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MULTI-TASK BOLASSO BASED AIRCRAFT DYNAMICS IDENTIFICATION

C. Rommel^{1,2}, J. F. Bonnans¹,
B. Gregorutti² and P. Martinon¹

CMAP Ecole Polytechnique - INRIA¹
Safety Line²

November 14th 2017



MOTIVATION

- 20 000 airplanes — 80 000 flights per day,

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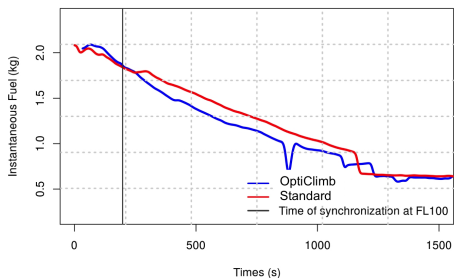
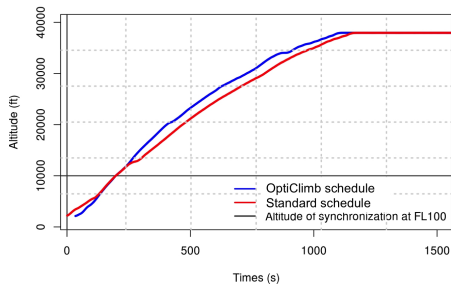
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EXAMPLE OF OPTIMIZED TRAJECTORY



Reference	Optimized
1311kg	1141kg

OPTIMAL CONTROL PROBLEM

$$\min_{(\mathbf{x}, \mathbf{u}) \in \mathbb{X} \times \mathbb{U}} \int_0^{t_f} C(t, \mathbf{u}(t), \mathbf{x}(t)) dt,$$

s.t. $\left\{ \right.$

OPTIMAL CONTROL PROBLEM

$$\begin{aligned} & \min_{(\mathbf{x}, \mathbf{u}) \in \mathbb{X} \times \mathbb{U}} \int_0^{t_f} C(t, \mathbf{u}(t), \mathbf{x}(t)) dt, \\ \text{s.t. } & \left\{ \begin{array}{l} \Phi(\mathbf{x}(0), \mathbf{x}(t_f)) \in K_\Phi, \end{array} \right. \end{aligned}$$

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OPTIMAL CONTROL PROBLEM

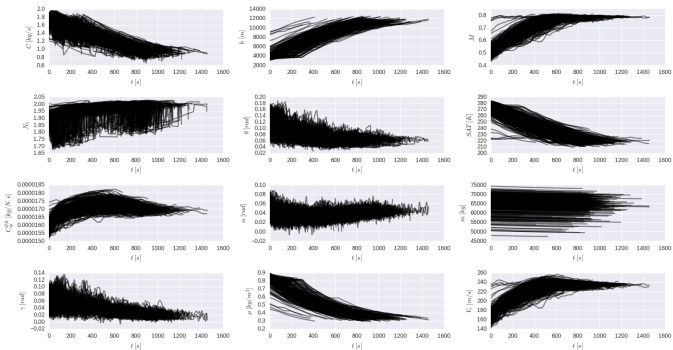
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QAR DATA

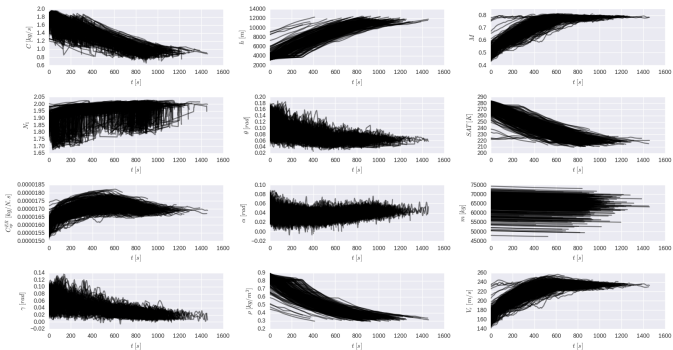
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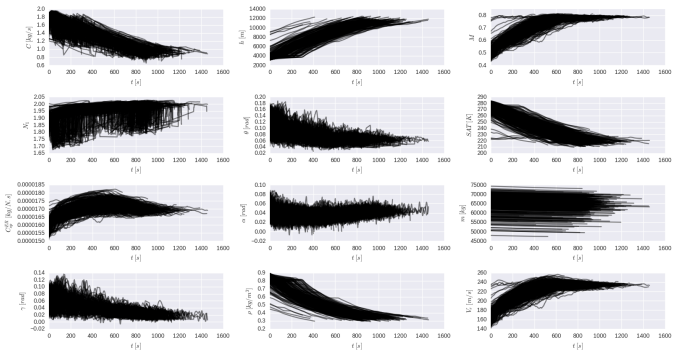


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Massive (> 1000 variables recorded every second),

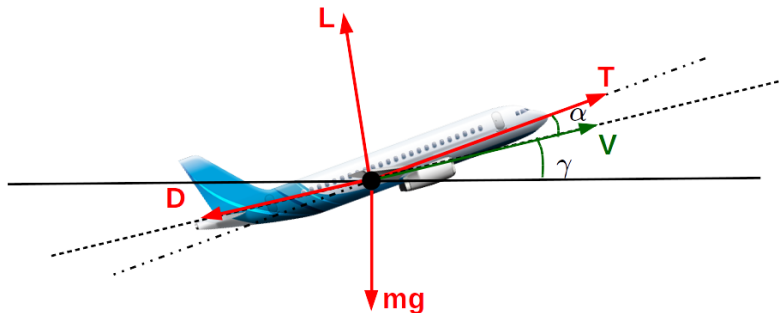
QAR DATA



Massive (> 1000 variables recorded every second),

$$\dot{\mathbf{x}} = \mathbf{g}(t, \mathbf{u}, \mathbf{x})$$

FLIGHT MECHANICS AND STATE EQUATION



Classic flight mechanics model

FLIGHT MECHANICS AND STATE EQUATION

$$\left\{ \begin{array}{l} \dot{h} = V \sin \gamma, \\ \dot{V} = \frac{T \cos \alpha - D - mg \sin \gamma}{m}, \\ \dot{\gamma} = \frac{T \sin \alpha + L - mg \cos \gamma}{mV}, \\ \dot{m} = -\frac{T}{I_{sp}}. \end{array} \right.$$

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FLIGHT MECHANICS AND STATE EQUATION

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Unknown functions of the state and control variables

MODEL REQUIREMENTS

$$\left\{ \begin{array}{l} T \text{ function of } (M, \rho, N_1), \\ D \text{ function of } (M, \rho, q), \\ L \text{ function of } (M, \rho, q), \\ I_{sp} \text{ function of } (M, h, SAT), \end{array} \right.$$

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$$\left\{ \begin{array}{l} T \text{ function of } (M, \rho, N_1) = \varphi_T(\mathbf{x}, \mathbf{u}), \\ D \text{ function of } (M, \rho, q) = \varphi_D(\mathbf{x}, \mathbf{u}), \\ L \text{ function of } (M, \rho, q) = \varphi_L(\mathbf{x}, \mathbf{u}), \\ I_{sp} \text{ function of } (M, h, SAT) = \varphi_{I_{sp}}(\mathbf{x}, \mathbf{u}), \end{array} \right.$$

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REGRESSION PROBLEMS

$$\begin{cases} \dot{h} &= V \sin \gamma \\ m\dot{V}_r &= T \cos \alpha - D - mg \sin \gamma \\ mV_r\dot{\gamma} &= T \sin \alpha + L - mg \cos \gamma \\ \dot{m} &= -\frac{T}{I_{sp}}. \end{cases}$$

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⇓

$$\left\{ \begin{array}{l} Y_1 = X_T \cos \alpha \cdot \theta_T - X_D \cdot \theta_D + \varepsilon_1 \\ Y_2 = X_T \sin \alpha \cdot \theta_T + X_L \cdot \theta_L + \varepsilon_2 \\ 0 = X_T \cdot \theta_T + \dot{m}X_{lsp} \cdot \theta_{lsp} + \varepsilon_3 \end{array} \right.$$

Targets to fit

Unknown

Random error

MULTI-TASK REGRESSION FRAMEWORK

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- Ensures all components of \hat{g} to share the same thrust \hat{T} ,
- Better predictive accuracy from tight coupling,
- Helps in high correlations setting.

FEATURE SELECTION

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EMPIRICAL RISK MINIMIZATION

$$\min_{\theta} \frac{1}{N} \sum_{i=1}^N \mathcal{L}(Y^i, X^i \theta),$$

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L^1 PENALIZATION

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Polynomial regression

FEATURE SELECTION

Let $\{(\mathbf{x}^i, \mathbf{u}^i, \dot{\mathbf{x}}^i)\}_{i=1}^N$ set of N observations, $\rightsquigarrow \{X^i, Y^i\}_{i=1}^N$,

Maybe not all monomials are relevant for T, D, L and/or I_{sp} model...

\rightsquigarrow **Overfitting...**

BLOCK SPARSE LASSO

$$\min_{\theta} \frac{1}{N} \sum_{i=1}^N \|Y^i - X^i \theta\|_2^2 + \lambda \|\theta\|_1,$$

\simeq Lasso [Tibshirani, 1994]

Polynomial regression \rightsquigarrow high correlations between elements of X^i

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\Rightarrow **Bolasso** [Bach, 2008]

BOOTSTRAP IMPLEMENTATION

Block sparse Bolasso

training data $\mathcal{T} = \{(X^i, Y^i)\}_{i=1}^N$,

Require: number of bootstrap replicates m ,
 L^1 penalization parameter λ ,

- 1: **for** $k = 1$ **to** m **do**
 - 2: Generate bootstrap sample \mathcal{T}^k ,
 - 3: Compute Block sparse Lasso estimate $\hat{\theta}^k$ from \mathcal{T}^k ,
 - 4: Compute support $J_k = \{j, \hat{\theta}_j^k \neq 0\}$,
 - 5: **end for**
 - 6: Compute $J = \bigcap_{k=1}^m J_k$,
 - 7: Compute $\hat{\theta}_J$ from $\mathcal{T}_J = \{(X_J^i, Y^i)\}_{i=1}^N$ using Least-Squares.
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- Consistency under high correlations proved in [Bach, 2008],
- Efficient implementations exists: LARS [Efron et al., 2004].

IDENTIFIABILITY ISSUES

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$$\min_{\theta} \frac{1}{N} \sum_{i=1}^N \|\tilde{Y}^i - \tilde{X}^i \theta\|_2^2 + \lambda_1 \|\theta\|_1,$$

$$\tilde{Y}^i = \begin{pmatrix} Y_1^i \\ Y_2^i \\ 0 \\ \lambda_2 \tilde{l}_{sp}^i \end{pmatrix}, \quad \tilde{X}^i = \begin{pmatrix} (X_{T1}^i)^\top & -(X_D^i)^\top & 0 & 0 \\ (X_{T2}^i)^\top & 0 & (X_L^i)^\top & 0 \\ (X_T^i)^\top & 0 & 0 & (X_{Ispm}^i)^\top \\ 0 & 0 & 0 & \lambda_2 (X_{Isp}^i)^\top \end{pmatrix},$$

Use prior $\tilde{l}_{sp} \rightsquigarrow \{\tilde{l}_{sp}^i = \tilde{l}_{sp}(\mathbf{x}^i, \mathbf{u}^i)\}$.

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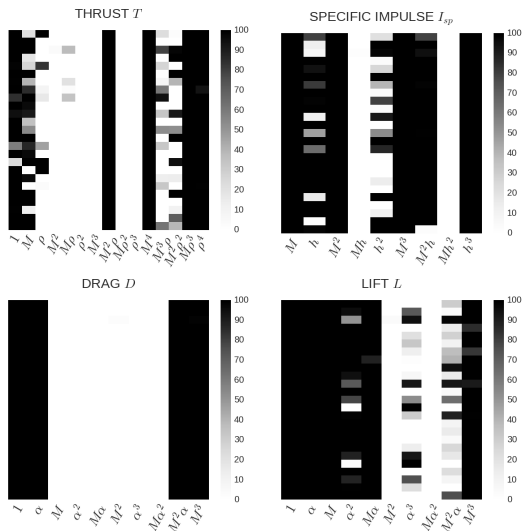
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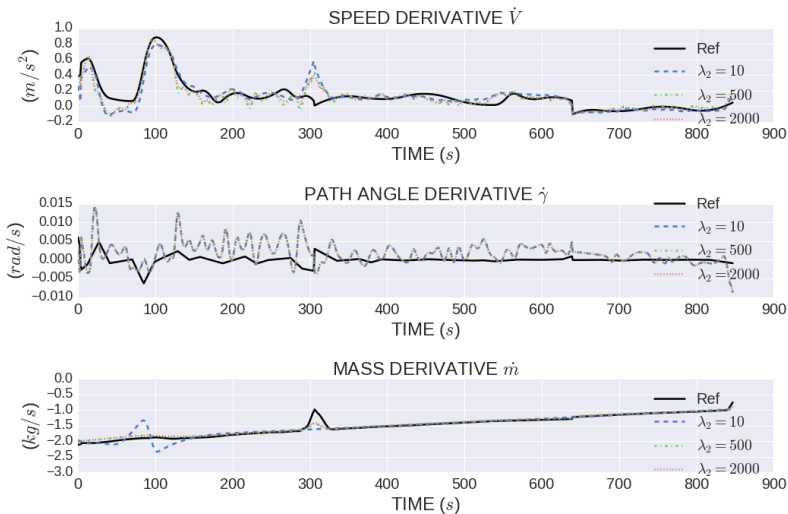
- 25 different B737-800,
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- We expect similar model structures,

FEATURE SELECTION RESULTS

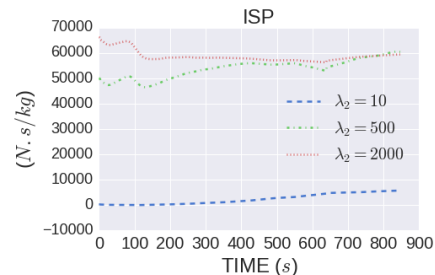
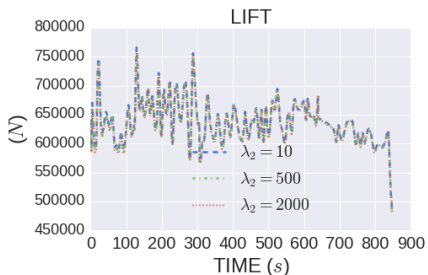
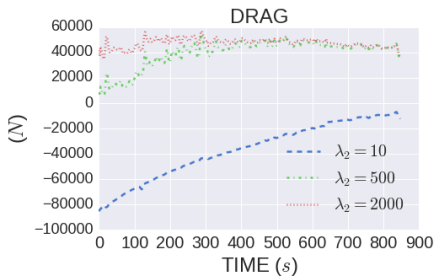
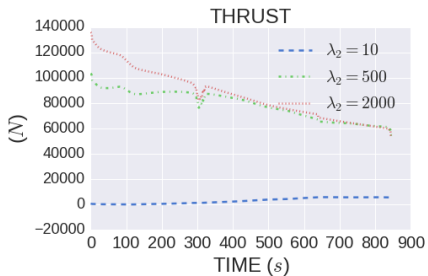


Feature selection results for the thrust, drag, lift and specific impulse models.

EFFECT OF λ_2 ON PREDICTED STATES DERIVATIVES



EFFECT OF λ_2 ON HIDDEN ELEMENTS



IDENTIFICATION RESULTS ASSESSMENT

$$\dot{\mathbf{x}} - \hat{\mathbf{g}}(\mathbf{x}, \mathbf{u})$$

IDENTIFICATION RESULTS ASSESSMENT

$$\mathbf{x} - \hat{\mathbf{x}}$$

$$\mathbf{u} - \hat{\mathbf{u}}$$

IDENTIFICATION RESULTS ASSESSMENT

$$\begin{aligned} \min_{\mathbf{x}, \mathbf{u} \in \mathbb{X} \times \mathbb{U}} \int_0^{t_f} (\|\mathbf{u}(t) - \mathbf{u}_m(t)\|_u^2 + \|\mathbf{x}(t) - \mathbf{x}_m(t)\|_x^2) dt \\ \text{s.t. } \dot{\mathbf{x}} = \hat{\mathbf{g}}(\mathbf{x}, \mathbf{u}) \end{aligned}$$

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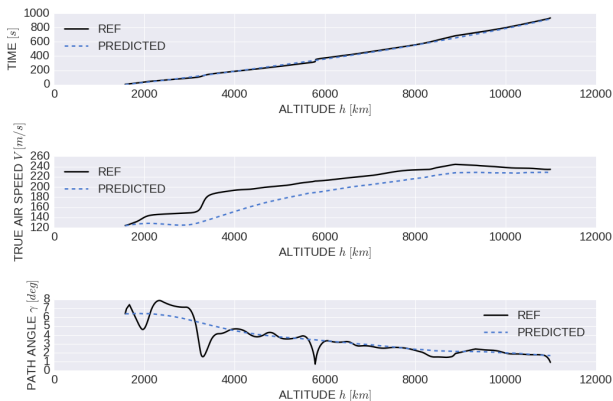


FIGURE : Obtained using BOCOP [Bonnans et al., 2017]

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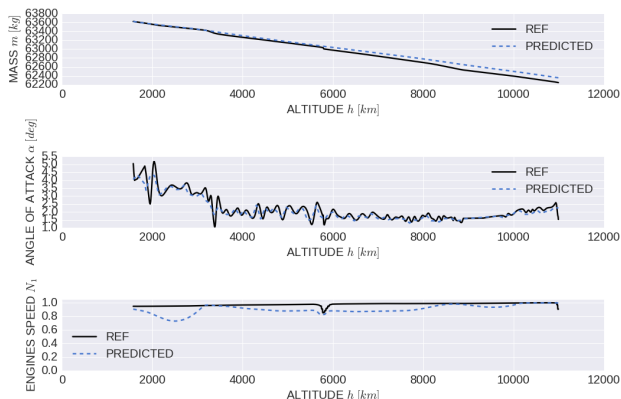






FIGURE : Obtained using BOCOP [Bonnans et al., 2017]

THANK YOU FOR YOUR ATTENTION !

REFERENCES

-  Bach, F. (2008).
Bolasso: model consistent Lasso estimation through the bootstrap.
pages 33–40. Proceedings of the 25th International Conference on
Machine Learning (ICML).
-  Bonnans, J. F., Giorgi, D., Grelard, V., Heymann, B., Maindrault, S.,
Martinon, P., Tissot, O., and Liu, J. (2017).
Bocop – A collection of examples.
Technical report, INRIA.
-  Efron, B. et al. (2004).
Least angle regression.
Annals of Statistics, 32:407–499.
-  Tibshirani, R. (1994).
Regression shrinkage and selection via the Lasso.
Journal of the Royal Statistical Society, 58:267–288.