

Multi-task Bolasso based aircraft dynamics identification

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Multi-task Bolasso based aircraft dynamics identification

C. Rommel^{1,2}, J. F. Bonnans¹, B. Gregorutti² and P. Martinon¹

CMAP Ecole Polytechnique - INRIA¹ Safety Line²

November 14th 2017





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AIRCRAFT DYNAMICS IDENTIFICATION

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• 20 000 airplanes — 80 000 flights per day,

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- 20 000 airplanes 80 000 flights per day,
- Should double until 2033,

- 20 000 airplanes 80 000 flights per day,
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- Responsible for 3% of CO_2 emissions,

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- 20 000 airplanes 80 000 flights per day,
- Should double until 2033,
- Responsible for 3% of CO₂ emissions,
- Accounts for 30% of operational cost for an airline,

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- 20 000 airplanes 80 000 flights per day,
- Should double until 2033,
- Responsible for 3% of CO₂ emissions,
- Accounts for 30% of operational cost for an airline,
- Rectilinear climb trajectories at full thrust.

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EXAMPLE OF OPTIMIZED TRAJECTORY



Reference Optimized 1141*kg* 1311*kg*

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s.t.
$$\begin{cases} \min_{(\mathbf{x},\mathbf{u})\in\mathbb{X}\times\mathbb{U}}\int_{0}^{t_{f}}C(t,\mathbf{u}(t),\mathbf{x}(t))dt,\\ \Phi(\mathbf{x}(0),\mathbf{x}(t_{f})) \in K_{\Phi}, \end{cases}$$

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s.t.
$$\begin{cases} \min_{(\mathbf{x},\mathbf{u})\in\mathbb{X}\times\mathbb{U}}\int_{0}^{t_{f}}C(t,\mathbf{u}(t),\mathbf{x}(t))dt, \\ \Phi(\mathbf{x}(0),\mathbf{x}(t_{f})) \in K_{\Phi}, \\ \mathbf{u}(t) \in U_{ad}, \text{ for a.e. } t\in[0,t_{f}], \end{cases}$$

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$$\min_{\substack{(\mathbf{x},\mathbf{u})\in\mathbb{X}\times\mathbb{U}\\\mathbf{x}(\mathbf{z})\in\mathbb{X}\times\mathbb{U}}} \int_{0}^{t_{f}} C(t,\mathbf{u}(t),\mathbf{x}(t))dt,$$

s.t.
$$\begin{cases} \Phi(\mathbf{x}(0),\mathbf{x}(t_{f})) \in K_{\Phi}, \\ \mathbf{u}(t) \in U_{ad}, \text{ for a.e. } t\in[0,t_{f}], \\ c_{j}(\mathbf{x}(t)) \leq 0, \quad j=1,\ldots,n_{c}, \text{ for all } t\in[0,t_{f}], \end{cases}$$

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$$\min_{\substack{(\mathbf{x},\mathbf{u})\in\mathbb{X}\times\mathbb{U}\\\mathbf{x},\mathbf{u})\in\mathbb{X}\times\mathbb{U}}} \int_{0}^{t_{f}} C(t,\mathbf{u}(t),\mathbf{x}(t))dt,$$
s.t.
$$\begin{cases} \Phi(\mathbf{x}(0),\mathbf{x}(t_{f})) \in K_{\Phi}, \\ \mathbf{u}(t) \in U_{ad}, \text{ for a.e. } t\in[0,t_{f}], \\ c_{j}(\mathbf{x}(t)) \leq 0, \quad j=1,\ldots,n_{c}, \text{ for all } t\in[0,t_{f}], \\ \dot{\mathbf{x}} = g(t,\mathbf{u},\mathbf{x}), \text{ for a.e. } t\in[0,t_{f}]. \end{cases}$$

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$$\min_{\substack{(\mathbf{x},\mathbf{u})\in\mathbb{X}\times\mathbb{U}\\\mathbf{x},\mathbf{u})\in\mathbb{X}\times\mathbb{U}}} \int_{0}^{t_{f}} C(t,\mathbf{u}(t),\mathbf{x}(t))dt,$$
s.t.
$$\begin{cases} \Phi(\mathbf{x}(0),\mathbf{x}(t_{f})) \in K_{\Phi}, \\ \mathbf{u}(t) \in U_{ad}, \text{ for a.e. } t\in[0,t_{f}], \\ c_{j}(\mathbf{x}(t)) \leq 0, \quad j=1,\ldots,n_{c}, \text{ for all } t\in[0,t_{f}], \\ \dot{\mathbf{x}} = g(t,\mathbf{u},\mathbf{x}), \text{ for a.e. } t\in[0,t_{f}]. \end{cases}$$

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Massive (> 1000 variables recorded every second),

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Massive (> 1000 variables recorded every second),

 $\dot{\mathbf{x}} = g(t, \mathbf{u}, \mathbf{x})$

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Classic flight mechanics model

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$$\begin{cases} \dot{h} = & V \sin \gamma, \\ \dot{V} = & \frac{T \cos \alpha - D - mg \sin \gamma}{m}, \\ \dot{\gamma} = & \frac{T \sin \alpha + L - mg \cos \gamma}{mV}, \\ \dot{m} = & -\frac{T}{I_{sp}}. \end{cases}$$

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$$\begin{cases} \dot{h} = V \sin \gamma, \\ \dot{V} = \frac{T \cos \alpha - D - mg \sin \gamma}{m}, \\ \dot{\gamma} = \frac{T \sin \alpha + L - mg \cos \gamma}{mV}, \\ \dot{m} = -\frac{T}{I_{sp}}. \end{cases}$$

State variables: $\mathbf{x} = [h, V, \gamma, m]$

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$$\begin{cases} \dot{h} = V \sin \gamma, \\ \dot{V} = \frac{T \cos \alpha - D - mg \sin \gamma}{m}, \\ \dot{\gamma} = \frac{T \sin \alpha + L - mg \cos \gamma}{mV}, \\ \dot{m} = -\frac{T}{I_{sp}}. \end{cases}$$

State variables: $\mathbf{x} = [h, V, \gamma, m]$ Control variables: $\mathbf{u} = [\alpha, N_1]$

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$$\begin{cases} \dot{h} = V \sin \gamma, \\ \dot{V} = \frac{T \cos \alpha - D - mg \sin \gamma}{m}, \\ \dot{\gamma} = \frac{T \sin \alpha + L - mg \cos \gamma}{mV}, \\ \dot{m} = -\frac{T}{l_{sp}}. \end{cases}$$

State variables: $\mathbf{x} = [h, V, \gamma, m]$ Control variables: $\mathbf{u} = [\alpha, N_1]$ Unknown functions of the state and control variables

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$$\begin{cases} \dot{h} = V \sin \gamma, \\ \dot{V} = \frac{T(\mathbf{x}, \mathbf{u}) \cos \alpha - D(\mathbf{x}, \mathbf{u}) - mg \sin \gamma}{m}, \\ \dot{\gamma} = \frac{T(\mathbf{x}, \mathbf{u}) \sin \alpha + L(\mathbf{x}, \mathbf{u}) - mg \cos \gamma}{mV}, \\ \dot{m} = -\frac{T(\mathbf{x}, \mathbf{u})}{I_{sp}(\mathbf{x}, \mathbf{u})}. \end{cases}$$

State variables: $\mathbf{x} = [h, V, \gamma, m]$ Control variables: $\mathbf{u} = [\alpha, N_1]$ Unknown functions of the state and control variables

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MODEL REQUIREMENTS

 $\begin{cases} T & \text{function of } (M, \rho, N_1), \\ D & \text{function of } (M, \rho, q), \\ L & \text{function of } (M, \rho, q), \\ I_{sp} & \text{function of } (M, h, SAT), \end{cases}$

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MODEL REQUIREMENTS

- $\begin{cases} T & \text{function of } (M, \rho, N_1) = \varphi_T(\mathbf{x}, \mathbf{u}), \\ D & \text{function of } (M, \rho, q) = \varphi_D(\mathbf{x}, \mathbf{u}), \\ L & \text{function of } (M, \rho, q) = \varphi_L(\mathbf{x}, \mathbf{u}), \\ I_{sp} & \text{function of } (M, h, SAT) = \varphi_{Isp}(\mathbf{x}, \mathbf{u}), \end{cases}$

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AIRCRAFT DYNAMICS IDENTIFICATION

$$\begin{cases} T & \text{function of } (M, \rho, N_1) = \varphi_T(\mathbf{x}, \mathbf{u}), \\ D & \text{function of } (M, \rho, q) = \varphi_D(\mathbf{x}, \mathbf{u}), \\ L & \text{function of } (M, \rho, q) = \varphi_L(\mathbf{x}, \mathbf{u}), \\ I_{sp} & \text{function of } (M, h, SAT) = \varphi_{Isp}(\mathbf{x}, \mathbf{u}), \end{cases}$$

• Need for smooth models,

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$$\begin{cases} T & \text{function of } (M, \rho, N_1) = \varphi_T(\mathbf{x}, \mathbf{u}), \\ D & \text{function of } (M, \rho, q) = \varphi_D(\mathbf{x}, \mathbf{u}), \\ L & \text{function of } (M, \rho, q) = \varphi_L(\mathbf{x}, \mathbf{u}), \\ I_{sp} & \text{function of } (M, h, SAT) = \varphi_{Isp}(\mathbf{x}, \mathbf{u}), \end{cases}$$

- Need for smooth models,
- Need for models which are fast to compute,

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$$\begin{cases} T & \text{function of } (M, \rho, N_1) = \varphi_T(\mathbf{x}, \mathbf{u}), \\ D & \text{function of } (M, \rho, q) = \varphi_D(\mathbf{x}, \mathbf{u}), \\ L & \text{function of } (M, \rho, q) = \varphi_L(\mathbf{x}, \mathbf{u}), \\ I_{sp} & \text{function of } (M, h, SAT) = \varphi_{Isp}(\mathbf{x}, \mathbf{u}), \end{cases}$$

- Need for smooth models,
- Need for models which are fast to compute,
- Need for interpretable models for safety,

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$$\begin{cases} T = X_T \cdot \theta_T, \\ D = X_D \cdot \theta_D, \\ L = X_L \cdot \theta_L, \\ I_{sp} = X_{lsp} \cdot \theta_{lsp}. \end{cases}$$

- Need for smooth models,
- Need for models which are fast to compute,
- Need for interpretable models for safety,

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$$\begin{cases} T = X_T \cdot \theta_T, & \text{with } X_T \neq \varphi_T(\mathbf{x}, \mathbf{u}), \\ D = X_D \cdot \theta_D, & \text{with } X_D \neq \varphi_D(\mathbf{x}, \mathbf{u}), \\ L = X_L \cdot \theta_L, & \text{with } X_L \neq \varphi_L(\mathbf{x}, \mathbf{u}), \\ I_{sp} = X_{lsp} \cdot \theta_{lsp}, & \text{with } X_{lsp} \neq \varphi_{lsp}(\mathbf{x}, \mathbf{u}). \end{cases}$$

- Need for smooth models,
- Need for models which are fast to compute,
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$$\begin{cases} T = X_T \cdot \theta_T, & \text{with } X_T \neq \varphi_T(\mathbf{x}, \mathbf{u}), \\ D = X_D \cdot \theta_D, & \text{with } X_D \neq \varphi_D(\mathbf{x}, \mathbf{u}), \\ L = X_L \cdot \theta_L, & \text{with } X_L \neq \varphi_L(\mathbf{x}, \mathbf{u}), \\ I_{sp} = X_{lsp} \cdot \theta_{lsp}, & \text{with } X_{lsp} \neq \varphi_{lsp}(\mathbf{x}, \mathbf{u}). \end{cases}$$

- Need for smooth models,
- Need for models which are fast to compute,
- Need for interpretable models for safety,
- Need for models which are rich enough.

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$$\begin{cases} T = X_T \cdot \theta_T, & \text{with } X_T = \Phi_d \circ \varphi_T(\mathbf{x}, \mathbf{u}), \\ D = X_D \cdot \theta_D, & \text{with } X_D = \Phi_d \circ \varphi_D(\mathbf{x}, \mathbf{u}), \\ L = X_L \cdot \theta_L, & \text{with } X_L = \Phi_d \circ \varphi_L(\mathbf{x}, \mathbf{u}), \\ I_{sp} = X_{Isp} \cdot \theta_{Isp}, & \text{with } X_{Isp} = \Phi_d \circ \varphi_{Isp}(\mathbf{x}, \mathbf{u}). \end{cases}$$

- Need for smooth models,
- Need for models which are fast to compute,
- Need for interpretable models for safety,
- Need for models which are rich enough.

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$$\begin{cases} T = X_T \cdot \theta_T, & \text{with } X_T = N_1(1, \rho, M, \rho^2, \rho M, M^2, ...), \\ D = X_D \cdot \theta_D, & \text{with } X_D = q(1, \alpha, M, \alpha^2, \alpha M, M^2, ...), \\ L = X_L \cdot \theta_L, & \text{with } X_L = q(1, \alpha, M, \alpha^2, \alpha M, M^2, ...), \\ I_{sp} = X_{Isp} \cdot \theta_{Isp}, & \text{with } X_{Isp} = (1, h, M, h^2, hM, M^2, ...). \end{cases}$$

- Need for smooth models,
- Need for models which are fast to compute,
- Need for interpretable models for safety,
- Need for models which are rich enough.

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$$\begin{cases} \dot{h} = V \sin \gamma \\ m \dot{V}_r = T \cos \alpha - D - mg \sin \gamma \\ m V_r \dot{\gamma} = T \sin \alpha + L - mg \cos \gamma \\ \dot{m} = -\frac{T}{I_{sp}}. \end{cases}$$

(CMAP, INRIA, SAFETY LINE)

AIRCRAFT DYNAMICS IDENTIFICATION

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AIRCRAFT DYNAMICS IDENTIFICATION

$$\begin{cases} \dot{h} = V \sin \gamma \\ m\dot{V}_r + mg \sin \gamma = T \cos \alpha - D \\ mV_r\dot{\gamma} + mg \cos \gamma = T \sin \alpha + L \\ \dot{m} = -\frac{T}{I_{sp}}. \end{cases}$$

(CMAP, INRIA, SAFETY LINE)

AIRCRAFT DYNAMICS IDENTIFICATION

$$\begin{cases} \dot{h} = V \sin \gamma \\ m\dot{V}_r + mg \sin \gamma = T \cos \alpha - D \\ mV_r\dot{\gamma} + mg \cos \gamma = T \sin \alpha + L \\ 0 = T + \dot{m}I_{sp}. \end{cases}$$

(CMAP, INRIA, SAFETY LINE)

AIRCRAFT DYNAMICS IDENTIFICATION

Targets to fit Unknown Random error

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$$\begin{cases} Y_1 = X_T \cos \alpha \cdot \theta_T - X_D \cdot \theta_D + \varepsilon_1 \\ Y_2 = X_T \sin \alpha \cdot \theta_T + X_L \cdot \theta_L + \varepsilon_2 \\ 0 = X_T \cdot \theta_T + \dot{m} X_{lsp} \cdot \theta_{lsp} + \varepsilon_3 \end{cases}$$

(CMAP, INRIA, SAFETY LINE)

AIRCRAFT DYNAMICS IDENTIFICATION

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(CMAP, INRIA, SAFETY LINE)

AIRCRAFT DYNAMICS IDENTIFICATION

$$\begin{cases} Y_1 = X_{T1} \cdot \theta_T - X_D \cdot \theta_D + \varepsilon_1 \\ Y_2 = X_{T2} \cdot \theta_T + X_L \cdot \theta_L + \varepsilon_2 \\ 0 = X_T \cdot \theta_T + X_{lspm} \cdot \theta_{lsp} + \varepsilon_3 \end{cases}$$

(CMAP, INRIA, SAFETY LINE)

AIRCRAFT DYNAMICS IDENTIFICATION

$$Y = X\theta + \varepsilon,$$

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AIRCRAFT DYNAMICS IDENTIFICATION

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AIRCRAFT DYNAMICS IDENTIFICATION

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 $Y = X\theta + \varepsilon,$



• Ensures all components of \hat{g} to share the same thrust \hat{T} ,

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AIRCRAFT DYNAMICS IDENTIFICATION

 $Y = X\theta + \varepsilon,$



- Ensures all components of \hat{g} to share the same thrust $\hat{\mathcal{T}}$,
- Better predictive accuracy from tight coupling,

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AIRCRAFT DYNAMICS IDENTIFICATION

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 $Y = X\theta + \varepsilon,$



- Ensures all components of \hat{g} to share the same thrust $\hat{\mathcal{T}}$,
- Better predictive accuracy from tight coupling,
- Helps in high correlations setting.

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AIRCRAFT DYNAMICS IDENTIFICATION

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FEATURE SELECTION Let $\{(\mathbf{x}^{i}, \mathbf{u}^{i}, \dot{\mathbf{x}}^{i})\}_{i=1}^{N}$ set of N observations,

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AIRCRAFT DYNAMICS IDENTIFICATION

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(CMAP, INRIA, SAFETY LINE)

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Empirical Risk Minimization

$$\min_{\theta} \frac{1}{N} \sum_{i=1}^{N} \mathcal{L}(Y^{i}, X^{i}\theta),$$

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AIRCRAFT DYNAMICS IDENTIFICATION

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LEAST SQUARES REGRESSION

$$\min_{\theta} \frac{1}{N} \sum_{i=1}^{N} \|Y^i - X^i\theta\|_2^2,$$

(CMAP, INRIA, SAFETY LINE)

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FEATURE SELECTION Let $\{(\mathbf{x}^{i}, \mathbf{u}^{i}, \dot{\mathbf{x}}^{i})\}_{i=1}^{N}$ set of N observations, $\rightsquigarrow \{X^{i}, Y^{i}\}_{i=1}^{N}$, Maybe not all monomials are relevant for T, D, L and/or I_{sp} model...

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→ Overfitting...

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(CMAP, INRIA, SAFETY LINE)

AIRCRAFT DYNAMICS IDENTIFICATION

→ Overfitting...

L^1 penalization

$$\min_{\theta} \frac{1}{N} \sum_{i=1}^{N} \|Y^i - X^i\theta\|_2^2 + \lambda \|\theta\|_1,$$

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AIRCRAFT DYNAMICS IDENTIFICATION

→ Overfitting...

L^1 PENALIZATION

$$\min_{\theta} \frac{1}{N} \sum_{i=1}^{N} \|Y^i - X^i\theta\|_2^2 + \lambda \|\theta\|_1,$$

 \simeq Lasso [Tibshirani, 1994]

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AIRCRAFT DYNAMICS IDENTIFICATION

→ Overfitting...

BLOCK SPARSE LASSO

$$\min_{\theta} \frac{1}{N} \sum_{i=1}^{N} \|Y^i - X^i\theta\|_2^2 + \lambda \|\theta\|_1,$$

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Polynomial regression

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Polynomial regression \rightsquigarrow high correlations between elements of X^i

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AIRCRAFT DYNAMICS IDENTIFICATION

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→ Unstable selections...

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AIRCRAFT DYNAMICS IDENTIFICATION

→ Overfitting...

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Polynomial regression \rightsquigarrow high correlations between elements of X^i

→ Unstable selections...

\Rightarrow **Bolasso** [Bach, 2008]

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AIRCRAFT DYNAMICS IDENTIFICATION

BOOTSTRAP IMPLEMENTATION

Block sparse Bolasso

training data $\mathcal{T} = \{(X^i, Y^i)\}_{i=1}^N$, **Require:** number of bootstrap replicates m, L^1 penalization parameter λ , 1: for k = 1 to m do 2: Generate bootstrap sample \mathcal{T}^k , 3: Compute Block sparse Lasso estimate $\hat{\theta}^k$ from \mathcal{T}^k , 4: Compute support $J_k = \{j, \hat{\theta}_j^k \neq 0\}$, 5: end for 6: Compute $J = \bigcap_{k=1}^m J_k$, 7: Compute $\hat{\theta}_J$ from $\mathcal{T}_J = \{(X_I^i, Y^i)\}_{i=1}^N$ using Least-Squares.

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AIRCRAFT DYNAMICS IDENTIFICATION

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• Consistency under high correlations proved in [Bach, 2008],

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AIRCRAFT DYNAMICS IDENTIFICATION

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- Consistency under high correlations proved in [Bach, 2008],
- Efficient implementations exists: LARS [Efron et al., 2004]. •

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AIRCRAFT DYNAMICS IDENTIFICATION

$$\begin{cases} Y_1 = X_{T1} \cdot \theta_T - X_D \cdot \theta_D + \varepsilon_1 \\ Y_2 = X_{T2} \cdot \theta_T + X_L \cdot \theta_L + \varepsilon_2 \\ 0 = X_T \cdot \theta_T + X_{lspm} \cdot \theta_{lsp} + \varepsilon_3 \end{cases}$$

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$$\min_{\theta} \frac{1}{N} \sum_{i=1}^{N} \|Y^{i} - X^{i}\theta\|_{2}^{2} + \lambda_{1} \|\theta\|_{1},$$

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AIRCRAFT DYNAMICS IDENTIFICATION

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$$\begin{split} \min_{\theta} \frac{1}{N} \sum_{i=1}^{N} \|Y^{i} - X^{i}\theta\|_{2}^{2} + \lambda_{1} \|\theta\|_{1}, \\ \Rightarrow \hat{\theta}_{\mathsf{T}} = \hat{\theta}_{\mathsf{lsp}} = \mathbf{0} \; ! \end{split}$$

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AIRCRAFT DYNAMICS IDENTIFICATION

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Use prior
$$\tilde{l}_{sp} \rightsquigarrow \{\tilde{l}_{sp}^{i} = \tilde{l}_{sp}(\mathbf{x}^{i}, \mathbf{u}^{i})\}$$
.
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AIRCRAFT DYNAMICS IDENTIFICATION

$$\begin{cases} Y_1 = X_{T1} \cdot \theta_T - X_D \cdot \theta_D + \varepsilon_1 \\ Y_2 = X_{T2} \cdot \theta_T + X_L \cdot \theta_L + \varepsilon_2 \\ 0 = X_T \cdot \theta_T + X_{lspm} \cdot \theta_{lsp} + \varepsilon_3 \end{cases}$$

$$\begin{split} \min_{\theta} \frac{1}{N} \sum_{i=1}^{N} \left(\|Y^{i} - X^{i}\theta\|_{2}^{2} + \lambda_{2} \|\tilde{I}_{sp}^{i} - X_{lsp}^{i} \cdot \theta_{lsp}\|_{2}^{2} \right) + \lambda_{1} \|\theta\|_{1}, \\ \Rightarrow \hat{\theta}_{\mathsf{T}} = \hat{\theta}_{\mathsf{lsp}} = \mathbf{0} \; ! \end{split}$$

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AIRCRAFT DYNAMICS IDENTIFICATION
IDENTIFIABILITY ISSUES

$$\begin{cases} Y_1 = X_{T1} \cdot \theta_T - X_D \cdot \theta_D + \varepsilon_1 \\ Y_2 = X_{T2} \cdot \theta_T + X_L \cdot \theta_L + \varepsilon_2 \\ 0 = X_T \cdot \theta_T + X_{lspm} \cdot \theta_{lsp} + \varepsilon_3 \\ \lambda_2 \tilde{l}_{sp} = \lambda_2 X_{lsp} \cdot \theta_{lsp} + \varepsilon_4 \end{cases}$$
$$\min_{\theta} \frac{1}{N} \sum_{i=1}^N \left(\|Y^i - X^i\theta\|_2^2 + \lambda_2 \|\tilde{l}_{sp}^i - X_{lsp}^i \cdot \theta_{lsp}\|_2^2 \right) + \lambda_1 \|\theta\|_1,$$
$$\Rightarrow \hat{\theta}_T = \hat{\theta}_{lsp} = \mathbf{0} \; !$$

Use prior
$$\tilde{l}_{sp} \rightsquigarrow \{\tilde{l}_{sp}^{i} = \tilde{l}_{sp}(\mathbf{x}^{i}, \mathbf{u}^{i})\}$$
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AIRCRAFT DYNAMICS IDENTIFICATION

IDENTIFIABILITY ISSUES

$$\begin{cases} Y_1 = X_{T1} \cdot \theta_T - X_D \cdot \theta_D + \varepsilon_1 \\ Y_2 = X_{T2} \cdot \theta_T + X_L \cdot \theta_L + \varepsilon_2 \\ 0 = X_T \cdot \theta_T + X_{lspm} \cdot \theta_{lsp} + \varepsilon_3 \\ \lambda_2 \tilde{l}_{sp} = \lambda_2 X_{lsp} \cdot \theta_{lsp} + \varepsilon_4 \\ \min_{\theta} \frac{1}{N} \sum_{i=1}^N \|\tilde{\boldsymbol{Y}}^i - \tilde{\boldsymbol{X}}^i \theta\|_2^2 + \lambda_1 \|\theta\|_1, \\ \Rightarrow \hat{\theta}_T = \hat{\theta}_{lsp} = \mathbf{0} \; ! \end{cases}$$

Use prior
$$\tilde{I}_{sp} \rightsquigarrow \{\tilde{I}_{sp}^{i} = \tilde{I}_{sp}(\mathbf{x}^{i}, \mathbf{u}^{i})\}$$
.
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AIRCRAFT DYNAMICS IDENTIFICATION

IDENTIFIABILITY ISSUES

$$\begin{split} \begin{cases} Y_1 &= X_{T1} \cdot \theta_T - X_D \cdot \theta_D + \varepsilon_1 \\ Y_2 &= X_{T2} \cdot \theta_T + X_L \cdot \theta_L + \varepsilon_2 \\ 0 &= X_T \cdot \theta_T + X_{lspm} \cdot \theta_{lsp} + \varepsilon_3 \\ \lambda_2 \tilde{I}_{sp} &= \lambda_2 X_{lsp} \cdot \theta_{lsp} + \varepsilon_4 \\ &\min_{\theta} \frac{1}{N} \sum_{i=1}^N \| \tilde{\boldsymbol{Y}}^i - \tilde{\boldsymbol{X}}^i \theta \|_2^2 + \lambda_1 \| \theta \|_1, \\ \tilde{\boldsymbol{Y}}^i &= \begin{pmatrix} Y_1^i \\ Y_2^i \\ 0 \\ \lambda_2 \tilde{I}_{sp}^i \end{pmatrix}, \quad \tilde{\boldsymbol{X}}^i = \begin{pmatrix} (X_{T1}^i)^\top - (X_D^i)^\top & 0 & 0 \\ (X_{T2}^i)^\top & 0 & (X_L^i)^\top & 0 \\ (X_T^i)^\top & 0 & 0 & (X_{lspm}^i)^\top \\ 0 & 0 & 0 & \lambda_2 (X_{lsp}^i)^\top \end{pmatrix}, \end{split}$$

Use prior
$$\tilde{l}_{sp} \rightsquigarrow \{\tilde{l}_{sp}^{i} = \tilde{l}_{sp}(\mathbf{x}^{i}, \mathbf{u}^{i})\}$$
.
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AIRCRAFT DYNAMICS IDENTIFICATION

• 25 different B737-800,

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AIRCRAFT DYNAMICS IDENTIFICATION

- 25 different B737-800,
- 10 471 flights = 8 261 619 observations,

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AIRCRAFT DYNAMICS IDENTIFICATION

- 25 different B737-800,
- 10 471 flights = 8 261 619 observations,
- Block sparse Bolasso used for T, D, L and Isp,

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- 25 different B737-800,
- 10 471 flights = 8 261 619 observations,
- Block sparse Bolasso used for T, D, L and I_{sp},
- We expect similar model structures,

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Feature selection results for the thrust, drag, lift and specific impulse models.

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Effect of λ_2 on predicted states derivatives



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Effect of λ_2 on hidden elements



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$$\dot{\mathbf{x}} - \hat{g}(\mathbf{x}, \mathbf{u})$$

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AIRCRAFT DYNAMICS IDENTIFICATION

$$x - \hat{x}$$

 $u - \hat{u}$

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AIRCRAFT DYNAMICS IDENTIFICATION

$$\min_{\mathbf{x},\mathbf{u}\in\mathbb{X}\times\mathbb{U}}\int_{0}^{t_{f}} \left(\|\mathbf{u}(t)-\mathbf{u}_{m}(t)\|_{u}^{2}+\|\mathbf{x}(t)-\mathbf{x}_{m}(t)\|_{x}^{2}\right) dt$$

s.t. $\dot{\mathbf{x}}=\hat{g}(\mathbf{x},\mathbf{u})$

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AIRCRAFT DYNAMICS IDENTIFICATION

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$$\min_{\mathbf{x},\mathbf{u}\in\mathbb{X}\times\mathbb{U}}\int_{0}^{t_{f}} \left(\|\mathbf{u}(t)-\mathbf{u}_{m}(t)\|_{u}^{2}+\|\mathbf{x}(t)-\mathbf{x}_{m}(t)\|_{x}^{2}\right) dt$$
s.t. $\dot{\mathbf{x}}=\hat{g}(\mathbf{x},\mathbf{u})$



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THANK YOU FOR YOUR ATTENTION !

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