

Handling spatio-temporal correlations in neuronal systems

Bruno Cessac

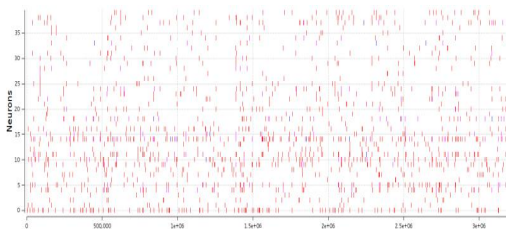
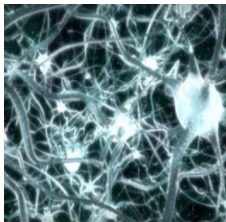
Biovision Team, INRIA Sophia Antipolis, France.

18-01-2017

- 1 Introduction
- 2 Mathematical setting
- 3 Handling data
- 4 Conclusion

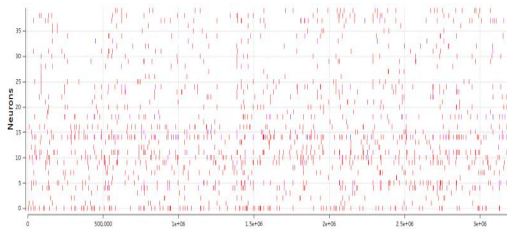
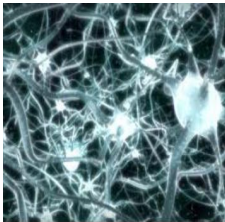
Introduction

Spike trains

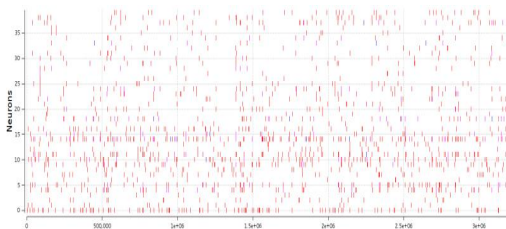
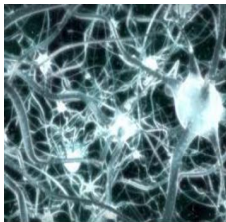


Spike activity results from neurons dynamics under stimulation.

Spike trains



Spike trains



Local dynamics

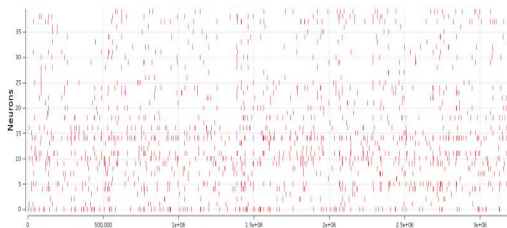
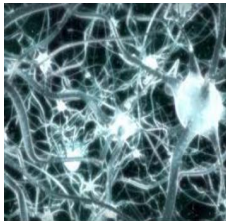
Network interactions

Stimulus

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specific spatio-temporal correlations

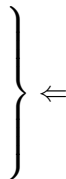
Spike trains



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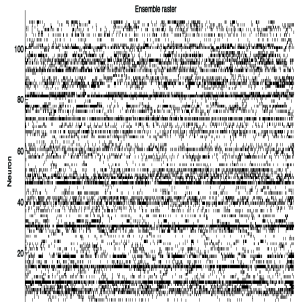
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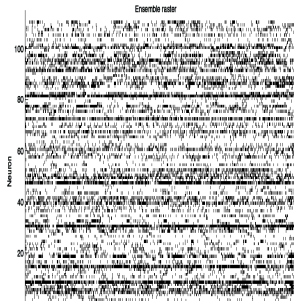
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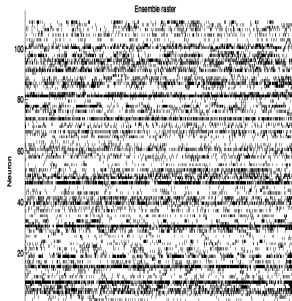
Mathematical setting





This spike train has been generated by an hidden *dynamics / stochastic process.*

Can we infer this process from the spike train's analysis ?

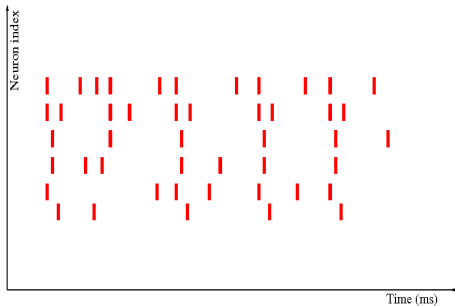


What do we learn about:

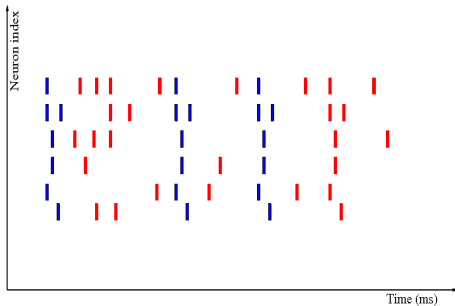
- the stimulus;
- the underlying network;
- the response of that network to a stimulus.

Handling temporality and memory

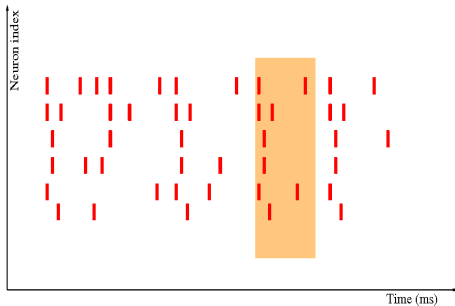
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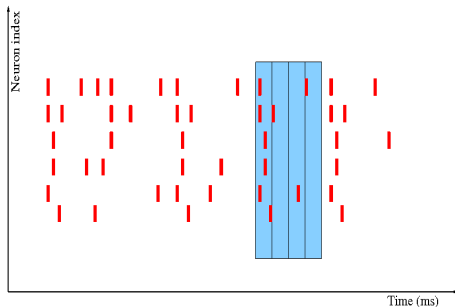
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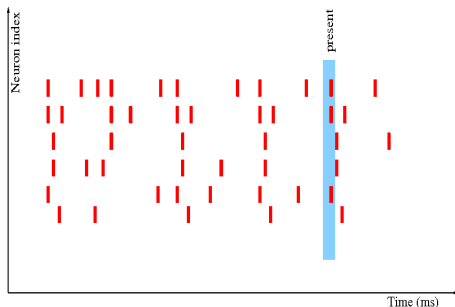


Handling temporality and memory



Models considering successive times as independent

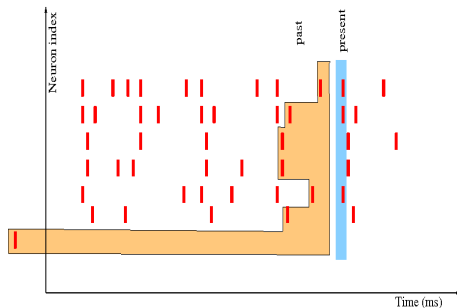
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Probability of characteristic spatio-temporal patterns

The probability of a spike pattern
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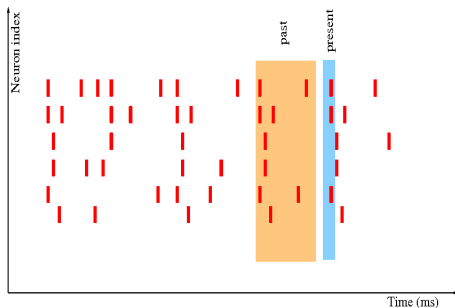
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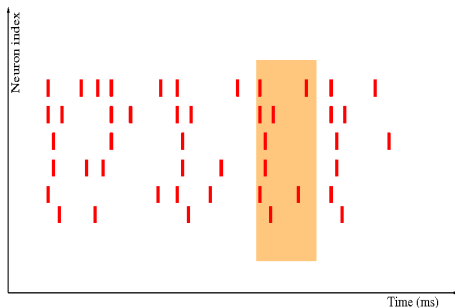
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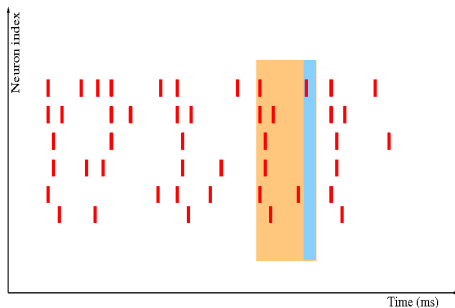
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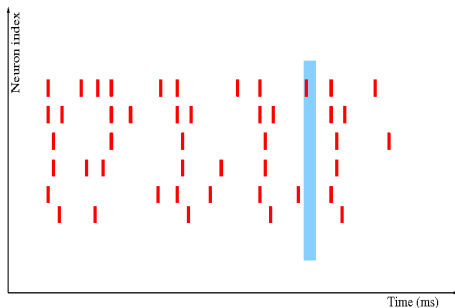
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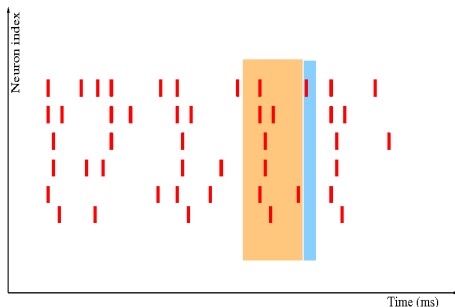
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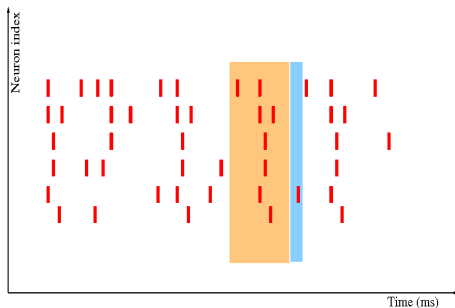
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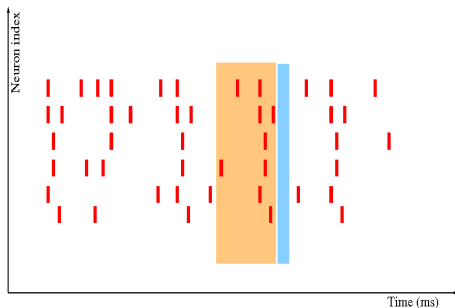
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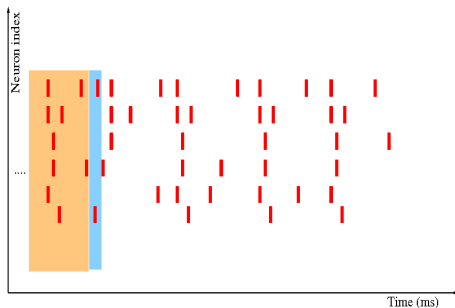
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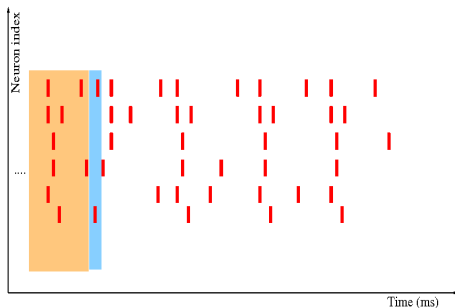
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Probability of characteristic spatio-temporal patterns

Given a set of hypotheses on transition probabilities there exists a mathematical framework to solve the problem.

Estimate:

$$P_n [\omega(n) \mid \omega_{n-D}^{n-1}]$$

where D can be

- Finite and constant (Markov chain).
- Finite and variable (Variable length Markov chain).
- Infinite (Chain with complete connection).

Estimating a probability in the spatio-temporal domain

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- 1 Conjecture the form of the transition probability as a function of network parameters and stimulus (Poisson, Hawkes, GLM).
- 2 Extract the form of the transition probability from spatio-temporal correlations (Maximum entropy).
- 3 Compute its form in simplified neural network models and extrapolate.

Example 1: The Generalized-Linear Model (GLM)

Paradigms of rates and receptive fields.

$$\lambda_k(t|H_t^{hist}) = f(b_k + (K_k * x)(t) + \sum_j (h_{kj} * y_j)(t))$$

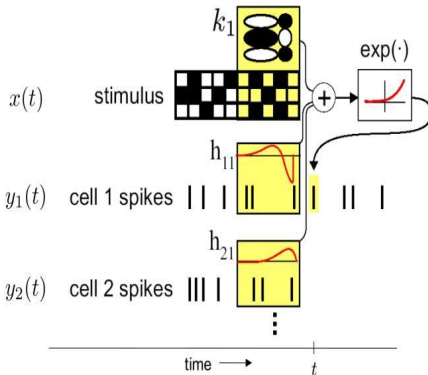
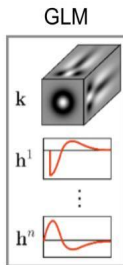
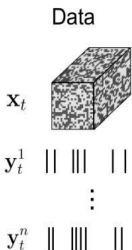
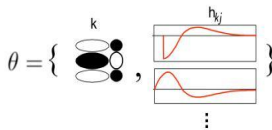


Figure: Generalized Linear Models.

Example 2: Maximum Entropy models

- Assume data is **stationary**.
- Choose a set of characteristic spike events.
- Compute their empirical mean.
 π_I : emp. prob. of spike event I .
- Find a probability distribution μ which *reproduces the empirical averages*:

$$\mu [\text{Spike event } I] = \pi_I \quad (C)$$

without any further assumption.



Maximize the entropy under constraints C .

- In good cases this probability exists and is unique. It is called a *Gibbs distribution*.

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$$\mathcal{H} = \sum_I h_I m_I$$

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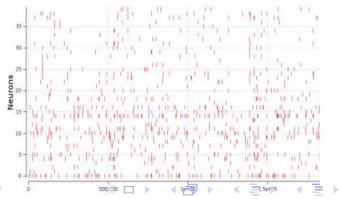
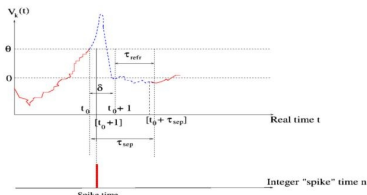
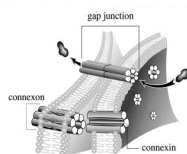
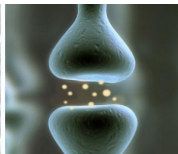
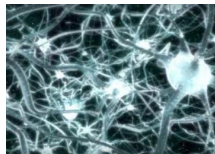
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Gibbs distribution extends to (i) non stationarity; (ii) infinite memory.

Example 3: An Integrate and Fire neural network model with chemical and electric synapses

R. Cofré, B. Cessac: "Dynamics and spike trains statistics in conductance-based Integrate-and-Fire neural networks with chemical and electric synapses", Chaos, Solitons and Fractals, 2013.
(Inspired from Rudolph, Destexhe, 2006).



Example 3: An Integrate and Fire neural network model with chemical and electric synapses

Sub-threshold dynamics:

$$C_k \frac{dV_k}{dt} = -g_{L,k}(V_k - E_L)$$

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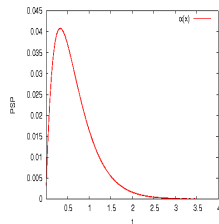


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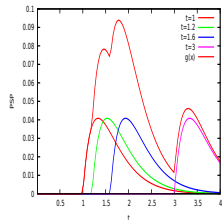


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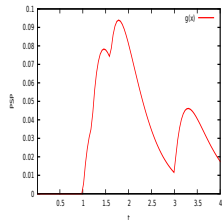


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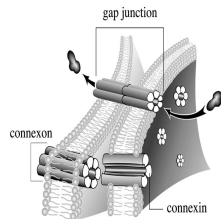
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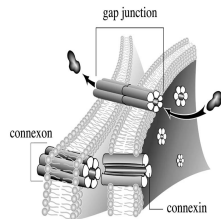
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Spike generation process

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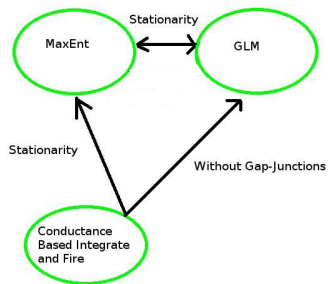
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- With gap-junctions the *conditional independence breaks down*. The explicit form of the transition probabilities has (not yet) been computed.
- The statistics of spike is described by a *Gibbs distribution* (even in the non stationary case). In the stationary case, it obeys a Maximum Entropy Principle.

Link between these approaches

B. Cessac and R. Cofr, Spike train statistics and Gibbs distributions, J. Physiol. Paris, Volume 107, Issue 5, Pages 360-368 (November 2013). Special issue: Neural Coding and Natural Image Statistics.



Maximum entropy potentials with spatio-temporal constraints are generic

Rodrigo Cofré, Bruno Cessac, "Exact computation of the maximum-entropy potential of spiking neural-network models", Phys. Rev. E 89, 052117 (2014).

Given a set of transition probabilities $P \left[\omega(D) \mid \omega_0^{D-1} \right] > 0$ there is a unique, up to a constant, equivalent MaxEnt potential, written as a linear combination of constraints (average of spike events) with a minimal number of terms. This potential can be explicitly (and algorithmically) computed.

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\Rightarrow

- Intractable determination of parameters;
- Stimulus dependent parameters;
- Overfitting.

Response to stimuli

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History dependence.

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History dependence, observable 

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History dependence, observable, network dynamics

Response to stimuli

B. Cessac, R. Cofré, Linear Response of Gibbs measures from Spiking Neuronal Network Models, in preparation

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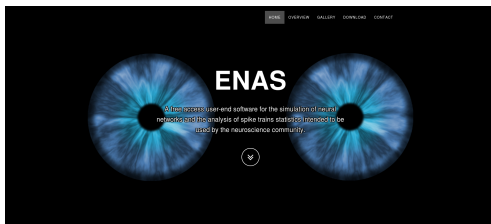
History dependence, (spontaneous) correlation between observable and network

Handling data

The Enas software

B Cessac, P. Kornprobst, S. Kraria, H. Nasser, D. Pamplona, G. Portelli, T. Viéville; *EnaS*: A New Software for Exploring Spike Trains at Single Cell and Population Levels, submitted.

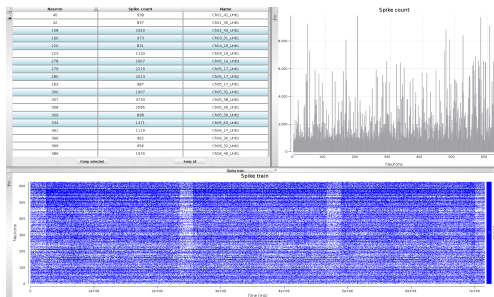
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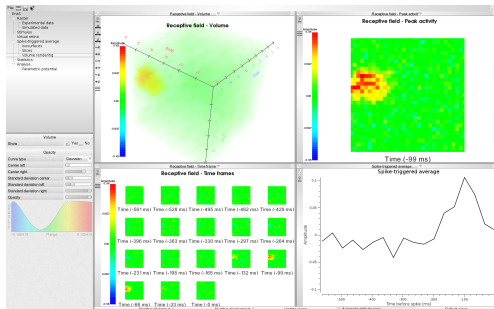
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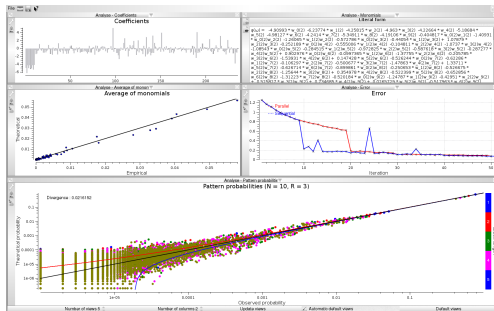
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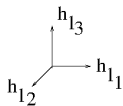
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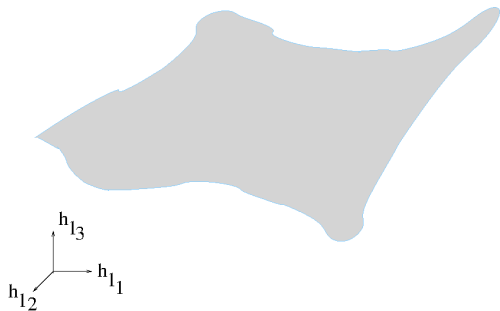
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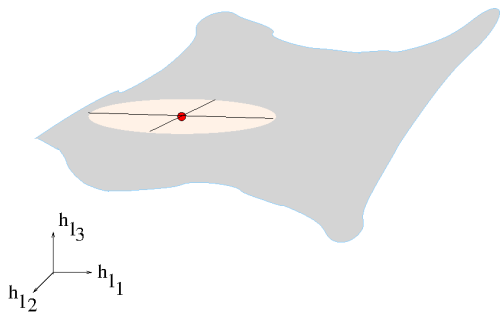
Parameters estimation and information geometry



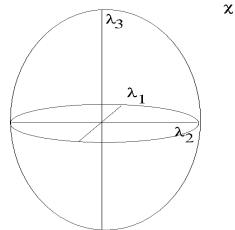
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χ Fisher metric

$$\chi_{ll'} = \frac{\partial^2 d_{KL}(\pi, \mu)}{\partial h_l \partial h_{l'}}$$

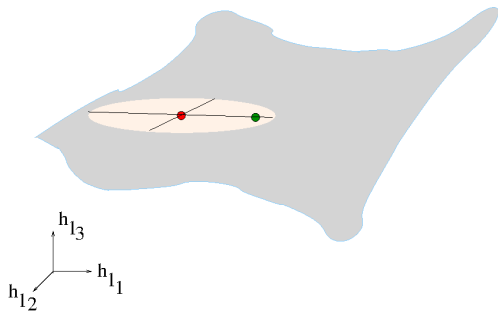
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$$\mathcal{P} = \sup_{\mu \in M_{inv}} h(\mu) + \mu(\mathcal{H})$$

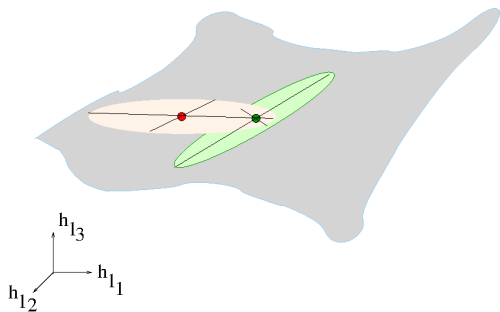
$$\chi_{ll'} = C_{ll'}(0) + \sum_{n=0}^{+\infty} C_{ll'}(n)$$

$$\delta \mu [m] = \chi \delta h \Rightarrow \delta h = \chi^{-1} \delta \mu [m]$$

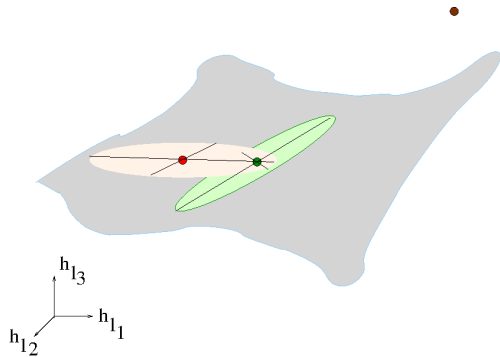
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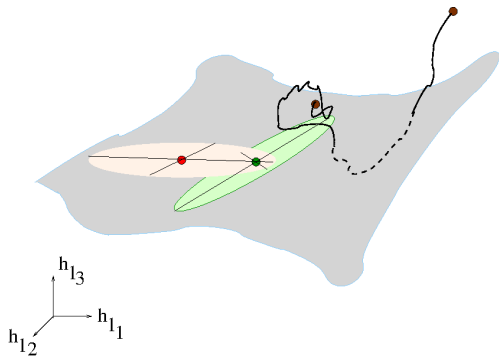
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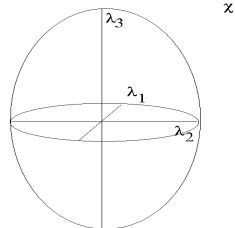
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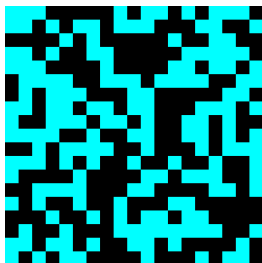
Dimension reduction

Dimensionality Reduction and Reliable Observations on Maximum Entropy Models on Spiking Networks, R. Herzog, B. Cessac, M.J. Escobar, A. Palacios, In preparation



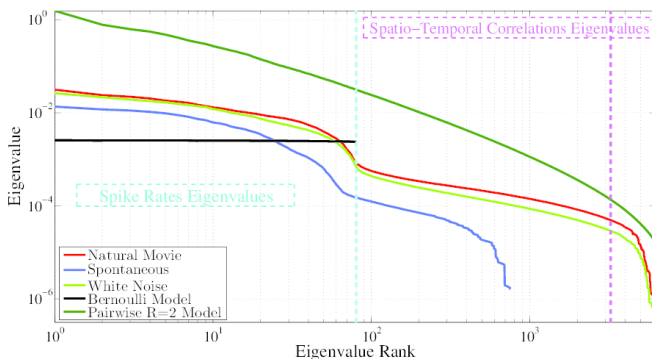
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Binning

B. Cessac, A. LeNy, E. Loecherbach, On the mathematical consequences of binning spike trains, *Neural Computation*, 2017

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- Consider a (stationary) Markov process generating spike trains, with transition probabilities $P \left[\omega(D) \mid \omega_0^{D-1} \right]$.

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NO !

Binning

Example. Take 1 neuron, $D = 1$, and bin with 2 time steps. Call ω the initial process and ϖ the binned process. Let $\mathbb{P}^{(b)}$ the probabilities in the binned process.

$$\varpi = 0 \Rightarrow (0, 0)$$

$$\varpi = 1 \Rightarrow (0, 1), (1, 0), (1, 1)$$

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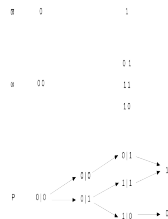
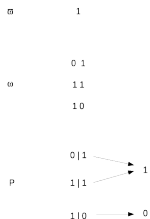
Let us first show that:

$$\mathbb{P}^{(b)}[\varpi(2) = 0 | \varpi(1) = 1, \varpi(0) = 1] \neq \mathbb{P}^{(b)}[\varpi(2) = 0 | \varpi(1) = 1].$$

Binning

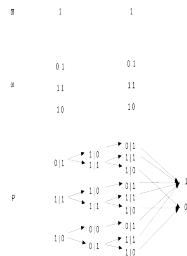
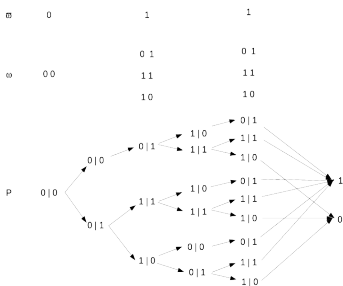
$$P^{(b)}[\varpi(2) = 0 | \varpi(1) = 1] = \frac{P^{(b)}[\varpi(2)=0, \varpi(1)=1]}{P^{(b)}[\varpi(1)=1]}$$

$$P^{(b)}[0|1] = \frac{P[0|0]P[0|0]P[0|1]P[1]+P[0|0]P[0|1]P[1]+P[0|1]P[1]+P[0|0]P[0|1]P[1|0]P[0]}{P[0|1]P[1]+P[1|1]P[1]+P[1|0]P[0]}$$



Binning

$$P^{(b)}[0|11] = \frac{P^{(b)}[011]}{P^{(b)}[11]}$$



Binning

But:

$$P^{(b)}[0|101] = P^{(b)}[0|10]$$

m	0	1	0	1
w	00	01 11 10	00	01 11 10

m	1	0	1
w	01 11 10	00	01 11 10



Binning

In general, the conditional probability

$$\mathbb{P}^{(b)}[\varpi(s+1)|\varpi_r^s] = \mathbb{P}^{(b)}[\varpi(s+1)|\varpi_l^s],$$

where l is the first occurrence of the symbol 0 when going from s to r (we set $l = r$, if ϖ does not contain the symbol 0).

\Rightarrow

The binned process is a Variable Length Markov Chain (VLMC) with unbounded memory.

Binning

Consequences:

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Proposition (Theorem 3.1 of chazottes-ugalde, 2011)

The transition kernel $\mathbb{P}^{(b)}$ of the binned chain is continuous with an exponential rate $\alpha(N) > 0$, depending on N , the number of neurons.

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Theorem

The law $\mathbb{P}^{(b)}$ of the binned chain is a Gibbs measure.

- 3 Nice measure when conditioning upon past, but also upon future (anticipation).
- 4 **Open question:** behaviour of the binned process when the number of neurons tends to ∞ (fallacious phase transitions ?)

Conclusion

A challenge: "Reading" "the" neural "code"

- **Mathematical challenge:** Improve existing mathematical tools / Elaborate new ones

A challenge: "Reading" "the" neural "code"

- **Fundamental challenge:** Understanding how visual system "encodes" / "decodes" "information"
- **Experimental challenge:** Elaborate better and better techniques to record neuronal activity
- **Mathematical challenge:** Improve existing mathematical tools / Elaborate new ones
- **Technological challenge:** New techniques in
 - Improving vision for visually impaired people (retinal prostheses, tools for reading, ...)
 - Computer vision
 -

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Extract a probabilistic model of $P[R | S]$, $P[S | R]$ from data.

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To predict is not to explain.

(René Thom)

Acknowledgment

Neuromathcomp-Biovision team

- Rodrigo Cofré
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