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Semiconductor laser Markov models in the micro-canonical, canonical and grand-canonical ensembles

Arthur Vallet, Laurent Chusseau, Fabrice Philippe, Alain Jean-Marie

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Semiconductor laser Markov models in the micro-canonical, canonical and grand-canonical ensembles

A semiconductor laser Markov picture

Arthur Vallet, Laurent Chusseau, Fabrice Philippe, Alain Jean-Marie

IES, Montpellier University

11/07/17





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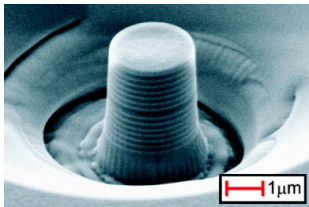
2 Semiconductor laser Markov picture

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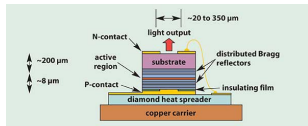
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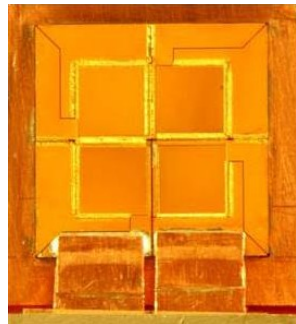
Micropillar with embedded colloidal CdSe/ZnS quantum dots

- $\approx 1 \mu m^2$
- $\approx 10 \mu W$



Vertical-Cavity Surface-Emitting Laser

- $\approx 0.01 mm^2$
- $\approx 1 mW$



High power VECSEL

- $\approx 1 cm^2$
- $\approx 1.5 kW$

[1] R. Bratschitsch and al., "Advances in Lasers and Electro Optic." *InTech*, 2010.



Photonics transitions

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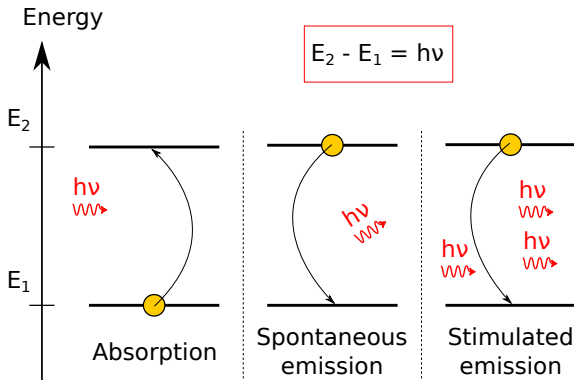
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Photonics transitions

Definition

- Absorption
 $\left(\frac{dN_2}{dt}\right)_{abs} = BN_1m$
- Stimulated emission
 $\left(\frac{dN_2}{dt}\right)_{e.st} = -BN_2m$
- Spontaneous emission
 $\left(\frac{dN_2}{dt}\right)_{e.sp} = AN_2$



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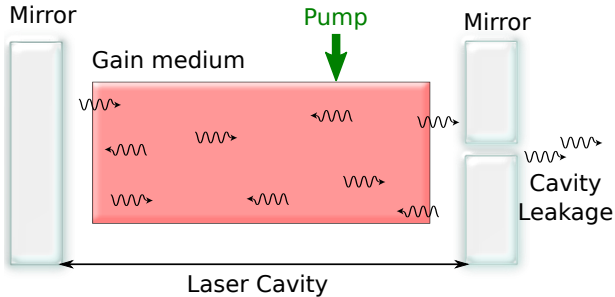
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Laser cavity scheme

Definition

- Laser = Gain medium + laser cavity
- population inversion by pump
- Cavity leakage : laser main output



Dual mode laser

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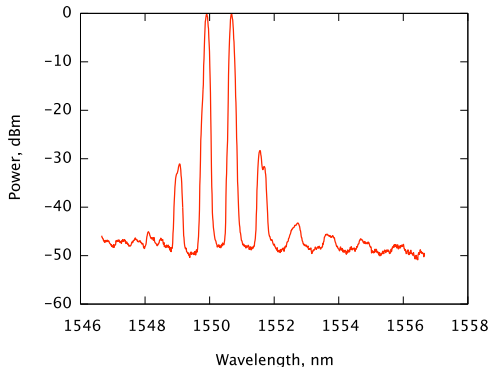
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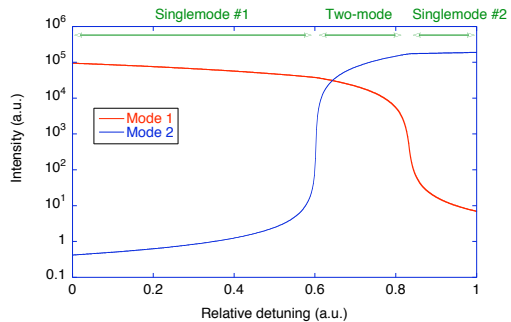
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Multimode laser



Modal competition

[2] L. Chusseau et al., "Four-sections semiconductor two-mode laser for THz generation," *Proc. SPIE* **6343**, 2006, pp. 1097–1106.



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Quantum simulations

- QED
- Few particles only
- Exact simulations
- Single-mode

Monte-Carlo simulations

- Trade-off between two approaches
- Small to big systems
- Multimode possibility
- Noise intrinsic in system

Rates equations resolution

- Analytic resolution
- No particles number limits
- Multimode possibility
- Fluctuations: Langevin noise

[3] M. Elk, "Numerical studies of the mesomaser," *Phys. Rev. A*, **54**, no. 5, pp. 4351–4358, 1996.

[4] G. P. Puccioni et al., "Stochastic Simulator for modeling the transition to lasing," *Opt. Express*, **23**, no. 3, p. 2369, 2015.

[5] A. Lebreton et al., "Stochastically sustained population oscillations in high- β nanolasers," *New J. Phys.*, **15**, 2013.

[6] L. a. Coldren et al., "Diode Lasers and Photonic Integrated Circuits", *Optical En.*, 1995.



Markov chains

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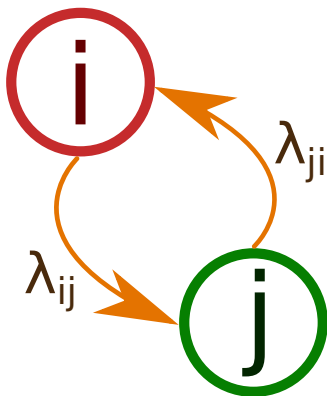
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Markov chain simple example

Algorithm 1: Gillespie algorithm : Monte-Carlo simulation by Markov chain

begin

```
1  t = 0
   while t < T do
2    random number r_t = U(0, 1) for the waiting time
      Λ = ∑_{i ≥ 1} λ_i
      τ = -ln(r_t) / Λ
3    random number r_e = U(0, 1) for the next event
      index = min i : ∑_{j=1}^i λ_j ≥ r_e Λ
4    update rates and state
      t = t + τ
```



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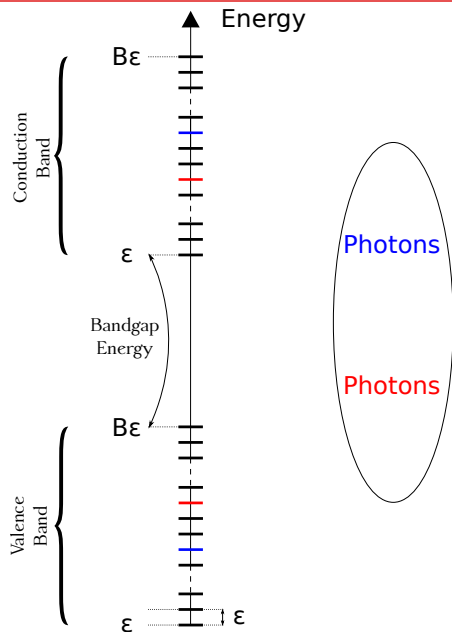
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States

- Photon reservoir
- B evenly-spaced energy states in each band
- B electrons in total
- 0 or 1 electron per level
- Photons interact with electrons only at lasing level



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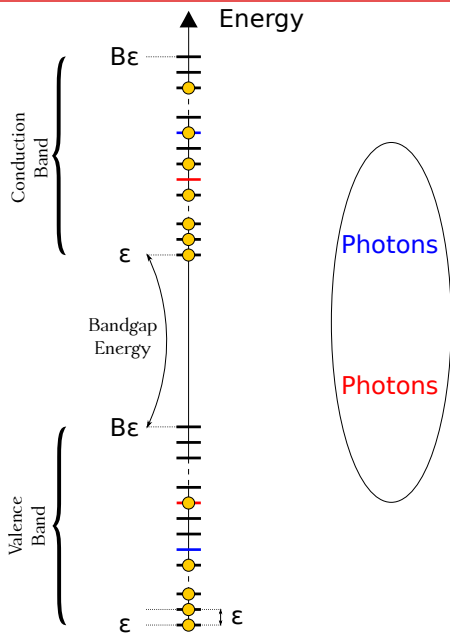
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Events

- Absorption
 $\bar{n}_c n_v m$
- Emission
 $n_c \bar{n}_v (m + 1)$
- Cavity output
 αm
- Thermalization
- Steady State :
Cavity output = pump



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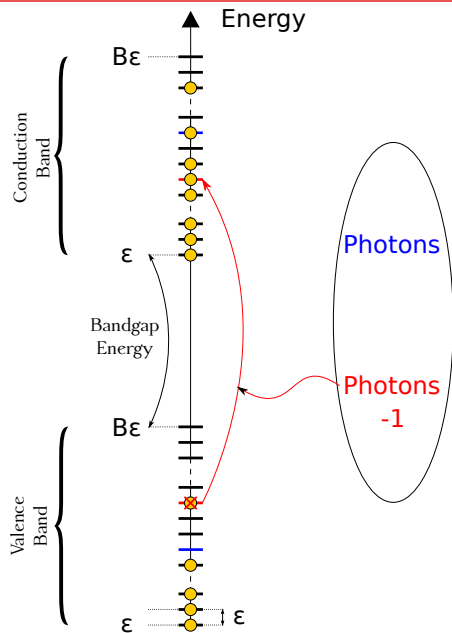
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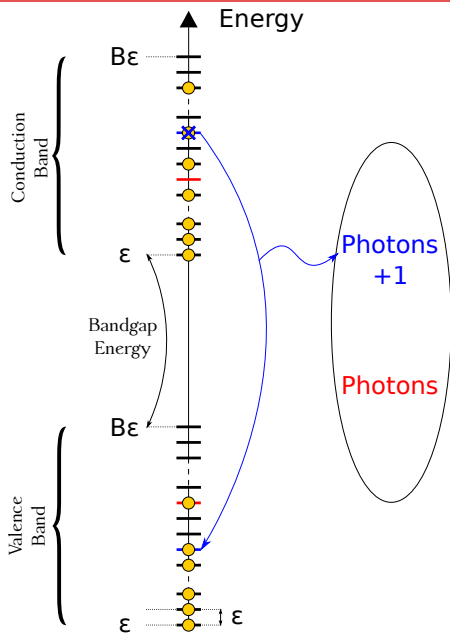
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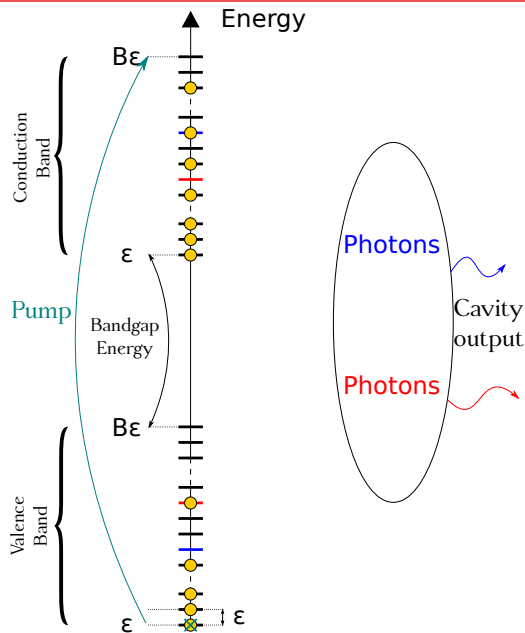
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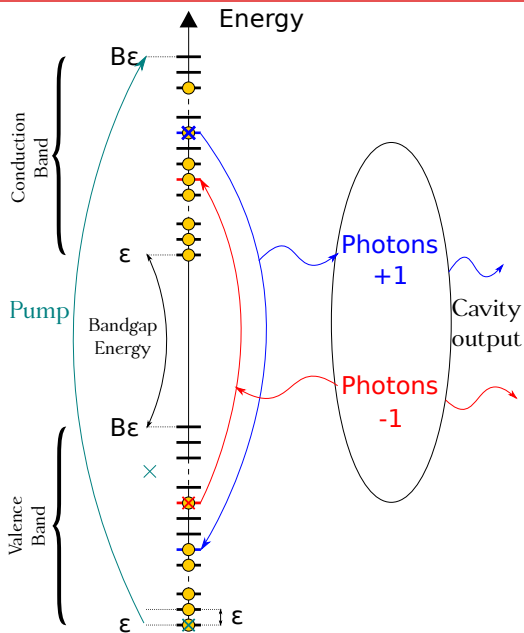
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Events

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Fermi-Dirac Distribution

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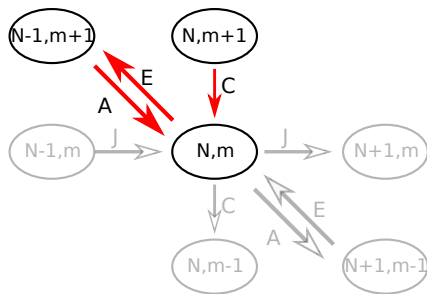
Conclusion

$$n_k = \frac{1}{e^{\frac{\epsilon}{k_B T}(k-\mu)} + 1}$$

Fermi-Dirac distribution

Problems

- Occupancy n_k and photon m are independent
- No fluctuation
- Thermal light variance \Rightarrow No laser
- Mean values calculable



$$\sum_{N=0}^B \Pi(N, m) n_k^2(N) = \sum_{N=0}^B \Pi(N, m+1) [\alpha + n_k^2(B - N)]$$

Detailed balance at m photons

$$\Pi(m) = \frac{\alpha + C_2}{C_1} \Pi(m+1)$$



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Micro-canonical bands repartition

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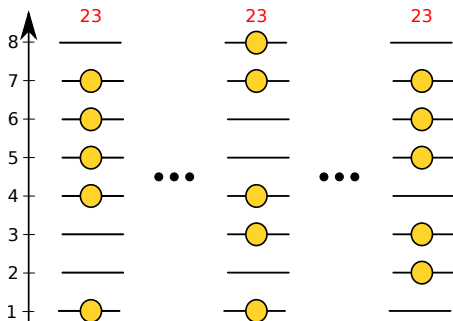
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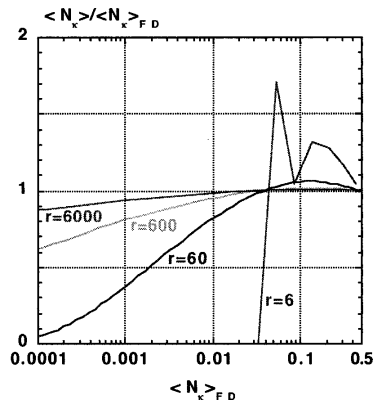
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Possible repartitions of electrons for a given energy



Ratio between Microcanonical and FD occupancies for the same energy r

$$m_{\kappa}(r) = - \sum_{i=1,2,\dots} (-1)^i W[r - i(\kappa + i/2)]$$

[7] J. Arnaud et al., "Illustration of the Fermi-Dirac statistics," *Am. J. Phys.*, **67**, no. 3, pp. 215-221, 1999.



Microscopic description

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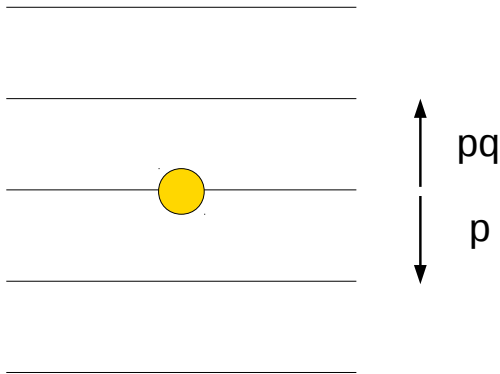
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Thermalization event

Definition

- All electron movement simulated
- $p \equiv$ electron movement speed
- Boltzmann thermalization
- $q \equiv e^{\frac{-\epsilon}{k_B T}}$



Microscopic application

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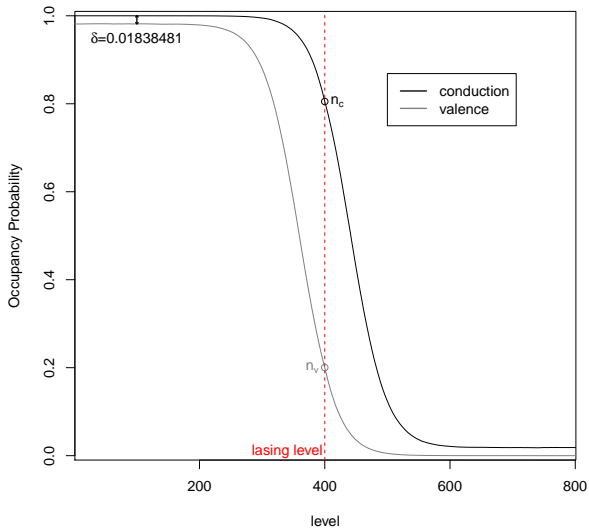
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Problems

- Occupancy shift
- Pump values biased
- $> 10^5$ thermalization events per photon event
- Very time consuming



Microscopic modal competition

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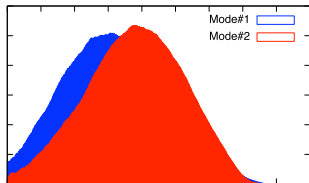
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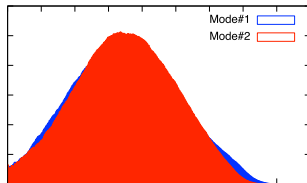
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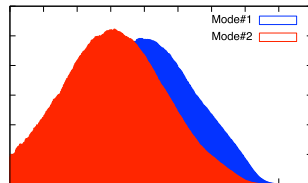
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$$\alpha_L = 0.615$$



$$\alpha_L = 0.616$$



$$\alpha_L = 0.617$$

Stable dual-mode solution exist but only a very tiny range of laser parameters

- [8] L. Chusseau et al., "Monte Carlo modeling of the dual-mode regime in quantum-well and quantum-dot semiconductor lasers," *Opt. Express*, 22, no. 5, pp. 5312–5324, 2014.



Hypothesis

We may use canonical occupancy at laser level.

$$n_k(N, q) = \frac{Z(N-1, q)q^k}{Z(N, q)} [1 - n_k(N-1, q)]$$

Canonical Occupancy

$$Z(N, q) = \prod_{i=1}^N \frac{q^{i-1} - q^B}{1 - q^i}$$

Partition function

$$n_k(N+1, q) = \frac{(1 - q^{N+1})q^k}{q^N - q^B} [1 - n_k(N, q)]$$

Recursive occupancy formula



Canonical vs. Microscopic difference

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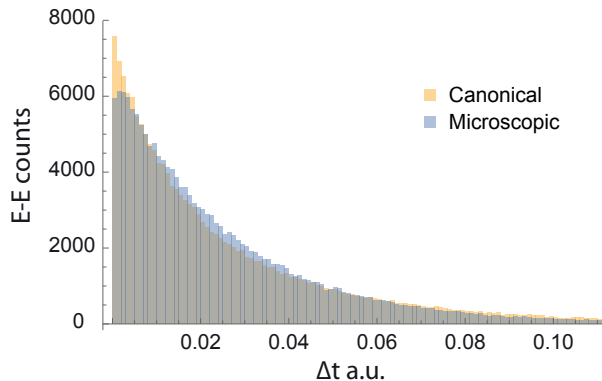
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Distributions of double emission events

Double emission events

- Canonical : Exponential distribution
- Microscopic : Non exponential distribution
- \exists Solution to restrain fast E-E within the Markov framework



Dual-mode photon statistics

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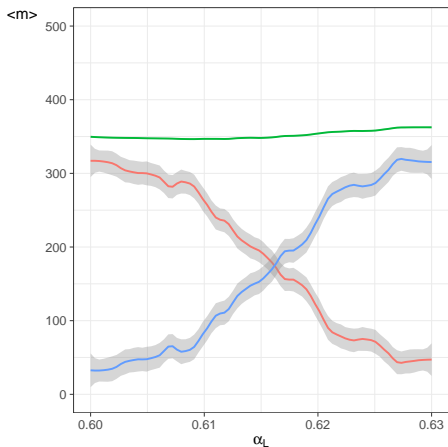
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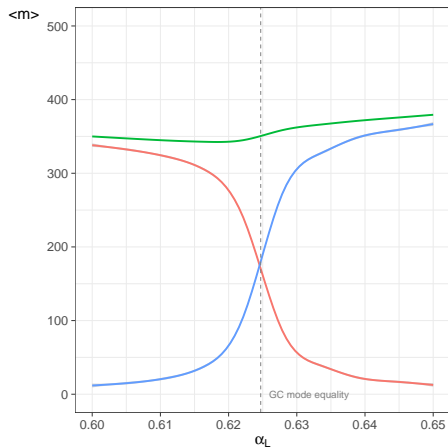
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Photon statistic for microscopic at pump rate 210

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Photon statistic for canonical at pump rate 210



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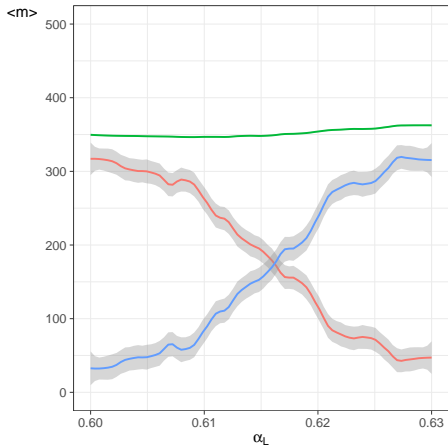
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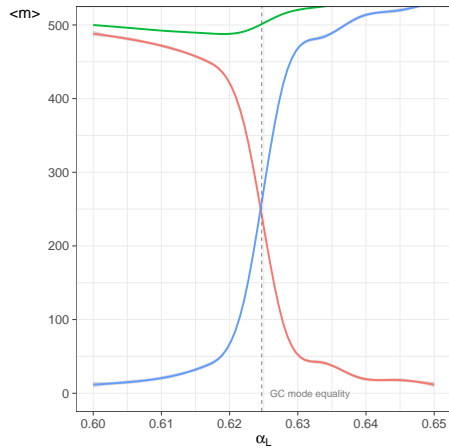
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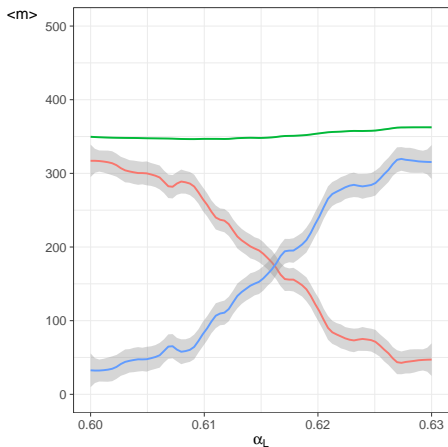
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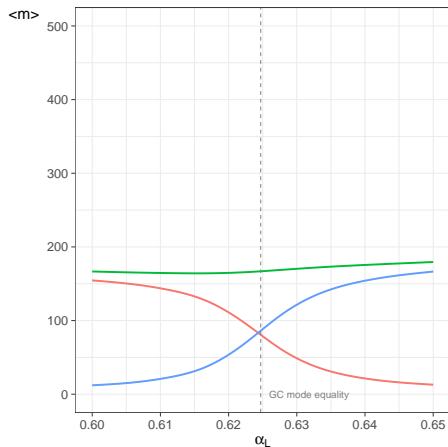
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Photon statistic for canonical at pump rate 100



Bimode photon Fano factor

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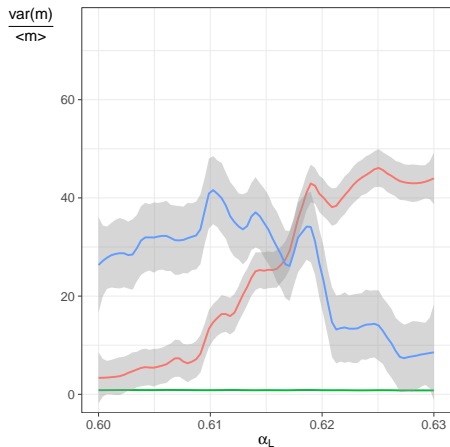
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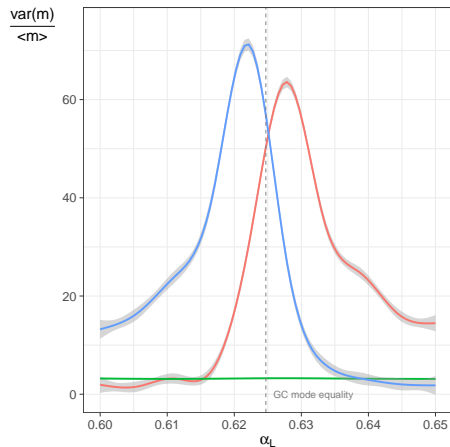
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Bimode photon Fano factor

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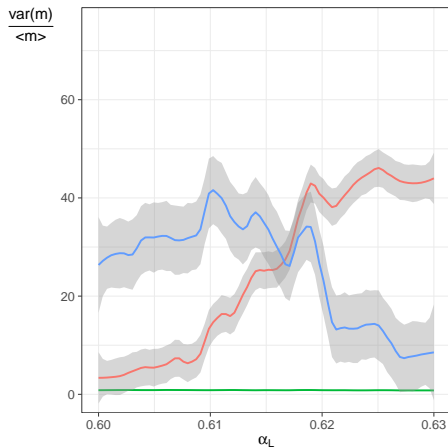
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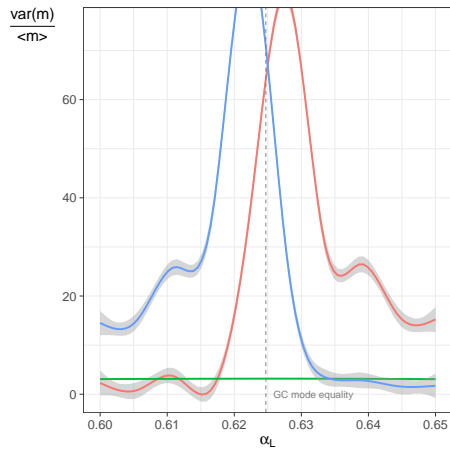
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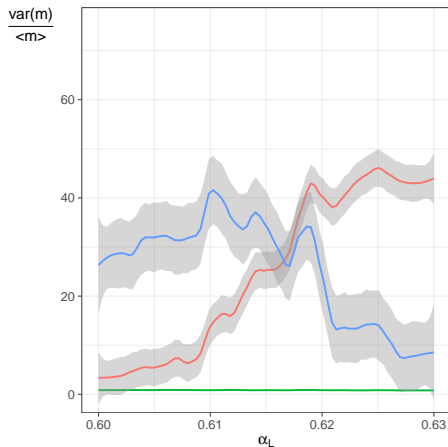
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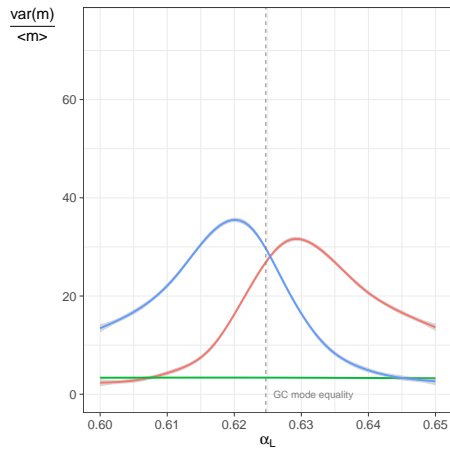
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Monte-Carlo semiconductor laser simulation based on Markov chain

- requires electron presence at lasing levels → different statistical ensembles.
- **Grand-canonical**: thermal light → nonsense for a laser.
- **Micro-canonical**: accurately, but considers only closed systems.
- **Microscopic**: take thermalization into account.
Satisfactory from physical point of view but was very slow.
- **Canonical**: shortcut in Markov chain
Speed up the program from 1h to 1s with the same physical accuracy.



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Events

Statistical specifications

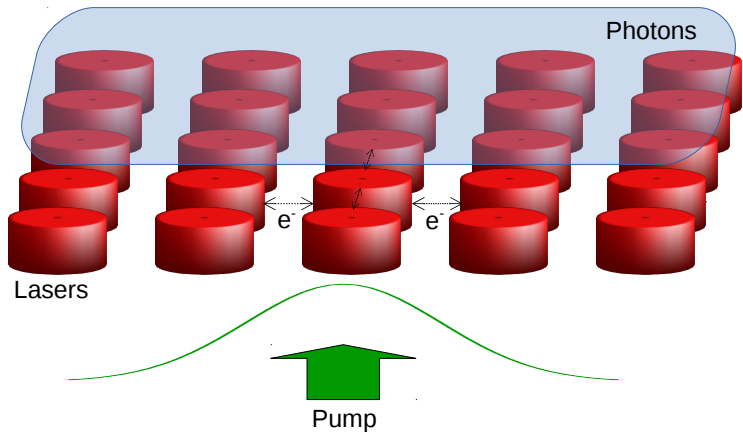
FD

MC

MS

Canon.

Conclusion



2D Semiconductor laser model



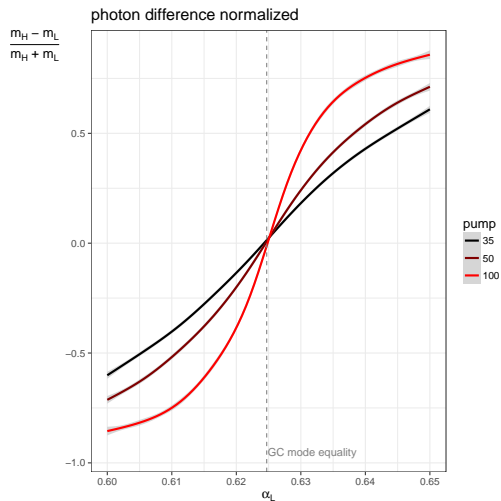
appendix



Modal competition

Laser Markov

Arthur Vallet



Modifications

- CPU time:
1 hour \rightarrow 1 sec
- No occupancy congestion
- Microscopic convergence $p \rightarrow \infty$

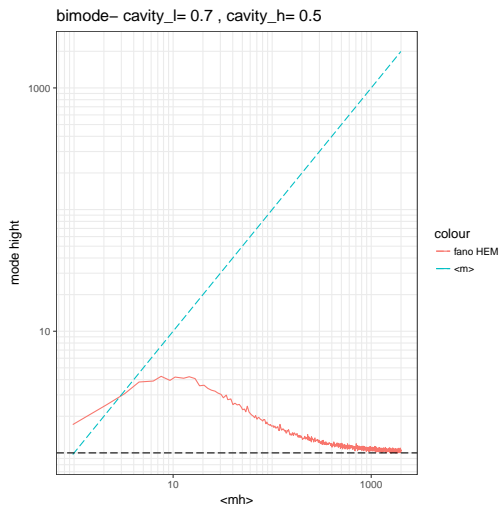
Canonical modal competition for different cavity output and pump rates



Single-mode threshold

Laser Markov

Arthur Vallet



Fano factor of a quasi-single-mode laser

Threshold definition

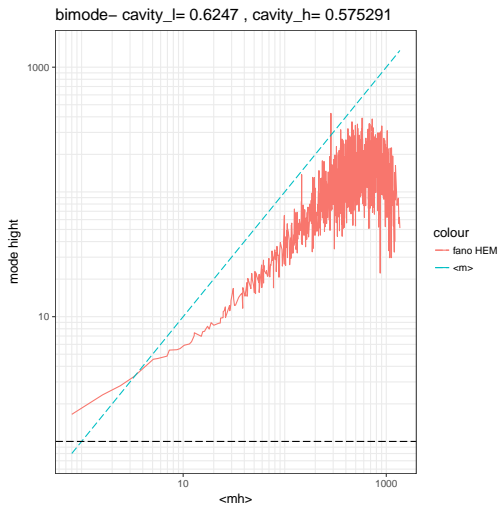
- Mode extinction
extinction frequency
extinction length
- Fermi level maximum
- Fano maximum



Threshold bi-mode

Laser Markov

Arthur Vallet



Threshold definition

- Mode threshold
- laser threshold
- noise dependence of modal competition

Fano factor of dual-mode with equality of photons mean