

# Grid Generation for Fusion Applications

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## Magnetized plasma

- An extremely anisotropic medium :
  - Transport coefficients in the parallel (to the magnetic field) and perpendicular directions differ by several order of magnitude :  $k_{//}/k_{\perp} \sim 10^9$
  - Dynamics is completely different in the parallel and perpendicular direction : incompressible in the perp direction, compressible (and supersonic in the SOL) in the parallel direction : plasma slides along the magnetic field lines
- Plasma - wall interaction (Bohm's boundary condition) : Plasma enter into the wall at supersonic velocity :

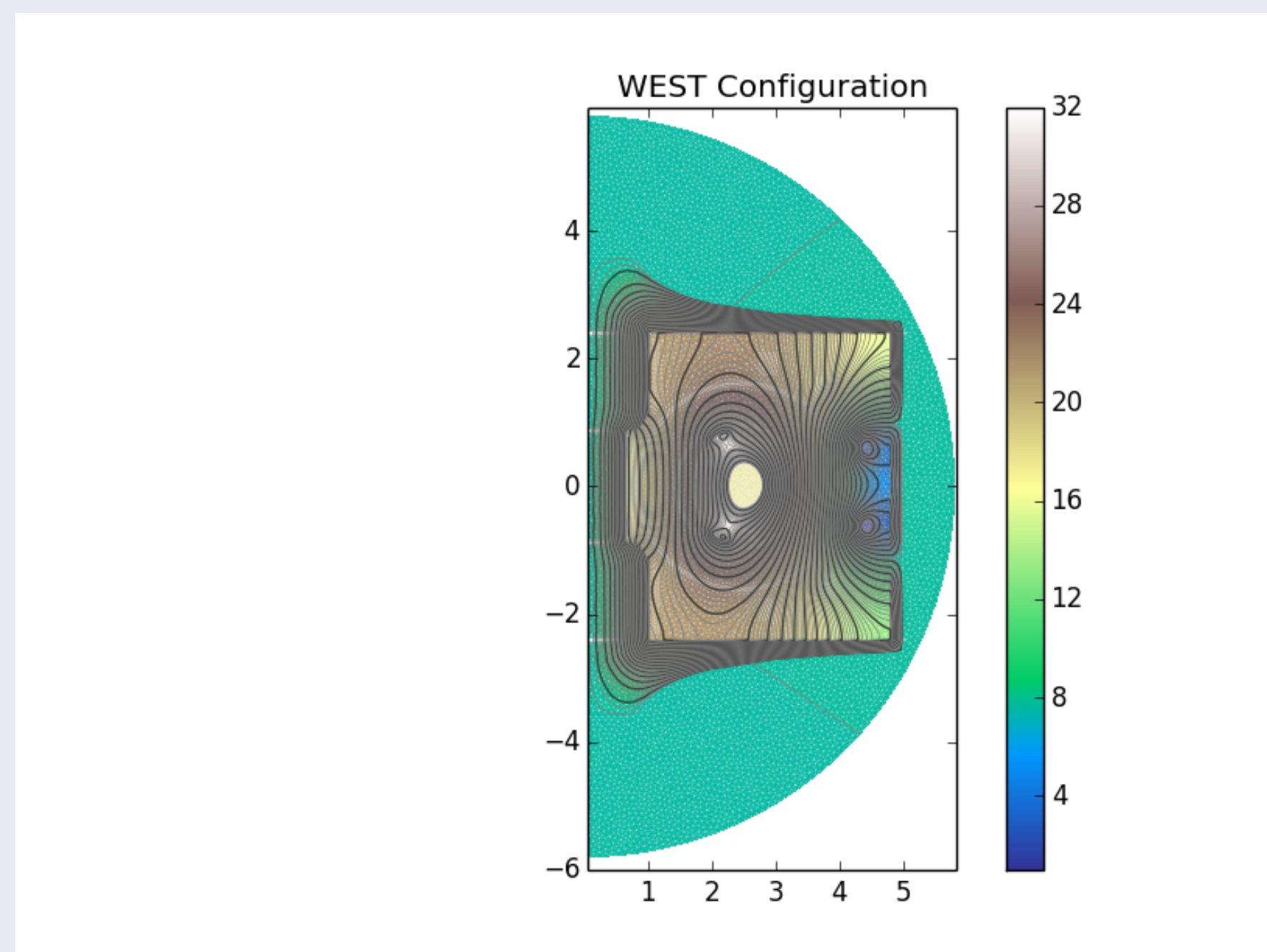
In magnetized plasma, plasma flows along the magnetic field lines

Meshes adapted to magnetic flux surfaces

## Grad-Shafranov solvers

For realistic geometries, magnetic field has to be computed by numerical Grad-Shafranov solvers

$$-\nabla \left( \frac{1}{\mu r} \nabla \psi \right) = \begin{cases} \mathbf{J}(x, \psi) & \text{plasma} \\ \mathbf{J}(\text{voltage}, \partial_t \psi) & \text{coils} \\ 0 & \text{elsewhere} \end{cases}$$



## Block-structured grid

- Block-structured meshes :
  - Identify the sub-domains (blocks-patches)  $\Omega_n$
  - Construct the mappings between logical grid  $[0, 1] \times [0, 1]$  and physical  $\Omega_n$
- How many blocks and definition of the patches ?  
This is a **segmentation problem** :  
Given a domain  $\Omega$  : Find a number  $n$  and  $n$  patches  $\Omega_n$  such that :  
$$\Omega = \cup \Omega_n \text{ with } \overset{\circ}{\Omega}_n \cap \overset{\circ}{\Omega}_m = \emptyset$$
and there exists a one to one mapping between  $\Omega_n$  and the the unit square :  
$$\exists \phi_n : [0, 1] \times [0, 1] \leftrightarrow \Omega_n$$
- for flux surface aligned grid, the segmentation problem can be solved by **Morse theory**

## Segmentation Problem and Morse Function

$f$  is a Morse function if all its critical points are regular.

**Critical points**  
Let  $\mathcal{C}^r$  be the space of  $r \geq 2$  differentiable scalar field defined on  $\Omega$ .  $\mathbf{p} \in \Omega$  is a critical point of  $f$  if  $\nabla f(\mathbf{p}) = 0$ .

**Regular Critical points**  
A critical point is regular if the Hessian of  $f$  at  $\mathbf{p}$  is invertible.

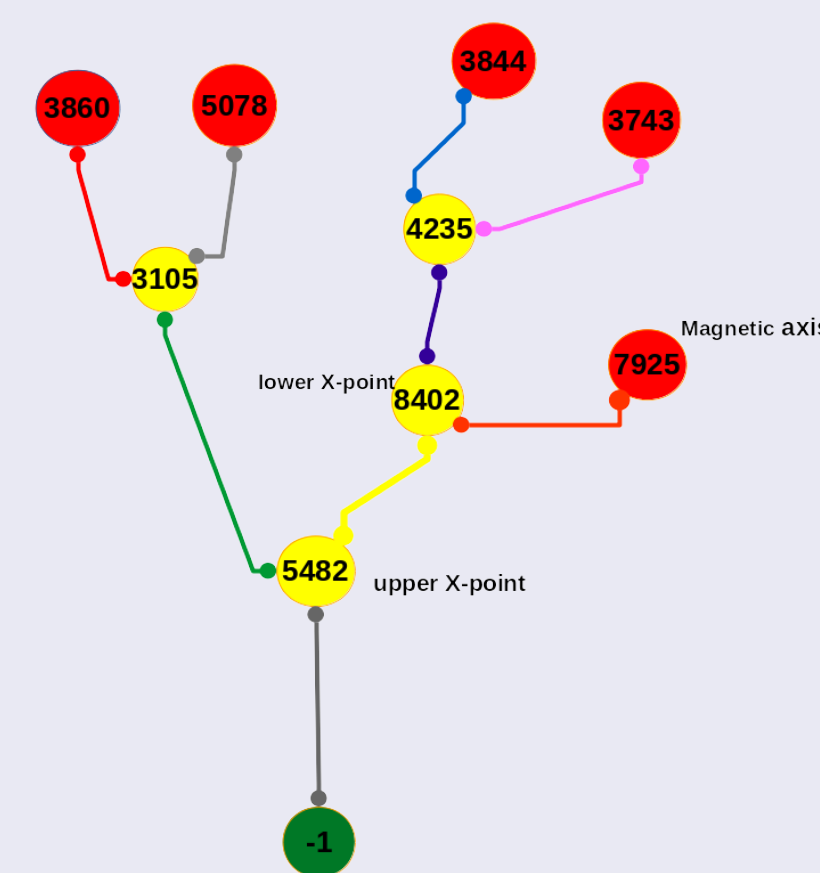
**In 2-D**  
the only possible critical points of a Morse function are :  
Maxima index = 2 Minima index = 0  
Saddles index = 1

**Topology of a Morse function** : The topological set of the iso-contours of  $f$  consists of connected components that are either

- Circle cells which are homeomorphic to open disks
- Circle bands which are homeomorphic to open annulus

## The Reeb graph

- Compute the connected components by computing the Reeb Graph



- Obtain the associated domain segmentation



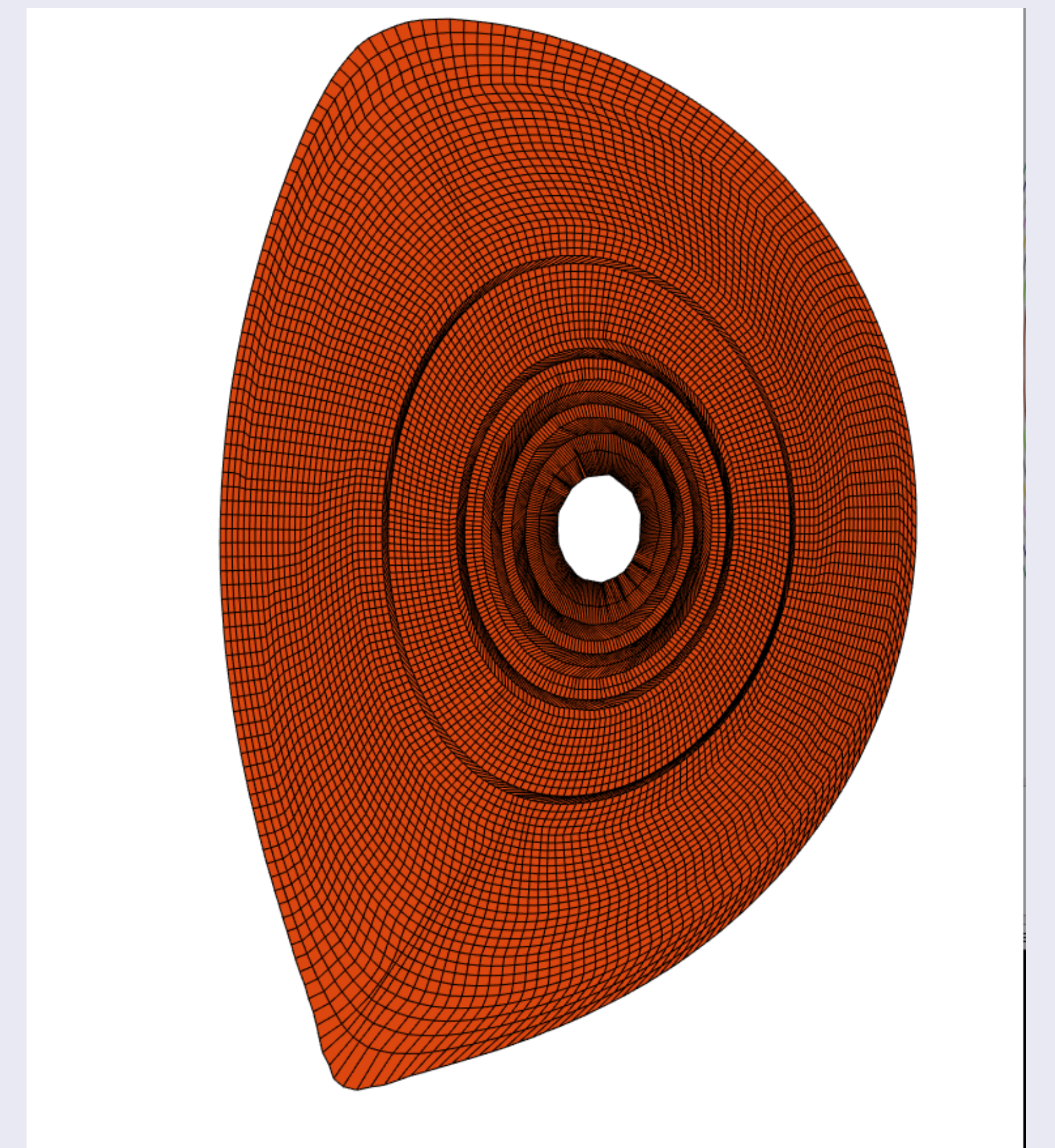
each subdomain correspond to an edge of the Reeb graph and in each subdomain, the function is monotone

## Construction of the mapping of $\Omega_n$

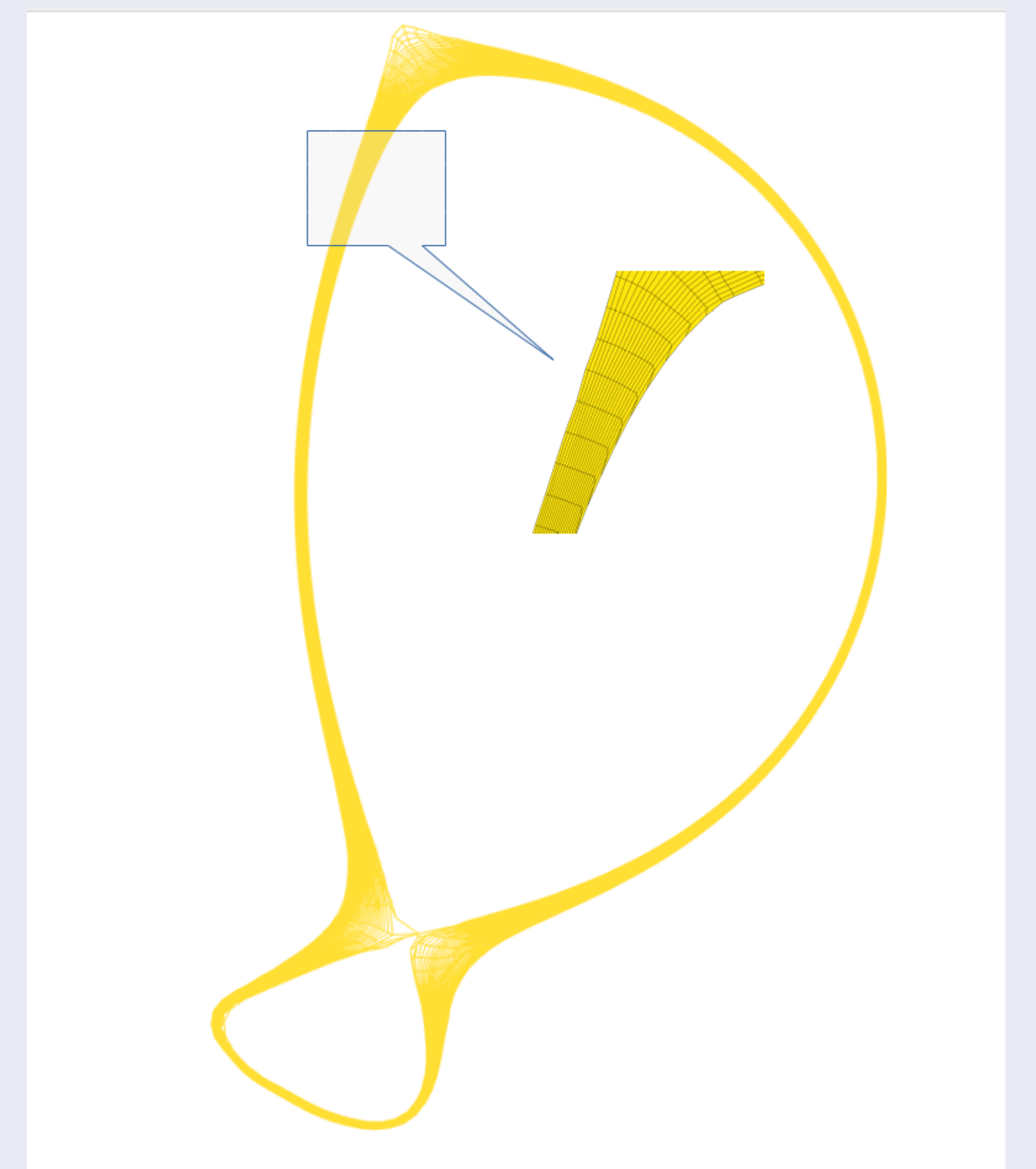
Orthogonal mesh constructed by intersecting

- The isolines
- Streamline integration of  $\frac{dx}{ds} = \nabla \psi$

Leaf of the Reeb Graph : homeomorphic to a disk  
Example : Reeb Graph edge : [7925-8402]



- Internal edge of the Reeb graph : homeomorphic to annulus  
Example : Reeb Graph edge : [7925-8402]



## Future works

- More accurate and smooth GS solvers
- Add additional mapping techniques : Elliptic solvers + equidistribution
- Topological simplification of the Reeb-Graph
- Construction of  $\mathcal{C}^1$  mappings
- Take into account the vacuum chamber boundary