$



Photonics transitions

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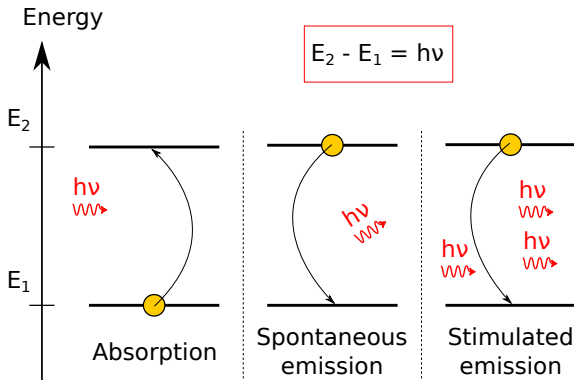
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Photonics transitions

Definition

- Absorption
 $\left(\frac{dN_2}{dt}\right)_{abs} = BN_1m$
- Stimulated emission
 $\left(\frac{dN_2}{dt}\right)_{e.st} = -BN_2m$
- Spontaneous emission
 $\left(\frac{dN_2}{dt}\right)_{e.sp} = AN_2$



Semiconductor laser description

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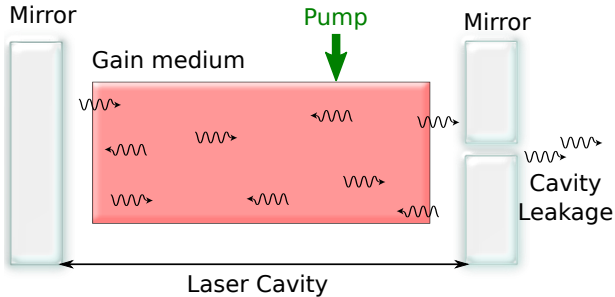
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Laser cavity scheme

Definition

- Laser = Gain medium + laser cavity
- population inversion by pump
- Cavity leakage : laser main output



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Dual mode laser

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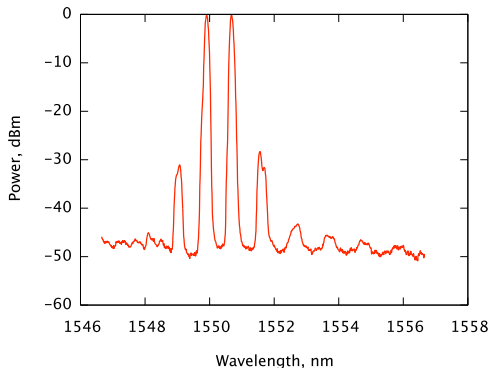
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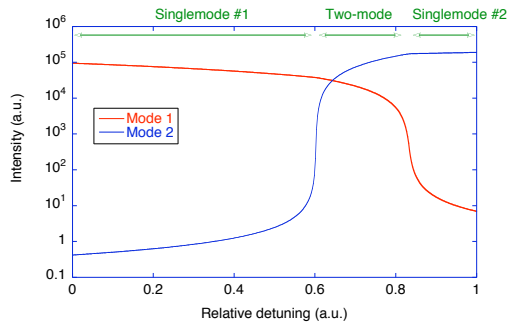
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Multimode laser



Modal competition

[2] L. Chusseau et al., "Four-sections semiconductor two-mode laser for THz generation," *Proc. SPIE* **6343**, 2006, pp. 1097–1106.



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Quantum simulations

- QED
- Few particles only
- Exact simulations
- Single-mode

Monte-Carlo simulations

- Trade-off between two approaches
- Small to big systems
- Multimode possibility
- Noise intrinsic in system

Rates equations resolution

- Analytic resolution
- No particles number limits
- Multimode possibility
- Fluctuations: Langevin noise

[3] M. Elk, "Numerical studies of the mesomaser," *Phys. Rev. A*, **54**, no. 5, pp. 4351–4358, 1996.

[4] G. P. Puccioni et al., "Stochastic Simulator for modeling the transition to lasing," *Opt. Express*, **23**, no. 3, p. 2369, 2015.

[5] A. Lebreton et al., "Stochastically sustained population oscillations in high- β nanolasers," *New J. Phys.*, **15**, 2013.

[6] L. a. Coldren et al., "Diode Lasers and Photonic Integrated Circuits", *Optical En.*, 1995.



Markov chains

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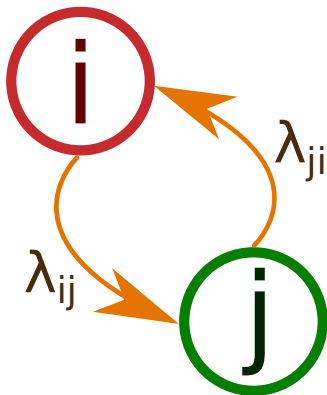
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Markov chain simple example

Algorithm 1: Gillespie algorithm : Monte-Carlo simulation by Markov chain

begin

```

1  t = 0
   while t < T do
2    random number rt = U(0, 1) for the waiting time
      Λ = ∑i ≥ 1 λi
      τ = -ln(rt) / Λ
3    random number re = U(0, 1) for the next event
      index = min i : ∑j=1i λj ≥ reΛ
4    update rates and state
      t = t + τ
  
```



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Model states

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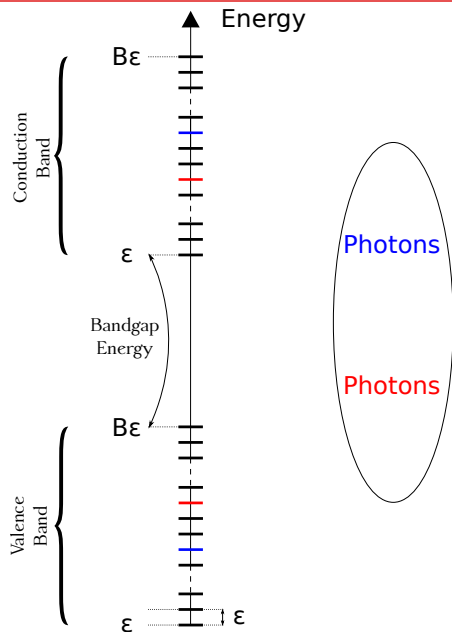
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States

- Photon reservoir
- B evenly-spaced energy states in each band
- B electrons in total
- 0 or 1 electron per level
- Photons interact with electrons only at lasing level



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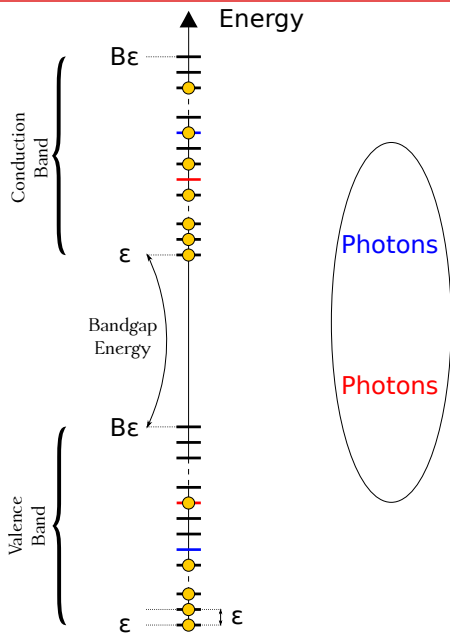
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Events

- Absorption
 $\bar{n}_c \bar{n}_v m$
- Emission
 $n_c \bar{n}_v (m + 1)$
- Cavity output
 αm
- Thermalization
- Steady State :
Cavity output = pump



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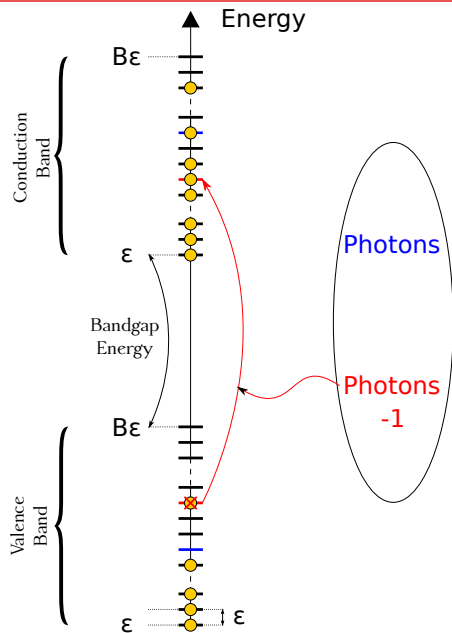
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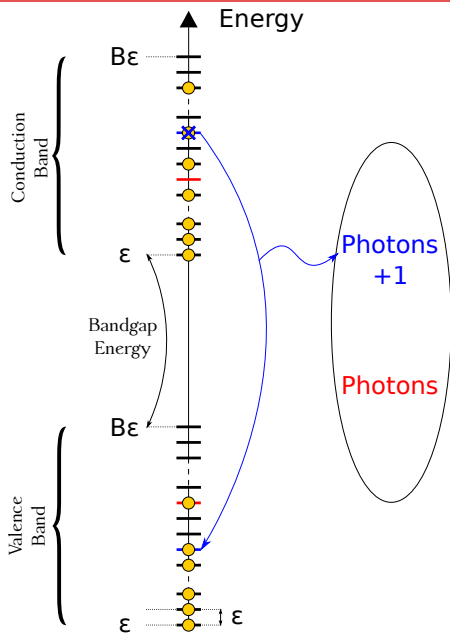
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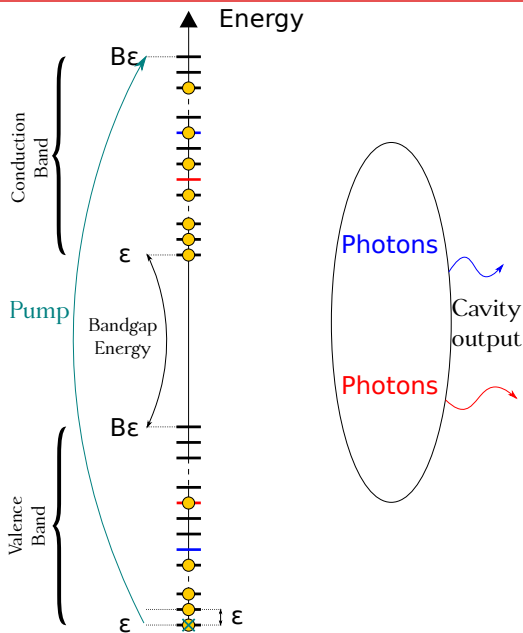
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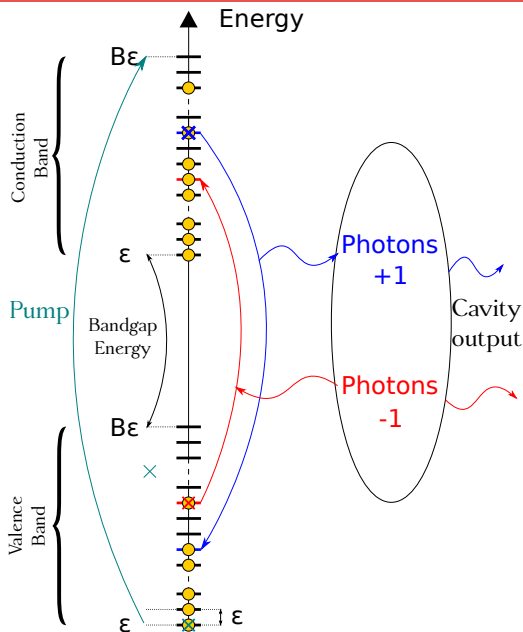
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Fermi-Dirac Distribution

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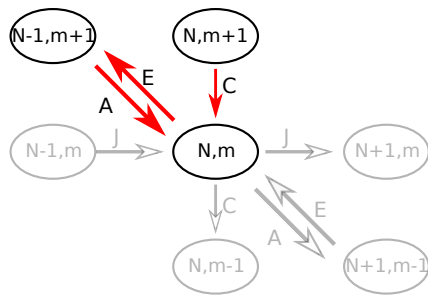
Conclusion

$$n_k = \frac{1}{e^{\frac{\epsilon}{k_B T}(k-\mu)} + 1}$$

Fermi-Dirac distribution

Problems

- Occupancy n_k and photon m are independent
- No fluctuation
- Thermal light variance \Rightarrow No laser
- Mean values calculable



$$\sum_{N=0}^B \Pi(N, m) n_k^2(N) = \sum_{N=0}^B \Pi(N, m+1) [\alpha + n_k^2(B - N)]$$

Detailed balance at m photons

$$\Pi(m) = \frac{\alpha + C_2}{C_1} \Pi(m+1)$$



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Micro-canonical bands repartition

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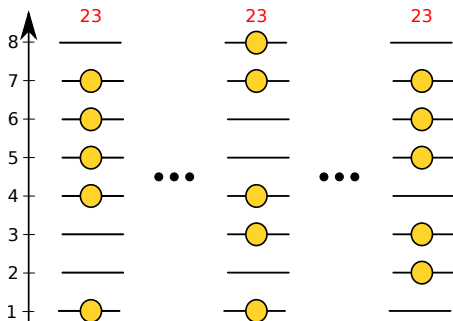
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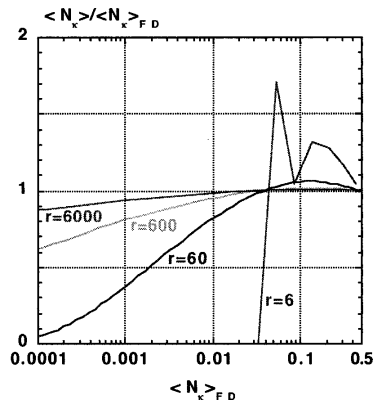
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Possible repartitions of electrons for a given energy



Ratio between Microcanonical and FD occupancies for the same energy r

$$m_{\kappa}(r) = - \sum_{i=1,2,\dots} (-1)^i W[r - i(\kappa + i/2)]$$

[7] J. Arnaud et al., "Illustration of the Fermi-Dirac statistics," *Am. J. Phys.*, **67**, no. 3, pp. 215-221, 1999.



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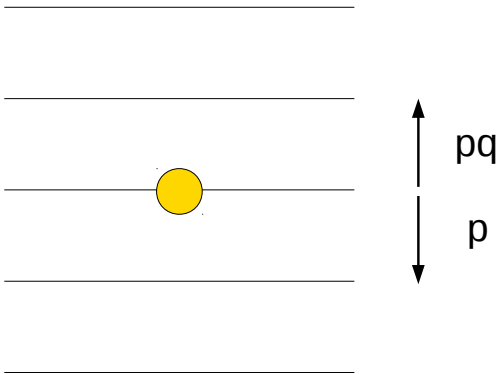
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Thermalization event

Definition

- All electron movement simulated
- $p \equiv$ electron movement speed
- Boltzmann thermalization
- $q \equiv e^{\frac{-\epsilon}{k_B T}}$



Microscopic application

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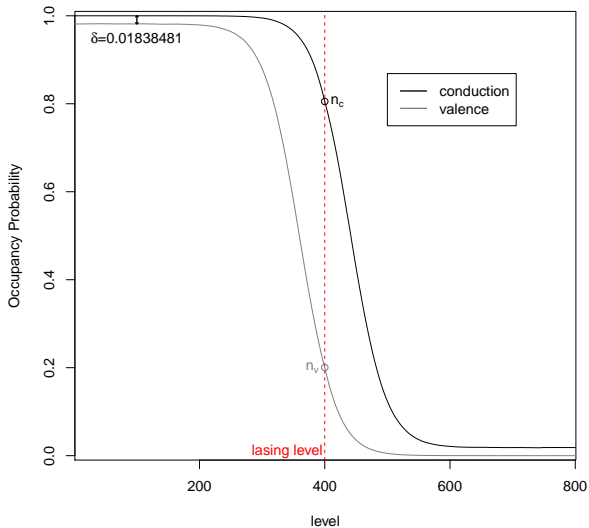
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Problems

- Occupancy shift
- Pump values biased
- $> 10^5$ thermalization events per photon event
- Very time consuming



Microscopic modal competition

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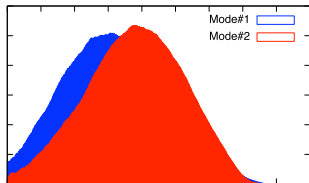
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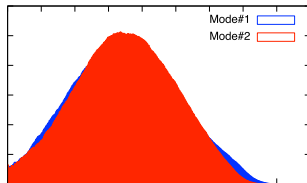
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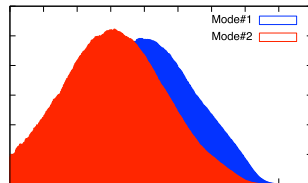
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$$\alpha_L = 0.615$$



$$\alpha_L = 0.616$$



$$\alpha_L = 0.617$$

Stable dual-mode solution exist but only a very tiny range of laser parameters

- [8] L. Chusseau et al., "Monte Carlo modeling of the dual-mode regime in quantum-well and quantum-dot semiconductor lasers," *Opt. Express*, 22, no. 5, pp. 5312–5324, 2014.



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Hypothesis

We may use canonical occupancy at laser level.

$$n_k(N, q) = \frac{Z(N-1, q)q^k}{Z(N, q)} [1 - n_k(N-1, q)]$$

Canonical Occupancy

$$Z(N, q) = \prod_{i=1}^N \frac{q^{i-1} - q^B}{1 - q^i}$$

Partition function

$$n_k(N+1, q) = \frac{(1 - q^{N+1})q^k}{q^N - q^B} [1 - n_k(N, q)]$$

Recursive occupancy formula



Canonical vs. Microscopic difference

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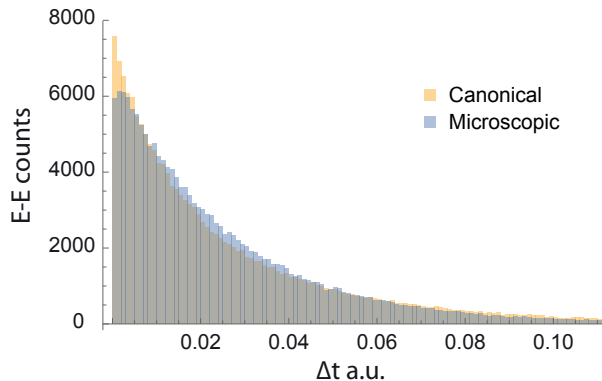
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Distributions of double emission events

Double emission events

- Canonical : Exponential distribution
- Microscopic : Non exponential distribution
- \exists Solution to restrain fast E-E within the Markov framework



Dual-mode photon statistics

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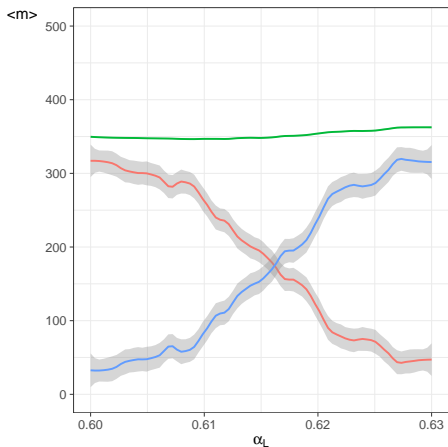
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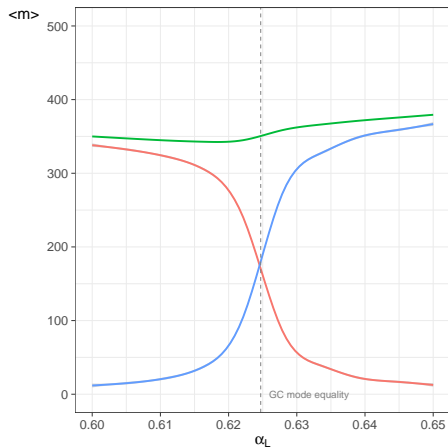
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Photon statistic for microscopic at pump rate 210

Canonical



Photon statistic for canonical at pump rate 210



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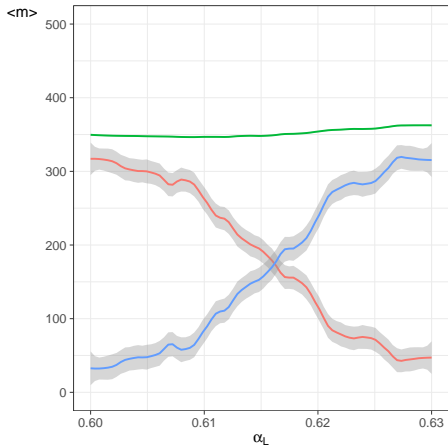
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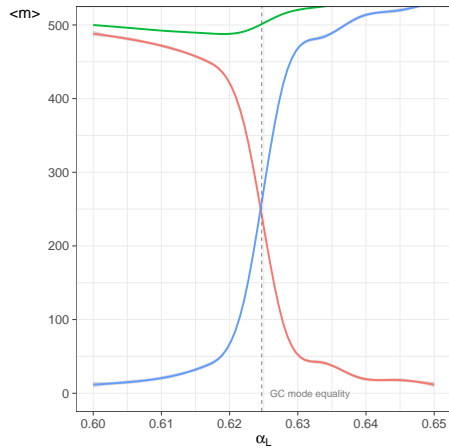
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Photon statistic for canonical at pump rate 300



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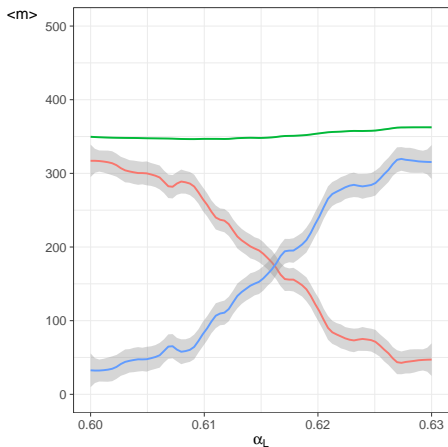
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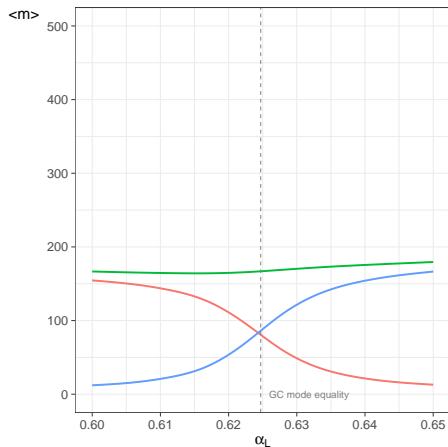
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Photon statistic for canonical at pump rate 100



Bimode photon Fano factor

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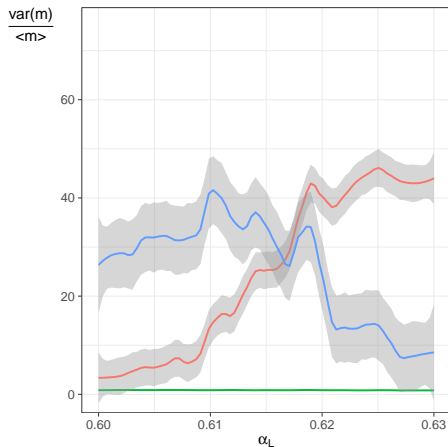
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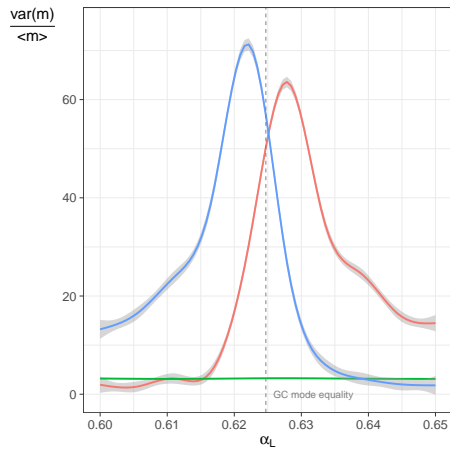
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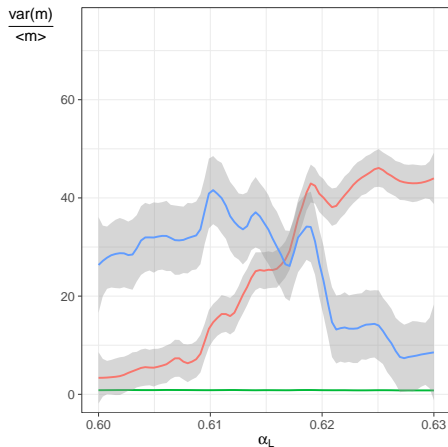
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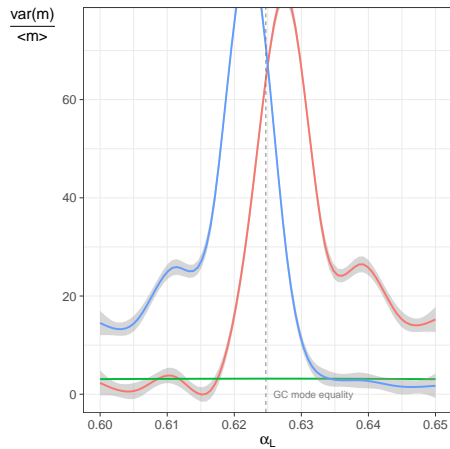
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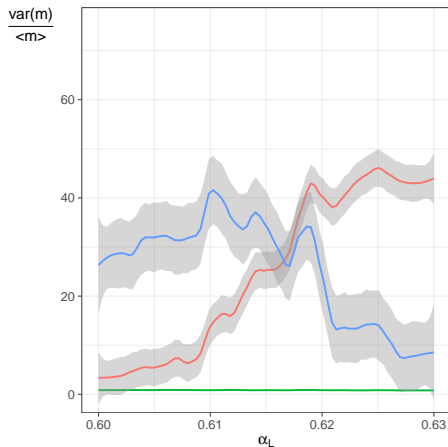
MC

MS

Canon.

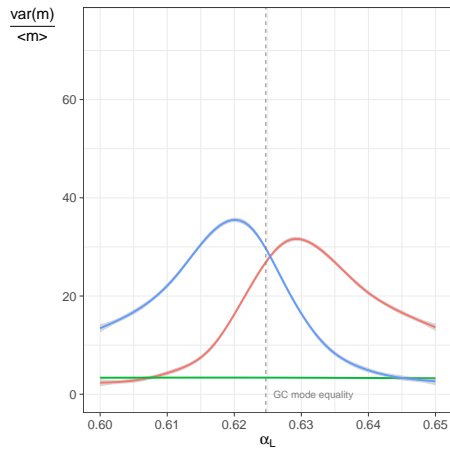
Conclusion

Microscopic



Photon Fano factor for microscopic at pump rate 210

Canonical



Photon Fano factor for canonical at pump rate 100



Laser Markov

Arthur Vallet

Introduction

Laser
overview

Multimode
Laser

Laser models

SC laser

States
Events

Statistical
specifications

FD
MC
MS
Canon.

Conclusion

Monte-Carlo semiconductor laser simulation based on Markov chain

- requires electron presence at lasing levels → different statistical ensembles.
- **Grand-canonical**: thermal light → nonsense for a laser.
- **Micro-canonical**: accurately, but considers only closed systems.
- **Microscopic**: take thermalization into account.
Satisfactory from physical point of view but was very slow.
- **Canonical**: shortcut in Markov chain
Speed up the program from 1h to 1s with the same physical accuracy.



Laser Markov

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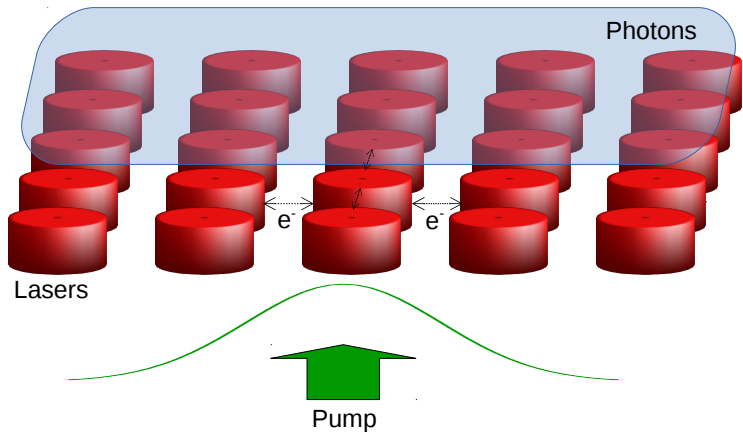
FD

MC

MS

Canon.

Conclusion



2D Semiconductor laser model



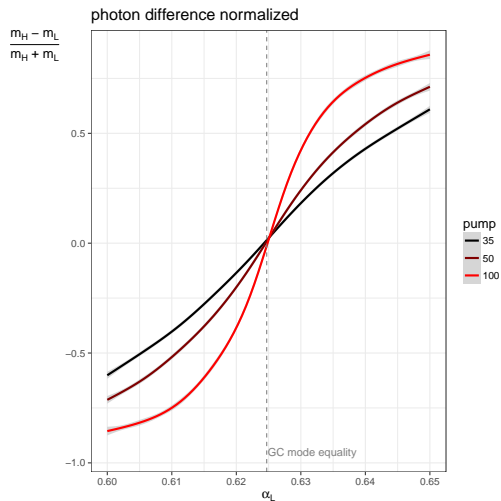
appendix



Modal competition

Laser Markov

Arthur Vallet



Modifications

- CPU time:
1 hour \rightarrow 1 sec
- No occupancy congestion
- Microscopic convergence $p \rightarrow \infty$

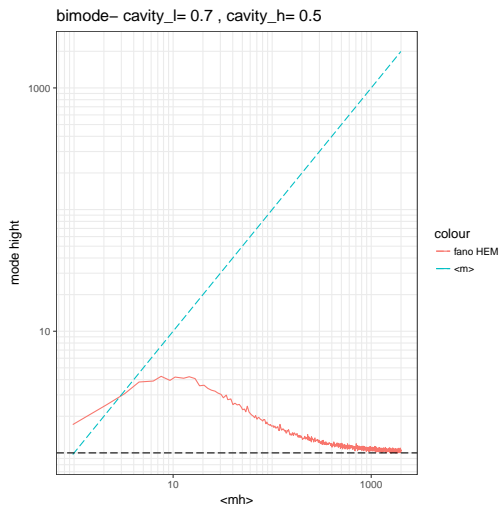
Canonical modal competition for different cavity output and pump rates



Single-mode threshold

Laser Markov

Arthur Vallet



Fano factor of a quasi-single-mode laser

Threshold definition

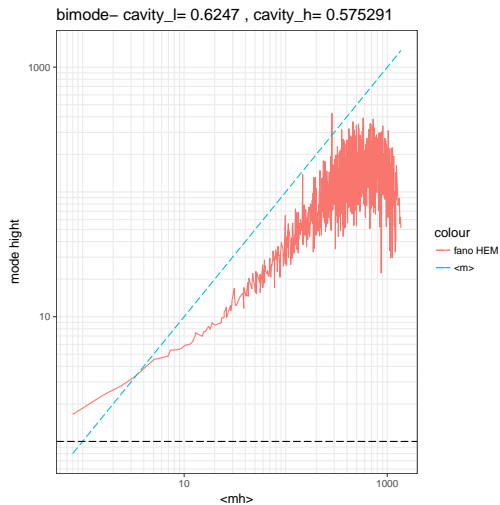
- Mode extinction
extinction frequency
extinction length
- Fermi level maximum
- Fano maximum



Threshold bi-mode

Laser Markov

Arthur Vallet



Threshold definition

- Mode threshold
- laser threshold
- noise dependence of modal competition

Fano factor of dual-mode with equality of photons mean