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Semiconductor laser Markov models in the micro-canonical, canonical and grand-canonical ensembles

A semiconductor laser Markov picture

Arthur Vallet, Laurent Chusseau, Fabrice Philippe, Alain Jean-Marie

IES, Montpellier University

11/07/17
Summary

1. Introduction
   - Laser overview
   - Multimode Laser
   - Laser models

2. Semiconductor laser Markov picture
   - States
   - Events

3. Statistical specifications
   - Grand-canonical ensemble
   - Micro-canonical ensemble
   - Microscopic picture
   - Canonical ensemble

4. Conclusion
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Semiconductor Laser overview

Micropillar with embedded colloidal CdSe/ZnS quantum dots

- \( \approx 1 \, \mu m^2 \)
- \( \approx 10 \, \mu W \)

Vertical-Cavity Surface-Emitting Laser

- \( \approx 0.01 \, mm^2 \)
- \( \approx 1 \, mW \)

High power VECSEL

- \( \approx 1 \, cm^2 \)
- \( \approx 1.5 \, kW \)

Photonics transitions

- **Absorption**: 
  \[ \left( \frac{dN_2}{dt} \right)_{abs} = BN_1 m \]
- **Stimulated emission**: 
  \[ \left( \frac{dN_2}{dt} \right)_{e.st} = -BN_2 m \]
- **Spontaneous emission**: 
  \[ \left( \frac{dN_2}{dt} \right)_{e.sp} = AN_2 \]

Energy levels and transitions:
- **Absorption**: \( E_2 \rightarrow E_1 \)
- **Spontaneous emission**: \( E_2 \rightarrow E_1 \)
- **Stimulated emission**: \( E_2 \rightarrow E_1 \)
Definition

- Laser = Gain medium + laser cavity
- Population inversion by pump
- Cavity leakage: laser main output
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Dual mode laser

Multimode laser

Modal competition

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Laser models overview

Quantum simulations
- QED
- Few particles only
- Exact simulations
- Single-mode

Monte-Carlo simulations
- Trade-off between two approaches
- Small to big systems
- Multimode possibility
- Noise intrinsic in system

Rates equations resolution
- Analytic resolution
- No particles number limits
- Multimode possibility
- Fluctuations: Langevin noise


**Algorithm 1**: Gillespie algorithm : Monte-Carlo simulation by Markov chain

begin

1. \( t = 0 \)

2. while \( t < T \) do
   - random number \( r_t = U(0, 1) \) for the waiting time
   - \( \Lambda = \sum_{i \geq 1} \lambda_i \)
   - \( \tau = -\frac{\ln r_t}{\Lambda} \)

3. random number \( r_e = U(0, 1) \) for the next event
   - \( index = \min i : \sum_{j=1}^{i} \lambda_j \geq r_e \Lambda \)

4. update rates and state
   - \( t = t + \tau \)

Markov chain simple example
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Model states

- Photon reservoir
- B evenly-spaced energy states in each band
- B electrons in total
- 0 or 1 electron per level
- Photons interact with electrons only at lasing level
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Model events

Events

- Absorption: \( \overline{n_c n_v m} \)
- Emission: \( n_c \overline{n_v} (m + 1) \)
- Cavity output: \( \alpha m \)
- Thermalization
- Steady State: Cavity output = pump
Model events

[Diagram of energy levels and transitions]

- **Absorption**: $\overline{n}_c n_v m$
- **Emission**: $n_c \overline{n}_v (m + 1)$
- **Cavity output**: $\alpha m$
- **Thermalization**
- **Steady State**: Cavity output = pump
Model events

Events

- Absorption: $n_c n_v m$
- Emission: $n_c n_v (m + 1)$
- Cavity output: $\alpha m$
- Thermalization
- Steady State: Cavity output $= \text{pump}$
Laser Markov
Arthur Vallet

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SC laser
States
Events

Statistical specifications
FD
MC
MS
Canon.

Conclusion

Model events

Events

- Absorption
  $\bar{n}_c n_v m$
- Emission
  $n_c \bar{n}_v (m + 1)$
- Cavity output
  $\alpha m$
- Thermalization
- Steady State:
  Cavity output = pump
Model events

Energy

Conduction Band

Valence Band

Pump

Bandgap Energy

Absorption

Emission

Cavity output

Photon +1

Photon -1

Cavity output = pump

Events

- Absorption
  \( \overline{n}_c n_v m \)

- Emission
  \( n_c \overline{n}_v (m + 1) \)

- Cavity output
  \( \alpha m \)

- Thermalization

Steady State:
Cavity output = pump

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Fermi-Dirac Distribution

\[ n_k = \frac{1}{e^{\frac{\epsilon}{k_B T} (k - \mu)} + 1} \]

Fermi-Dirac distribution

Problems

- Occupancy \( n_k \) and photon \( m \) are independent
- No fluctuation
- Thermal light variance \( \Rightarrow \) No laser
- Mean values calculable

\[
\sum_{N=0}^{B} \Pi(N, m) n_k^2(N) = \sum_{N=0}^{B} \Pi(N, m+1) \left[ \alpha + n_k^2(B - N) \right]
\]

Detailed balance at \( m \) photons

\[
\Pi(m) = \frac{\alpha + C_2}{C_1} \Pi(m + 1)
\]
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Micro-canonical bands repartition

Possible repartitions of electrons for a given energy

Ratio between Microcanonical and FD occupancies for the same energy \( r \)

\[
m_k(r) = - \sum_{i=1,2,\ldots} (-1)^i W[r - i(k + i/2)]
\]

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Microscopic description

Thermalization event

Definition

- All electron movement simulated
- \( p \equiv \) electron movement speed
- Boltzmann thermalization
- \( q \equiv e^{-\frac{\epsilon}{k_BT}} \)

\( p = pq \)
Microscopic application

Problems

- Occupancy shift
- Pump values biased
- $> 10^5$ thermalization events per photon event
- Very time consuming
Microscopic modal competition

Stable dual-mode solution exist but only a very tiny range of laser parameters

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Hypothesis

We may use canonical occupancy at laser level.

\[ n_k(N, q) = \frac{Z(N - 1, q)q^k}{Z(N, q)} [1 - n_k(N - 1, q)] \]

Canonical Occupancy

\[ Z(N, q) = \prod_{i=1}^{N} \frac{q^{i-1} - q^B}{1 - q^i} \]

Partition function

\[ n_k(N + 1, q) = \frac{(1 - q^{N+1})q^k}{q^N - q^B} [1 - n_k(N, q)] \]

Recursive occupancy formula
Canonical vs. Microscopic difference

Double emission events

- Canonical: Exponential distribution
- Microscopic: Non exponential distribution
- Solution to restrain fast E-E within the Markov framework

Distributions of double emission events
Dual-mode photon statistics

**Microscopic**

Photon statistic for microscopic at pump rate 210

**Canonical**

Photon statistic for canonical at pump rate 210
Laser Models

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Dual-mode photon statistics

Microscopic

Photon statistic for microscopic at pump rate 210

Canonical

Photon statistic for canonical at pump rate 300
Dual-mode photon statistics

Microscopic

Photon statistic for microscopic at pump rate 210

Canonical

Photon statistic for canonical at pump rate 100
Bimode photon Fano factor

Microscopic

Photon Fano factor for microscopic at pump rate 210

Canonical

Photon Fano factor for canonical at pump rate 210
Bimode photon Fano factor

Microscopic

Photon Fano factor for microscopic at pump rate 210

Canonical

Photon Fano factor for canonical at pump rate 300
Bimode photon Fano factor

Photon Fano factor for microscopic at pump rate 210

Photon Fano factor for canonical at pump rate 100
Monte-Carlo semiconductor laser simulation based on Markov chain

- requires electron presence at lasing levels $\rightarrow$ different statistical ensembles.
- **Grand-canonical**: thermal light $\rightarrow$ nonsense for a laser.
- **Micro-canonical**: accurately, but considers only closed systems.
- **Microscopic**: take thermalization into account. Satisfactory from physical point of view but was very slow.
- **Canonical**: shortcut in Markov chain
  Speed up the program from 1h to 1s with the same physical accuracy.
Future

2D Semiconductor laser model
appendix
Modal competition

Canonical modal competition for different cavity output and pump rates

Modifications

- CPU time: 1 hour $\rightarrow$ 1 sec
- No occupancy congestion
- Microscopic convergence $p \rightarrow \infty$
Single-mode threshold

Fano factor of a quasi-single-mode laser

Threshold definition

- Mode extinction
- Extinction frequency
- Extinction length
- Fermi level maximum
- Fano maximum
Threshold bi-mode

Fano factor of dual-mode with equality of photons mean

Threshold definition
- Mode threshold
- Laser threshold
- Noise dependence of modal competition