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Abstract

The local approach to linear parameter varying (LPV) system identification consists in interpolating individually estimated local linear time invariant (LTI) models corresponding to fixed values of the scheduling variable. It is shown in this paper that, without any global structural assumption of the considered LPV system, individually estimated local state-space LTI models do not contain sufficient information for determining similarity transformations making them coherent. It is possible to estimate these similarity transformations from input-output data under appropriate excitation conditions.

Key words: System identification, LPV model, coherent local linear models.

1 Introduction

Linear parameter varying (LPV) models provide an effective approach to handling nonlinear control systems [14, 9, 6, 11]. Some successful methods for LPV system identification have been reported recently [17, 8, 5, 13, 21, 10]. In the local approach to LPV system identification, interpolation is essential to establishing global models from a collection of locally estimated linear time invariant (LTI) models [14, 2, 1]. As LTI state-space models can be estimated in an arbitrary state basis, it is necessary to use a coherent collection of local models for the purpose of interpolation.

This paper is focused on the problem of making local state-space models coherent, *without treating the interpolation step*. Only *state-space* models are considered in this paper, as local model coherence is not relevant for other models. For shorter expressions, the words "state-space" will be omitted from terms like "local state-space model" and "LTI state-space model". In practice, interpolation is based on a finite set of local LTI models, each corresponding to a specific value of the scheduling variable $p(t)$, the main discussion of this paper is thus about the case where $p(t)$ evolves within a finite set, but its motivation is indeed with the perspective of interpolation for continuous values of $p(t)$.

It seems natural to transform all the local LTI models to some canonical form in order to make them coherent. *The main purpose* of this paper is to point out the fact that, in the local approach to LPV system identification, *structurally independent* local LTI models *themselves* do *not* contain sufficient information to determine similarity transformations making them coherent. However, locally estimated LTI models can be made coherent by making use of the information contained in some input-output data sequences across all the working points, notably with an algorithm initially introduced in the framework of piecewise linear hybrid systems [18, 20].

Preliminary results of this work have been presented in [20], which are completed in the present paper with a rigorous proof of the main result.

2 Problem statement

Let $u(t) \in \mathbb{R}^q$ and $y(t) \in \mathbb{R}^s$ be respectively the input and the output at discrete time instant $t = 0, 1, 2, \ldots, p(t)$ be the scheduling variable evolving within a compact set $\mathbb{\Pi}$. An LPV system is described by the state-space model

$$x(t+1) = A(p(t))x(t) + B(p(t))u(t) + w(t) \quad (1a)$$
$$y(t) = C(p(t))x(t) + D(p(t))u(t) + v(t) \quad (1b)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $A(p(t)), B(p(t)), C(p(t)), D(p(t))$ are matrices of appropriate sizes depending on $p(t) \in \mathbb{\Pi}$, and $w(t) \in \mathbb{R}^n, v(t) \in \mathbb{R}^s$ are state and output noises with covariance matrices $Q(p(t)), R(p(t))$.

Based on the fact that the LPV system (1) becomes an LTI system when the scheduling variable $p(t)$ is maintained at a fixed value, the following definitions aim at establishing a link between LPV and LTI models.

Consider a set of m LTI models indexed by the integer i:

$$x(t+1) = A_i x(t) + B_i u(t) + w(t) \qquad (2a)$$
$$y(t) = C_i x(t) + D_i u(t) + v(t) \qquad (2b)$$

characterized by matrices A_i, B_i, C_i, D_i of appropriate sizes, and noise covariance matrices Q_i and R_i.

The notation

$$\sigma_i \triangleq (A_i, B_i, C_i, D_i, Q_i, R_i) \qquad (3)$$

will be used to denote the *matrices* characterizing the i-th local LTI model (2), or the LTI model itself *by abuse of notation*. The set of LTI models will be denoted by

$$\Sigma = \{\sigma_i : i = 1, 2, \ldots, m\}. \qquad (4)$$

Definition 1 *A set of local LTI models*

$$\Sigma^* = \{(A_i^*, B_i^*, C_i^*, D_i^*, Q_i^*, R_i^*) : i = 1, 2, \ldots, m\} \quad (5)$$

is called a multi-snapshot *of the LPV system* (1) *for*

$$p(t) \in \mathbb{P} = \{p_1, \ldots, p_m\} \subset \mathbb{\Pi}, \qquad (6)$$

if

$$A_i^* = A(p_i), \; B_i^* = B(p_i), \; C_i^* = C(p_i),$$
$$D_i^* = D(p_i), \; Q_i^* = Q(p_i), \; R_i^* = R(p_i). \qquad (7)$$

$\square$

In the local approach to LPV system identification [14, 2, 1], a set of locally estimated LTI models are interpolated to obtain a global model. As such local LTI models are typically estimated up to *different and arbitrary similarity transformations*, they do not constitute a multi-snapshot of the underlying LPV system in the sense of Definition 1. It is thus important to make the local models "coherent" before the interpolation step.

What does mean a "coherent" set of local LTI models with the perspective of their interpolation? The following auxiliary definition will be helpful.

Definition 2 *The input-output behavior of a set of local LTI models $\Sigma = \{\sigma_1, \ldots, \sigma_m\}$ is the input-output behavior of the* multi-model switching system *consisting of the same set of LTI models, such that the i-th LTI model σ_i is active when $p(t) = p_i \in \mathbb{P} = \{p_1, \ldots, p_m\}$, and at every transition between two LTI models, the initial state of the new active model is equal to the final state of the previous active LTI model.* $\square$

For example, following this definition, the input-output behavior of a multi-snapshot Σ^* (see Definition 1) of an LPV system (1) is identical to input-output behavior the LPV system when $p(t)$ evolves within the restricted set $\mathbb{P}$.

If the interpolation of a set of local LTI models is expected to describe correctly an LPV system for all sequences of $p(t)$ within $\mathbb{\Pi}$, it should also be true in the particular case where $p(t)$ evolves within the restricted set $\mathbb{P} = \{p_1, \ldots, p_m\} \subset \mathbb{\Pi}$, including when $p(t)$ switches between different values within $\mathbb{P}$. It means that a "coherent" set of local LTI models should have the same input-output behavior as the underlying LPV system when $p(t) \in \mathbb{P}$, in the sense of Definition 2. This requirement will be satisfied by Definition 4 through Property 1.

Definition 3 *Two LTI models $\sigma \triangleq (A, B, C, D, Q, R)$ and $\tilde{\sigma} \triangleq (\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}, \tilde{Q}, \tilde{R})$ are related by a* similarity transformation *characterized by an invertible matrix $T \in \mathbb{R}^{n \times n}$ and denoted as*

$$\sigma \xrightarrow{T} \tilde{\sigma}, \qquad (8)$$

if the matrices characterizing the two LTI models satisfy

$$\tilde{A} = TAT^{-1}, \quad \tilde{B} = TB, \quad \tilde{C} = CT^{-1}, \qquad (9a)$$
$$\tilde{D} = D, \quad \tilde{Q} = TQT^T, \quad \tilde{R} = R. \qquad (9b)$$

$\square$

Definition 4 *A set of LTI models*

$$\tilde{\Sigma} = \{\tilde{\sigma}_i : i = 1, 2, \ldots, m\}$$

is said coherent *with another set*

$$\Sigma = \{\sigma_i : i = 1, 2, \ldots, m\},$$

if there exits an invertible transformation matrix $T \in \mathbb{R}^{n \times n}$, common to the local models, such that

$$\sigma_i \xrightarrow{T} \tilde{\sigma}_i \quad \text{for all } i = 1, \ldots, m. \qquad (10)$$

This relationship between Σ and $\tilde{\Sigma}$ is then denoted by

$$\Sigma \xRightarrow{T} \tilde{\Sigma}. \qquad (11)$$

The local LTI models $\tilde{\sigma}_i$ are simply said coherent *when the reference model set Σ is obvious.* $\square$

The relevance of this definition is justified by the following property, as discussed before Definition 3.

Property 1 *If a set of local models $\tilde{\Sigma}$ is coherent with the multi-snapshot Σ^* of the LPV system (1), then $\tilde{\Sigma}$ has the same input-output behavior (in the sense of Definition 2) as that of the LPV system (1) with the scheduling variable $p(t)$ restricted to the finite set $\{p_1, \ldots, p_m\}$ and with appropriate initial states.* $\square$

The proof of this property is trivial: under the assumed conditions, the LPV system (1) and its multi-snapshot Σ^* have the same input-output behavior, and the states of Σ^* and $\tilde{\Sigma}$ are related by the transformation matrix T.

Property 1 is a *necessary* condition that a relevant definition of local model coherence should satisfy. It does

not exclude other possible definitions, notably those based on p-dependent state transformation matrices $T(p)$. Such considerations, related to LPV system equivalent state-space representations as investigated in [3], would be out of the scope of this technical communiqué.

In practice, local LTI models are estimated from a finite data sample subject to random uncertainties, thus the definition of coherent local models is understood in an approximative sense. If the estimation of each local model is consistent, then Definition 4 can also be understood for the limiting models when the data size for each local model estimation tends to infinity.

If some global structural assumptions of the matrix functions $A(p), B(p)$, etc. were assumed, then they could be used to make estimated local LTI models coherent. This paper is focused on *structurally independent local LTI models*, as defined below.

Definition 5 *A set of local LTI models σ_i are structurally dependent if their parametrizations are such that fixing the matrices $A_i, B_i, C_i, D_i, Q_i, R_i$ reduces the degrees of freedom of the matrices $A_j, B_j, C_j, D_j, Q_j, R_j$, for $j \neq i$; otherwise they are structurally independent.* □

For instance, if a set of local LTI models is parametrized such that all the matrices A_i share an equal entry at the same position, say $A_i(1,1) = A_j(1,1)$ for all $i, j = 1, 2, \ldots, m$, then the local models are structurally *dependent*. A less trivial counterexample will be given in Section 3.2 with equation (27).

3 Making local LTI models coherent

Apply *different* similarity transformations $\hat{T}_i$ to a set of local LTI models Σ^*, resulting in a transformed set of models $\hat{\Sigma}$, which is *not* coherent with Σ^*. Assume that neither $\hat{T}_i$ nor Σ^* is known, is it possible to derive, *solely from $\hat{\Sigma}$*, a model set $\tilde{\Sigma}$ coherent with Σ^*?

3.1 Structurally independent local LTI models

Proposition 1 *Given a set of structurally independent LTI models*

$$\Sigma^* = \{\sigma_i^* : i = 1, 2, \ldots, m\}, \tag{12}$$

assume that a set of arbitrary unknown invertible matrices

$$\{\hat{T}_i \in \mathbb{R}^{n \times n} : i = 1, 2, \ldots, m\} \tag{13}$$

transform the LTI models

$$\sigma_i^* \xrightarrow{\hat{T}_i} \hat{\sigma}_i, \tag{14}$$

resulting in a set of transformed models

$$\hat{\Sigma} = \{\hat{\sigma}_i : i = 1, 2, \ldots, m\}, \tag{15}$$

then it is impossible to determine invertible matrices $\tilde{T}_i \in \mathbb{R}^{n \times n}$ (not necessarily equal to $\hat{T}_i$) for $i = 1, \ldots, m$, solely *based on the set of transformed models $\hat{\Sigma}$, such that the further transformed set of models*

$$\tilde{\Sigma} = \{\tilde{\sigma}_i : i = 1, 2, \ldots, m\} \tag{16}$$

through

$$\hat{\sigma}_i \xrightarrow{\tilde{T}_i} \tilde{\sigma}_i \tag{17}$$

is coherent with the original set Σ^*, or more precisely,

$$\Sigma^* \xRightarrow{T} \tilde{\Sigma} \tag{18}$$

for some unknown $T \in \mathbb{R}^{n \times n}$. □

Proof. The proof proceeds *by contradiction*, starting by assuming *the contrary* of Proposition 1, in which the word "impossible" is replaced by "possible", or more compactly, "*solely* based on $\hat{\Sigma}$, the original model set Σ^* *can* be determined up to an unknown transformation matrix T common to all local LTI models". As the matrix $T \in \mathbb{R}^{n \times n}$ can be *any* invertible matrix, what is determined from $\hat{\Sigma}$, is a set of (infinitely many) sets of LTI models, which will be *denoted by* $\mathbb{S}(\hat{\Sigma})$ in what follows. The *contrary* of Proposition 1 then means

$$\mathbb{S}(\hat{\Sigma}) = \{\Sigma : \Sigma^* \xRightarrow{T} \Sigma, T \in \mathbb{R}^{n \times n}, \det(T) \neq 0\}. \tag{19}$$

Within this set, the particular element Σ^* corresponds to the identity transformation matrix T, hence

$$\Sigma^* \in \mathbb{S}(\hat{\Sigma}). \tag{20}$$

Consider a set of m invertible and *pairwise distinct* matrices $S_i \in \mathbb{R}^{n \times n}$, $i = 1, 2, \ldots, m$ and a model set

$$\Sigma' = \{\sigma_i' : i = 1, 2, \ldots, m\} \tag{21}$$

such that

$$\sigma_i' \xrightarrow{S_i} \sigma_i^* \tag{22}$$

for $i = 1, 2, \ldots, m$. The two transformations (22) and (14) then imply

$$\sigma_i' \xrightarrow{\hat{T}_i'} \hat{\sigma}_i \tag{23}$$

with $\hat{T}_i' = \hat{T}_i S_i$, for $i = 1, 2, \ldots, m$. This result means that the model set $\hat{\Sigma}$ originating from Σ^* through transformations (14) could also have originated from Σ' through transformations (23). Therefore, following the same reasoning as at the beginning of this proof, that has led to $\Sigma^* \in \mathbb{S}(\hat{\Sigma})$, it yields $\Sigma' \in \mathbb{S}(\hat{\Sigma})$. This result

and (19) imply that $\Sigma^* \overset{T}{\Longrightarrow} \Sigma'$ for some invertible T. In terms of the LTI models composing Σ^* and Σ', it means

$$\sigma_i^* \overset{T}{\longrightarrow} \sigma_i' \tag{24}$$

with a common T for all $i = 1, 2, \ldots, m$. This result is clearly in contradiction with (22), where S_i are *pairwise distinct* matrices. This contradiction invalidates the assumption made at the beginning of this proof, hence proving Proposition 1. $\qquad\square$

3.2 Examples based on global structural assumptions

The result of Proposition 1 may seem in contradiction with some known methods for making local LTI models coherent. In fact, these methods assume (implicitly) some particular structure of the matrix-valued functions $A(p), B(p)$ etc., therefore, the local LTI models are not *structurally independent* in the sense of Definition 5. To better clarify the situations, some examples are recalled below, by pointing out their particular global structural assumptions.

3.2.1 Coherent LTI models based on canonical forms

In order to make local LTI models coherent, a *natural idea* is to find similarity transformations of the local LTI models leading to some LTI canonical state-space form, or to estimate local LTI models directly in such a form [19, 12, 7, 2]. This practice *assumes* that the local LTI models are coherent when they are all transformed into the same LTI canonical form [1].

For the sake of presentation simplicity, consider the case of single-input-single-output (SISO) models. In the controllable form, the m local models involve, for $i = 1, 2, \ldots, m$,

$$\tilde{A}_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ a_i^{(1)} & a_i^{(2)} & a_i^{(3)} & \cdots & a_i^{(n)} \end{bmatrix}, \quad \tilde{B}_i = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}. \tag{25}$$

The assumption that the local LTI models in this canonical form are coherent implies that there exists a common invertible matrix $T \in \mathbb{R}^{n \times n}$ such that, for all $i = 1, 2, \ldots, m$, $\tilde{A}_i = T A_i^* T^{-1}$. Let $\sqcap(M)$ denote the sub matrix of M excluding its last row, then

$$\sqcap \left(T A_i^* T^{-1} \right) = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}, \quad \forall i = 1, 2, \ldots, m. \tag{26}$$

By assuming that $\sqcap \left(T A_i^* T^{-1} \right)$ are equal to the same matrix for all $i = 1, 2, \ldots, m$, the local LTI models are not structurally independent in the sense of Definition 5.

3.2.2 Coherent LTI models based on the observability matrix

In [1] another method is proposed to make local LTI models coherent, in a framework more general than the one of the present paper. In the particular case of SISO observable systems, this method consists in finding different similarity transformations so that the m transformed local LTI models all have the same observability matrix. It implies the assumption that the local LTI systems composing a multi-snapshot of the underlying LPV system all have the same observability matrix, or more explicitly,

$$C_i A_i^s = C_j A_j^s \tag{27}$$

for all $i, j \in \{1, 2, \ldots, m\}$ and $s \in \{0, 1, \ldots, n-1\}$. Again the local LTI models are not structurally independent in the sense of Definition 5.

Remark. This observability matrix-based method is *incompatible* with the previously presented example of canonical form-based method, in the sense that in general the two resulting model sets are not coherent with each other. Moreover, the canonical *observable* form-based method is incompatible with the canonical *controllable* form-based method, and similarly the *observability* matrix-based method is incompatible with the *controllability* matrix-based method. These incompatibilities between "natural" methods illustrate the fact that there is no *generally natural* global structural assumption for making estimated local LTI models coherent. $\qquad\square$

4 Data-based transformations for coherent LTI models

In order to make local LTI models coherent, as an alternative to global structural assumptions, the information contained in some data sequences can be used.

The idea is based on Property 1 that is satisfied by coherent local LTI models. In particular, the state trajectory "continuity" in Definition 2 is essential: when the scheduling variable $p(t)$ switches from one value to another within $\mathbb{P}$, the local LTI model switches accordingly, but the initial state of the new active LTI model is equal to the final state of the previous active LTI model.

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[1] Transforming the local LTI models to the same LTI canonical form does not lead to a canonical form of the underlying LPV system, as pointed out in [15].

Given a data set $\mathbb{D} = \{(p(t), u(t), y(t)) : t = 1, 2, \ldots, N\}$ with $p(t) \in \mathbb{P}$, start by segmenting the data sequence into pieces of constant $p(t)$ value. If the local LTI models are estimated from pieces of this data set, the whole state trajectory on each data segment can be obtained as a by-product of the identification procedure [16]. If the local LTI models have been estimated from another data set, then the initial state and the whole state trajectory on each data segment of $\mathbb{D}$ can be estimated with the already estimated local LTI models [4]. Similarity transformations of the local LTI models are then determined so that, when the local LTI model switches, the initial state of the new active LTI model is equal to the final state of the previous active LTI model.

A complete data-based algorithm for making local LTI models coherent has been introduced in [18] in the framework of piecewise linear hybrid systems. A variant algorithm has been presented in [20].

5 Conclusion

It has been shown in this paper that structurally independent local LTI models estimated from data collected with fixed scheduling variable of an LPV system do not contain sufficient information to make themselves coherent with the perspective of LPV model interpolation. Nevertheless, well designed data sequences can be used to address this problem.

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