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Low-Dimensional Motor Control Representations in Throwing Motions

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1 Statement

The authors declares that there is no conflict of interest regarding the publication of this paper.

2 Abstract

In this study we identified a low-dimensional representation of control mechanisms in throwing motions from a variety of subjects and target distances. The control representation was identified at the kinematic level in task and joint spaces respectively, and at the muscle activation level using the theory of muscle synergies. Representative features of throwing motions in all of these spaces were chosen to be investigated. Features were extracted using factorization and clustering techniques from the muscle data of unexperienced subjects (with different morphologies and physical conditions) during a series of throwing tasks. Two synergy extraction methods were tested to assess their consistency. For the task features, the degrees of freedom (DoF) and the muscles under study, the results can be summarized as: 1) a control representation across subjects consisting of only two synergies at the activation level and of representative features in the task and joint spaces, 2) a reduction of control redundancy (since the number of synergies are less than the number of actions to be controlled), 3) links between the synergies triggering intensity and the throwing distance, and finally 4) consistency of the extraction methods. Such results are useful to better represent mechanisms hidden behind such dynamical motions, and could offer a promising control representation for synthesizing motions with muscle-driven characters.

3 Introduction

Understanding how humans control motion is an important aspect in a variety of fields, ranging from neuroscience, to robotics, and animation [1]. Several theories have been proposed which aim at unveiling the efficient and powerful mechanisms behind human motion generation. In neuroscience and biomechanics some of the objectives of identifying such mechanisms are to validate an existing motor control theory, to diagnose and treat pathologies, or to enhance athletic performance. In animation and robotics, identifying such mechanisms is the key to enhance the realism and efficiency of the motions in virtual humans and robots, since it would allow the development of more realistic motion controllers, reflecting a global control of motion [2]. More realistic motions imply a higher degree of similarity to humans, at the visual, kinematic, and dynamic level.

Our motivation lies in the domains of neuroscience and animation. In the animation field, characters with more detailed actuators (or muscles) are starting to be used for motion synthesis. The use of muscle-based characters, entails several advantages such as smoother torque generation [3], more realistic responses to perturbations [4, 5], and an ease to simulate pathologies and fatigue [6, 7]. However, the use of muscles complicates the control problem by augmenting non-linearity and redundancy, as at least two muscles are necessary to actuate each degree of freedom [8]. Furthermore, computationally expensive optimization-based solutions, which are unlikely to represent how humans control motions, are used to compute a high number of control signals.

Thus, it is necessary to define compact control schemes reducing the complexity of the control for such applications. Neuroscience provides several interesting ways to circumvent this issue, such

47 as the theory of muscle synergies [9, 1], that tries to reduce redundancy by identifying a simple and
48 generic control representation of given tasks. This theory is based on an interesting hypothesis of
49 how redundancy is handled by the central nervous system (CNS): it assumes the existence of links
50 between the ensemble of muscle control signals during the performance of a task, which reduces
51 redundancy. Thus, through synergies, the muscles are controlled as groups and not individually.
52 There is support for a neural organization of these synergies [10, 11] while remaining an open
53 question [12, 13]. Even if the interpretation of the low-dimensionality revealed by decomposition
54 methods is subject to debate [14, 15], these methods allow the creation of a compact and low
55 dimensional control representation based on experimental data for simple and complex motions.
56 Indeed, electromyography (EMG) signals processing strategies (Principal Components Analysis,
57 Non-negative Matrix Factorisation) are able to extract compact features from EMG signals, even
58 if these extracted synergies may be a consequence of a more complex mechanism [16, 17]. Many
59 studies have extracted synergies during a variety of simple upper body human motions (such as
60 pointing and reaching [18, 19]). However, to the authors' knowledge, only few studies have dealt
61 with complex, unconstrained (or free), and dynamic motion [20, 21] and, among them, overhead
62 throwing is interesting to analyze through this theory.

63 An overhead throwing motion consists in launching an object forward and above the shoulder
64 by using one arm. This is the type of motion with which humans can throw with speed and
65 accuracy [22]. Unlike simple manipulation tasks such as reaching, lifting, pulling and pushing, this
66 task is more complex, requiring a higher coordination, accuracy, and skill. Thus, it is a highly
67 redundant and nonlinear task, which involves a dynamic manipulation [23]. It is highly redundant
68 because there exists an infinite number of solutions or movements that achieve the same target hit.
69 It is highly non-linear due to the fact that positions and velocities are coupled, and that in order
70 to hit the desired target, at the moment of ball release the hand velocity, position, and the object's
71 time of flight should satisfy the parabolic projectile equation. Finally, it is also dynamic because
72 of the high accelerations and momentum at certain motion phases. Thus, the aim of this work is
73 to identify a compact representation of the control mechanism behind overhead throwing motions,
74 in order to: 1) validate a control representation extraction methodology, and 2) produce a low-
75 dimensional control representation which could later be exploited to synthesize throwing motions
76 in animation. Such representations could later be extracted from other types of throwing motions
77 (such as sidehand and underhand throws) and simpler arm motions such as pointing, to feed a
78 controller library that may produce variety of motions from these compact representations [24].

79 The generic representation should contain a reduced set of control variables (less than the
80 number of joint actions in study). It should also encode important temporal and spatial control
81 trends, invariant over a variety of morphologies. Finally, it should show some of the links between
82 these control variables and task space goals or features, that has to be used as controller inputs in
83 motion synthesis tools. One of our previous work [25] has shown that the control strategy could
84 be represented as synergies during throwing motions. However, this analysis comprehended solely
85 the activation space and a unique subject. In this paper, we propose to extend this analysis by
86 extracting control strategies from a variety of subjects, and by analyzing their relationships with
87 kinematic goals.

88 For this purpose, we present an analysis to identify generic representations of control strategies
89 starting at the task space level, joint space level, until finally reaching the activation (or actuation)
90 space level, where we extract the synergies or basis control functions. First, the experimental
91 setup used to extract the motion and muscle data is presented. Next, the control variables or
92 features at the task, joint, and activation spaces are defined. This is then followed by a detailed
93 explanation of the methods used to extract the control representation, which encompass clustering
94 and matrix factorization techniques. Two methods were used to extract the muscle synergies and
95 the consistency of their results is assessed. Finally, the generic control strategies representations
96 are illustrated and explained. Results show the existence of a generic control representation at
97 the activation level for a variety of morphologies during throwing tasks, and its relationship with
98 task space goals and features. Our new model could be later used to control a larger variety of
99 characters and a larger family of motions involving similar task space goals.

100 4 Materials and Methods

101 4.1 Experimental Setup

102 4.1.1 Subjects

103 Ten healthy men (age 29.8 ± 5.6 years old; weight 72.4 ± 9.9 kg; height 1.77 ± 0.07 m) volunteered
104 for the experiments. The subjects were all right handed and all except one (subject 3) had never
105 suffered injuries in the right arm. Furthermore, none of the participants were professional athletes,
106 and they all had different physical conditions (with a mean number of hours of sport activity per
107 week of 3.85 ± 3.07). Each subject provided a written informed consent form before participa-
108 tion. The experimentation was conducted in accordance with the Declaration of Helsinki (1964).
109 The study was approved by the ethics committee of the M2S laboratory of University of Rennes 2.

110 4.1.2 Task

111 A series of experiments were conducted where the task consisted of a right-hand overhead throw
112 to a static target placed at different distances from a fixed throwing site. The target was placed at
113 2 m, 4 m, and 7 m along a straight line from the throwing site. The target was a hole of diameter
114 0.7 m placed at 1.5 m from the ground. The ball was a standard american football ball, 0.28 m
115 long and 0.15 m large, weighing 0.4 kg. Before beginning the experiments subjects underwent a
116 short training where they practiced long distance throws for 5 to 10 minutes. Once the training
117 was finished the experiments began. During these experiments, the throwing order was randomized
118 (to reduce learning effects) and for each distance the subjects performed 6 throws for a total of 18
119 throws. A description of the motion and the experimental setup are featured in Figures 1 and 2.
120 The overhead throw is composed of four main stages [26]: Starting position, cocking, acceleration,
121 release and follow-through. In the starting position the thrower positions his body sideways with
122 respect to the intended target. The cocking phase consists of the motion between the starting
123 position until maximum external rotation is reached, before the ball starts to move forward. The
124 acceleration begins as the ball is moved forward and finishes when the ball is released. This phase
125 is known as the explosive phase since the velocity of the ball changes from zero to its maximum in
126 a short time period. Finally, the release and follow-through phase consists in a deceleration of the
127 throwing arm once the ball is released.

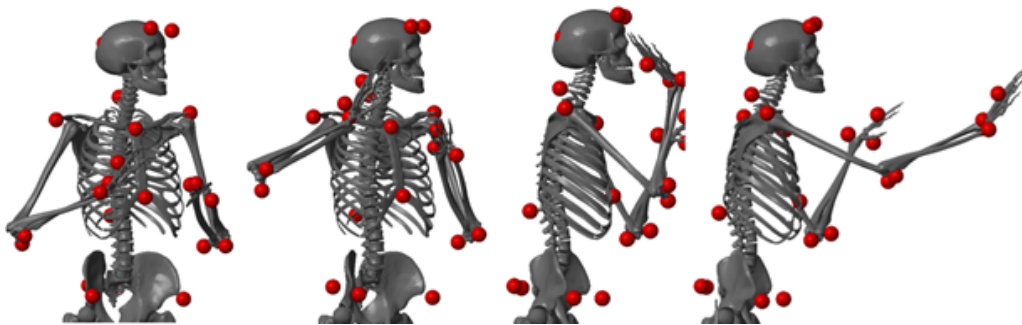


Figure 1: **Overhead throwing motion.** Example of an overhead throwing motion to a 4 m target (bone graphics issued from [27]). Representative posture of each phase of the motion are shown.

128 For each throw, the subject stood in starting position (see Figure 1). Recording began (onset)
129 when a motion of the hand was detected (threshold at 0.05 m/s) and ended when the ball was
130 released (offset).

131 4.1.3 Data Acquisition and Processing

132 During the throwing task the activity of several muscles of the right arm and body kinematics
133 were recorded. For this study we focused on studying 4 degrees of freedom (DoF), and 6 mus-
134 cles. The Segmental Interaction Principle states that energy can be transferred between segments,

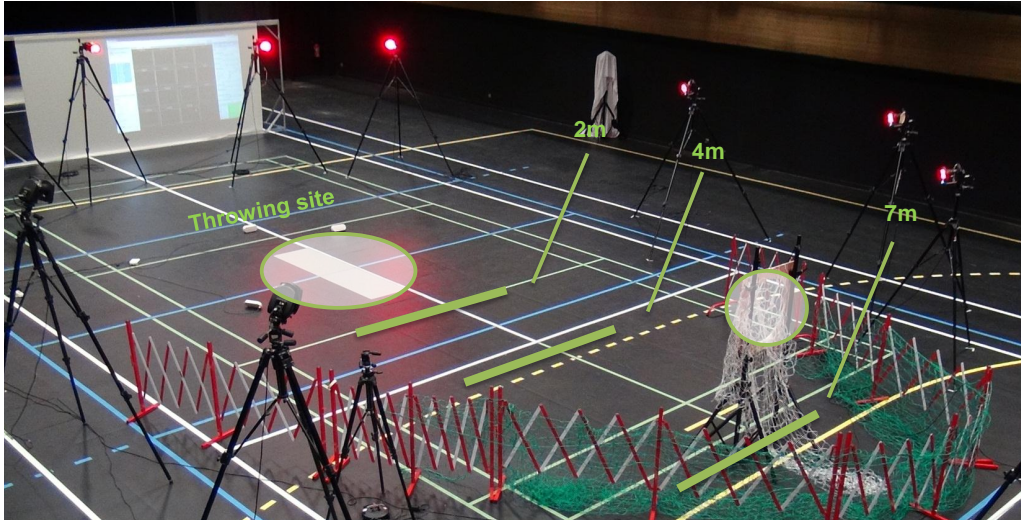


Figure 2: **Experimental Setup.** The setup consisted in a throwing site and a target that could be placed at 2, 4 and 7m from the thrower. Motion capture was done thanks to 16 cameras and EMG measurements were done through a wireless EMG system.

135 and in both simultaneous and sequentially coordinated movements, energy is transferred through
 136 the linked segment system of the body [28]. However, studies have shown that unlike baseball
 137 throwing, in football throwing (or passing) the rotations or contributions of the legs, pelvis, and
 138 upper torso are limited [29]. Thus, we decided to first focus on the arm’s kinematic chain (be-
 139 ginning at the glenohumeral joint), and on the degrees of freedom with the highest contribution
 140 to throwing per segment. These degrees of freedom were the shoulder internal/external rotation,
 141 and shoulder, elbow and wrist flexion/extension. The shoulder internal/external rotation and el-
 142 bow flexion/extension were especially selected due to the fact that they are the major upper limb
 143 actions during throwing [23], mainly during the acceleration phase [30].

144 Next, we selected a set of muscles which contained: 1) at least an agonist and antagonist muscle
 145 per each DoF under study, 2) muscles with important contributions during throwing [31, 32], and
 146 3) muscles which could be reached by surface EMG electrodes [33]. Thus, the recorded muscles were
 147 the deltoid posterior and anterior, the biceps, the triceps long, and the wrist extensors (extensor
 148 digitorum, extensor carpi radialis and ulnaris) and flexors (flexor digitorum, flexor carpi radialis
 149 and ulnaris), which were recorded as a group.

150 The muscle activity was collected using wireless surface EMG electrodes (Cometa Waveplus
 151 EMG system) and well known electrode placement protocols [33, 34]. This activity was then pro-
 152 cessed using a standard protocol [35]: The EMGs were amplified (gain 1000), digitized (1kHz),
 153 band-pass filtered (10-450 Hz, 4th order butterworth filter with no phase shift), rectified, and
 154 low-pass filtered (6 Hz, 4th order butterworth filter with no phase shift [36]). Additionally, elec-
 155 trocardiogram (ECG) artifacts were removed using an ICA-based filtering procedure [37]. Motion
 156 was captured using a Vicon system (16 cameras, 100Hz sampling rate) and reflective markers. The
 157 markers were placed on bony landmarks (49 markers) as recommended by the International Society
 158 of Biomechanics (ISB) [38, 39, 40, 41], around the target (6 markers), and on the ball (9 markers)
 159 (Figure 3). Each of the marker trajectories was low pass filtered (10 Hz, 4th order Butterworth
 160 low pass filter with no phase shift).

161 4.2 Control Features

162 The throwing motions can be characterized at three different levels: in the task space, in the joint
 163 space, and in the activation space (muscular space). The following sections aim at characterizing
 164 the motion at each of these levels through the definition of a set of kinematic and muscular features.

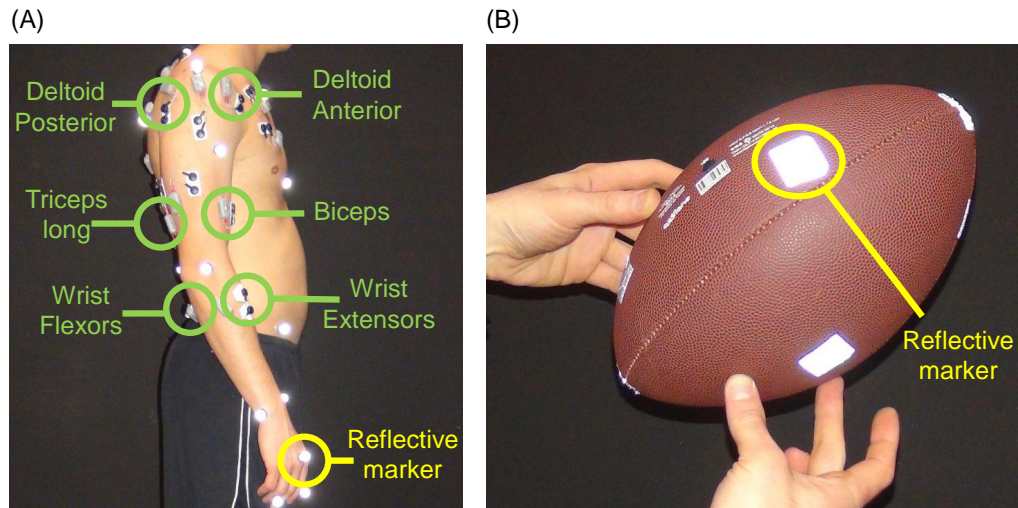


Figure 3: **Marker and EMG placement.** [33, 34] recommendations were followed for the EMG placement, whereas reflective markers were placed following the ISB recommendations with small adjustments [38, 39, 40, 41].

165 4.2.1 Task Space Features

166 Overhead throwing motions are horizontal projectile motions which are determined by three factors:
 167 velocity of release, height of release, and angle of release [42]. Based on this observation a set of task
 168 space features was defined and analyzed across task space conditions. These task features were: the
 169 hand velocity of release and the hand release height which was normalized by the subject's height,
 170 as shown in Figure 4. The angle of release was not considered due to its difficult estimation caused
 171 by marker occlusion - some of the markers of the arm were lost at the time of release of the ball,
 172 and the release angle computation was very sensitive to the methods used to reconstruct missing
 173 trajectories of the markers. Nevertheless, studies have shown that the most important parameter
 174 when determining range of throwing is the release speed [43, 44]. This is also evidenced by the
 175 equations of projectile motion, which show that the range is roughly proportional to the square of
 176 the release speed.

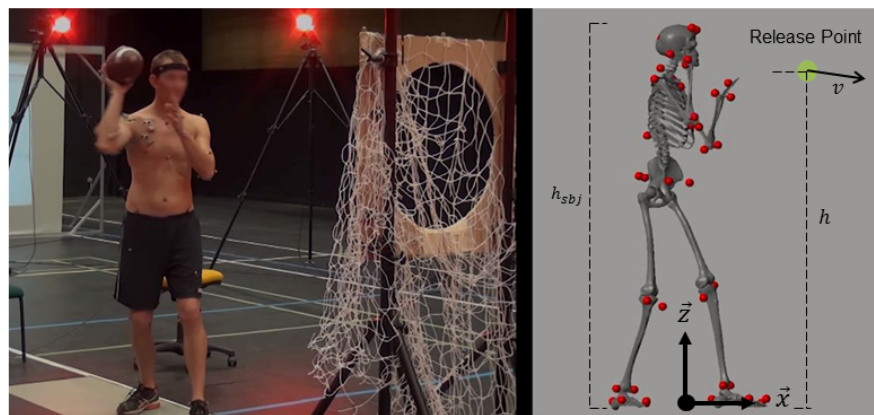


Figure 4: **Subject at release time and corresponding task space features.** The features consists in the hand release height and the hand velocity of release. Marker occlusion prevented the definition of the angle of release as a reliable feature.

177 The time of release (t_{rel}) was computed as the instant at which maximum hand velocity was
 178 reached, since it is known that this event occurs almost in parallel to the ball release in the

179 acceleration phase. For this purpose, a reflective marker was placed on the outer side of the
 180 hand (third metacarpal bone) and its position was recorded. After the derivation of this marker's
 181 trajectory, the maximum hand velocity or velocity at release (v) was computed as follows:

$$v = \max \left(\sqrt{\mathbf{v}_x(t)^2 + \mathbf{v}_y(t)^2 + \mathbf{v}_z(t)^2} \right) \quad (1)$$

182 Where $\mathbf{v}_x(t)$, $\mathbf{v}_y(t)$ and $\mathbf{v}_z(t)$ are the velocity components of the hand marker in the global
 183 coordinates frame, as defined in Figure 4 (\vec{x} front, \vec{z} up). The hand height at release (h) was
 184 determined as follows and divided by the subject's height to allow an inter-subject comparison:

$$h = \frac{\mathbf{h}_z(t_{rel})}{h_{sbj}} \quad (2)$$

185 Where \mathbf{h}_z is the hand marker's position component along z , and h_{sbj} is the height of the subject.
 186 These two features were computed for each subject and repetition. Then, they were grouped
 187 per throwing distance (d), which could be 2 m, 4 m or 7 m, yielding a total of six vectors. The
 188 means and standard deviations of these vectors were later calculated, resulting in one task feature
 189 vector for subject j :

$$\mathbf{f}_{T,j} = [\bar{v}_{2m,j} \quad \bar{v}_{4m,j} \quad \bar{v}_{7m,j} \quad \bar{h}_{2m,j} \quad \bar{h}_{4m,j} \quad \bar{h}_{7m,j}] \quad (3)$$

190 4.2.2 Joint Space Features

191 The joint space features consisted in the joint positions and velocities. The joints positions were
 192 estimated from motion capture, with an inverse kinematics method allowing the segment lengths
 193 and marker positions to be calibrated [45]. The joint velocities were computed by deriving the joint
 194 position trajectories. The joint space analysis focused on the following degrees of freedom of the
 195 throwing arm: shoulder internal/external rotation, and shoulder, elbow, and wrist flexion/extension
 196 ($\mathbf{q}_1(t)$, $\mathbf{q}_2(t)$, $\mathbf{q}_3(t)$ and $\mathbf{q}_4(t)$ respectively).

197 An average trajectory was computed for each subject, each degree of freedom, and each throwing
 198 distance. These trajectories were later time normalized across subjects in order to allow the inter-
 199 subject comparison in section 4.3.1. A joint space feature vector per subject j was constructed,
 200 containing the mean joint position \bar{Q}_d matrices per throwing distance:

$$\mathbf{f}_{Q,j} = [\bar{Q}_{2m,j}(t) \quad \bar{Q}_{4m,j}(t) \quad \bar{Q}_{7m,j}(t)] \quad (4)$$

201 Where, each joint position matrix contains the average positions ($\bar{\mathbf{q}}(t)$) per DoF,

$$\bar{Q}_d = [\bar{\mathbf{q}}_1(t) \quad \bar{\mathbf{q}}_2(t) \quad \bar{\mathbf{q}}_3(t) \quad \bar{\mathbf{q}}_4(t)] \quad (5)$$

202 4.2.3 Activation Space Features

203 The control done at the muscle level is the one that interests us the most, since it is the actuation
 204 space. This control can be described via muscle activations. However, an activation representation
 205 is redundant since there are more muscles than degrees of freedom and each muscle needs its own
 206 activation signal. A simpler and less redundant representation of these signals can be achieved via
 207 muscle synergies [46, 18].

208 One way to represent such synergies is via the time-invariant synergy model [25, 8]. In this
 209 model a synergy \mathbf{w}_i is defined as a $M \times 1$ vector of coefficients, specifying the relative activation
 210 level of M-muscles. Each synergy is paired with a time-varying combination coefficient vector $\mathbf{c}_i(t)$
 211 ($1 \times T$), which determines its temporal evolution. A set of N-synergies can be linearly combined
 212 to generate M-muscle activation patterns $A(t)$:

$$A(t) = WC(t) = [\mathbf{w}_1 \quad \mathbf{w}_2 \quad \dots \quad \mathbf{w}_N] \begin{bmatrix} \mathbf{c}_1(t) \\ \mathbf{c}_2(t) \\ \dots \\ \mathbf{c}_N(t) \end{bmatrix} \quad (6)$$

213 Where, $A(t)$ is the $M \times T$ samples matrix containing the recorded muscle activations patterns,
 214 W is the $M \times N$ muscle synergy matrix, and $C(t)$ is the $N \times T$ samples combination coefficient
 215 matrix. To separate and highlight the contribution of each synergy \mathbf{w}_i and its coefficient $\mathbf{c}_i(t)$ to
 216 the muscle activation patterns, the previous equation can also be written as:

$$A(t) = \sum_{i=1}^N \mathbf{w}_i \mathbf{c}_i(t) \quad (7)$$

217 Based on this model, the time-invariant activation space feature \mathbf{f}_A was defined as the matrix
 218 W , and the time-variant activation space feature $\mathbf{f}_{A_t}(t)$ was defined as the matrix $C(t)$:

$$\mathbf{f}_A = W \quad (8)$$

$$\mathbf{f}_{A_t}(t) = C(t) \quad (9)$$

219 Where,

$$C(t) = [C_{2m}(t) \quad C_{4m}(t) \quad C_{7m}(t)]$$

220 Each sub-matrix $C_d(t)$ is of dimensions $N \times T_d$, where T_d is the total number of samples
 221 contained in the throws to distance d .

222 In our case, this model was used in two methods (see section 4.3.2). Their results were com-
 223 pared to test their robustness and consistency. The first method consisted in extracting a synergy
 224 model ($W, C(t)$) per subject, and the second method consisted in extracting one synergy model
 225 representative of all subjects. Thus, in the first case a variety of W matrices were generated repre-
 226 senting each subject’s throw, and in the second case a single W matrix was generated representing
 227 all subjects and throws.

228 For both models the combination coefficient matrix $C(t)$ encoded the temporal evolution of
 229 the synergies during each throw. These coefficients will be further described in terms of: 1) their
 230 shapes, 2) how their energy changes with throwing distance, and 3) their triggering order. In
 231 general, the average image of the energy $\overline{E}_{c_i,d}$ of each combination coefficient $\mathbf{c}_{i,d}$ contained in
 232 matrix $C_d(t)$ was computed as follows:

$$\overline{E}_{c_i,d} = \frac{\sum_{s=1}^{T_d} |\mathbf{c}_{i,d}(t_s)|^2}{n} \quad (10)$$

233 Where n is the number of trials per throwing distance, and t_s the current time sample.

234 4.3 Control Representations Extraction

235 Once the features to analyze were defined at each level (task, joint, and activation spaces) and for
 236 each subject j , control representations based on these features were extracted. The objective of
 237 such an extraction was to verify if a generic control representation existed for overhead throwing
 238 across subjects. Such representations are denoted by an index All , which generalizes the subject
 239 feature vectors (j) of the previous sections to all participants.

240 The identification of these representations was made through clustering algorithms for the
 241 time-invariant features and averaging and cross-correlation for the time-variant features.

242 Clustering is a technique that consists in the assignment of features into groups or subsets based
 243 on a similarity criteria. In the next sections we will see that the existence of a generic representation
 244 in each space will depend on the number of clusters or groups found with these techniques.

245 The first step before using the clustering algorithms is feature scaling. This preprocessing step
 246 is necessary due to the fact that clustering algorithms use distances to classify features. Thus,
 247 features should be standardized such that they have contributions of equal importance in the
 248 distance measurements.

249 Two different types of clustering algorithms were used to extract control representations from
 250 the time-invariant features: a centroid-based clustering (k-means) algorithm and a connectivity-
 251 based clustering algorithm (hierarchical clustering). These two algorithms were used in order
 252 to verify if different techniques yielded similar control representations. Furthermore, the specific

253 interest in using hierarchical clustering was to verify if the chosen number of clusters of the k-means
 254 algorithm matched the natural divisions in the data.

255 K-means clustering is an iterative algorithm for data partitioning that assigns or classifies
 256 features into one of k clusters defined by centroids. The main steps of the algorithm are the
 257 following, given k : 1) select k initial cluster centroids, 2) compute the distances between each
 258 feature to each cluster centroid, 3) assign the features to the cluster with the closest centroid all
 259 at once (phase 1), and individually reassign points if it reduces the sum of distances (phase 2), 4)
 260 obtain new centroids by averaging the features in each cluster, and 5) repeat steps 2-4 until the
 261 assignments do not change or the iterations reach their maximum. The k-means++ algorithm in
 262 MATLAB was used with a squared euclidean norm to compute distances. The advantage of this
 263 algorithm is that it uses the heuristic in [47] to find centroid seeds for k-means clustering. This
 264 induces a faster convergence to higher quality solutions, or to a lower sum-of-squares point-to-
 265 cluster centroid distances (within each cluster). Finally, in order to assess the k-means clustering
 266 quality an indicator called cluster silhouette was computed [48, 49]. This indicator enables us to
 267 distinguish clear-cut clusters from weak ones. It measures how similar the features are to features
 268 in their own cluster, when compared to features in other clusters, and is computed as follows:

$$Sil = \frac{(b_j - g_j)}{\max(b_j, g_j)} \quad (11)$$

269 Where, g_j is the average distance from the j th feature to the other features in the same cluster
 270 as j , and b_j is the minimum average distance from the j th point to points in a different cluster,
 271 minimized over clusters. The silhouette value can range from -1 to 1 . By averaging the silhouette
 272 values of each feature in the cluster, an average silhouette (\overline{Sil}) can be obtained for the entire
 273 cluster. A subjective interpretation for this value was proposed by the authors of [49] to assess the
 274 clustering quality, as shown in Table 1.

Table 1: **Assessment of k-means clustering quality [49].**

Subjective Interpretation of the average silhouette value	
\overline{Sil}	Proposed Interpretation
0.71-1.00	A strong structure has been found
0.51-0.70	A reasonable structure has been found
0.25-0.50	The structure is weak and could be artificial; try additional methods on data set
≤ 0.25	No substantial structure has been found

275 This interpretation was used to select with which number k of clusters the data was well
 276 separated ($\overline{Sil} \geq 0.71$) or if no separations could be made, in which case only a single cluster
 277 exists.

278 To complete this assessment, hierarchical clustering was also used to partition the feature space
 279 into groups. Hierarchical clustering is an algorithm for cluster analysis that aims at grouping fea-
 280 tures at different levels using a cluster tree or dendrogram. In agglomerative hierarchical clustering
 281 each feature starts in its own cluster, these clusters are then combined via a metric and a linkage
 282 criterion. The metric defines a distance between pairs of features, and the linkage criterion defines
 283 the distance between sets by computing the pairwise distances between features. An advantage of
 284 this strategy is that it does not need an initial indication of the number of clusters, and therefore,
 285 it reveals the natural divisions in the data. For its implementation, the hierarchical algorithm tools
 286 in MATLAB were used with the euclidean distance as metric, and an unweighted average distance
 287 (euclidean) for the linkage.

288 The following sections present how these methods and the synergy extractions [9] were used to
 289 extract control representations across subjects.

290 4.3.1 Task and Joint Space Control Representation Extraction

291 First, we determined if a common representation existed across subjects in task space. Thus, the
 292 feature vectors $\mathbf{f}_{T,j}$ in Eq(3) were first standardized, and then given as inputs to the clustering
 293 algorithms. First, the k-means algorithm was applied by varying the number of clusters and

294 checking how well the clusters were separated thanks to the clusters' silhouette values. In this
 295 space, since each subject is characterized by a single vector, we expect a common representation
 296 to exist when $k = 1$. In other words when the features are so similar, that well separated clusters
 297 cannot be formed. If this was the case, then the common strategy was defined by averaging the
 298 task feature vectors across subjects:

$$\mathbf{f}_{T,All} = [\bar{v}_{2m,All} \quad \bar{v}_{4m,All} \quad \bar{v}_{7m,All} \quad \bar{h}_{2m,All} \quad \bar{h}_{4m,All} \quad \bar{h}_{7m,All}] \quad (12)$$

299 To further verify the results of k-means, hierarchical clustering was then applied. This algorithm
 300 does not need an initial estimate of the desired number of clusters, thus, it was used in order to
 301 determine if the natural cluster divisions of the data agreed with the results provided by k-means.
 302 In other words, if no natural cluster divisions were found and a common task control strategy across
 303 subjects existed. Finally, a Wicoxon rank sum test was performed on the features across subjects
 304 to detect significant changes in their values with regard to the throwing distance (confidence level
 305 below 0.05).

306 At the joint space level, the features $\mathbf{f}_{Q,j}$ used to represent the motion are all time varying
 307 (average joint positions per subject). Therefore, cross-correlation was used to evaluate the similarity
 308 of the joint trajectories and velocities across subjects and throwing distances. A high correlation
 309 signified that the joint trajectories or velocities were similar among subjects. Low correlations
 310 signified very low kinematic similarities. The common joint space strategy was then defined by
 311 averaging the joint trajectories and velocities across subjects:

$$\mathbf{f}_{Q,All} = [\bar{Q}_{2m,All}(t) \quad \bar{Q}_{4m,All}(t) \quad \bar{Q}_{7m,All}(t)] \quad (13)$$

312 4.3.2 Activation Space Control Representation Extraction

313 The synergies and their combination coefficients (section 4.2.3) were extracted via a NMF (non-
 314 negative matrix factorization) [50] algorithm. This algorithm decomposes a non-negative matrix
 315 into a non-negative linear combination of basis vectors, by solving the following optimization prob-
 316 lem:

$$\begin{aligned} & \underset{W,C}{\text{minimize}} && \frac{1}{2} \|A(t) - WC(t)\|_F^2 \\ & \text{subject to} && W, C(t) \geq 0 \end{aligned} \quad (14)$$

317 When applying this algorithm, the synergy model order or number of synergies to extract
 318 should be defined. To do this we used two criteria. The first criteria consisted in choosing a
 319 number of synergies N less than the number of recorded muscles $M = 6$ in order to obtain
 320 a lower dimensional control representation. The second criteria attempted to preserve a good
 321 quality in the reconstruction of the original activations. Therefore, a criterion based on the average
 322 coefficient of determination r^2 between the original and reconstructed muscle patterns [9, 18] was
 323 used. This criterion states that the chosen number of synergies should correspond to the sharpest
 324 change in the slope of the r^2 curve (Figure 10)). This change in slope is interpreted as the point
 325 separating "structured" from noise-dependent variability. After this point, additional synergies
 326 start to capture only the small residual noise-dependent variability, therefore, this can be used to
 327 define the minimum number of synergies that capture the task-related features [51, 52, 46]. We
 328 highlight the fact that these criteria guarantee that the number of control variables N will be less
 329 than the number of muscles or actuators, however there is no guarantee that they will be less than
 330 the number of DoF. This is a possible added value of a representation through synergies. The
 331 NMF algorithm used was the one developed in [53] and the update rule used was the non-negative
 332 least squares one.

333 We employed two methods for identifying a representative synergy (or time invariant features)
 334 using this extraction algorithm. The first is based on k-means [54] and hierarchical clustering,
 335 and the second one is based on the identification procedure in [9]. The comparison of the results
 336 extracted from both methods was useful to test the consistency and robustness of these extraction
 337 methodologies.

338 The first method consisted of 3 stages: 1) extraction of individual subject synergy models, 2)
339 standardization of \mathbf{w}_i vectors, and 3) application of k-means and hierarchical clustering algorithms.
340 In the first stage matrix $A(t)$ (6 muscles x 3600 samples) was constructed by concatenating the ac-
341 tivation signals for all the trials of individual subjects. This method enabled us to take into account
342 intra-subject variability in the synergy extraction. The concatenated signals were normalized by
343 their maximal value to obtain activations framed between 0 and 1. Next, NMF was applied on
344 this matrix to obtain one N-synergy model ($W, C(t)$) per subject. Once a model was obtained
345 for each subject, the synergy matrices W were standardized for their use in the clustering algo-
346 rithms. Essentially, each synergy \mathbf{w}_i of each subject was a feature vector containing the relative
347 activation levels of the muscles. These vectors were treated individually and without specifying
348 their correspondence to a specific subject. They were used to create a synergy pool on which
349 k-means and hierarchical clustering were applied in order to identify common features among this
350 synergy pool. The k-means algorithm was applied first while varying the number of clusters k . We
351 expected a unique strategy to exist when $k = N$, in other words when the number of clusters is
352 equal to the number of synergies extracted for each subject. If this was the case, then the centroids
353 of these clusters represented the mean synergy vectors, or the representative activation control
354 representation for all subjects through method I ($W_{I,AU}$).

$$\mathbf{f}_{A,AU} = W_{I,AU} \quad (15)$$

355 Finally, hierarchical clustering was applied. This algorithm was used in order to determine if
356 the natural cluster divisions of the data corresponded to the number of k-means centroids.

357 The second synergy extraction method consisted in the direct identification of a common ac-
358 tivation control strategy for all subjects, based on [9]. In this method, the NMF algorithm was
359 applied on a matrix $A(t)$ (6 muscles x 36000 samples), constructed by concatenating the activation
360 signals for all trials of all subjects. Therefore, by applying NMF on this pool of EMG signals one
361 common synergy model ($W_{II,AU}$) was found for all subjects.

$$\mathbf{f}_{A,AU} = W_{II,AU} \quad (16)$$

362 However, the coefficients ($C_{II}(t)$) in this method encoded how much and when each synergy
363 was triggered for each repetition and subject. Therefore, to identify a common time-varying control
364 representation for all subjects and repetitions ($\overline{C}_{II,AU}(t)$), averaging and correlation computation
365 were used. First, the mean combination coefficients per subject, per throwing distance were com-
366 puted. Next, cross-correlation was used to make comparisons across subjects at each throwing
367 distance. Thus, a common combination coefficient was computed by making a second averaging
368 across all subjects .

$$\mathbf{f}_{A_t,AU}(t) = \overline{C}_{II,AU}(t) \quad (17)$$

369 Where $\overline{C}_{II,AU}(t)$ contains the coefficient matrices per throwing distance,

$$\overline{C}_{II,AU}(t) = [\overline{C}_{II,2m,AU}(t) \quad \overline{C}_{II,4m,AU}(t) \quad \overline{C}_{II,7m,AU}(t)] \quad (18)$$

370 5 Results and Discussion

371 5.1 Global Considerations

372 The motion, as defined in the task description above, had an average duration of 1.67 ± 0.27 s for
373 all the throws made by all the subjects. Thus, the standard deviation seemed sufficiently low to
374 compare the different throws and normalize them against time, as it has been done for some of the
375 processings of extraction. Subjects had a global performance higher than 80%, meaning that the
376 task was quite easy to perform and reproducible from one trial to one other. The following sections
377 detail the representations extracted from the experimental data in the task, joint and activation
378 spaces. For all the cross-correlation we performed, the mean value of the lag was about less than
379 $10^{-15}\%$ of the signal length, meaning that most of the signal shapes were comparable directly.
380 Therefore, we did not present the lags related to cross-correlation results in the corresponding
381 tables.

382 5.2 Task Space Control Representation

383 The subject task features $f_{T,j}$ were collected and a representative task space control representation
 384 $f_{T,All}$ was extracted as described in section 4.3.1. The task feature vector for each subject is
 385 featured in Figure 5. A gradual increase in hand release velocity and height can be seen across
 386 subjects as the throwing distance increases. Moreover, as evidenced by the Wilcoxon rank sum
 387 test, this increment is statistically relevant in 9/10 subjects for the hand velocity, and in 7/10
 388 subjects for the hand height.

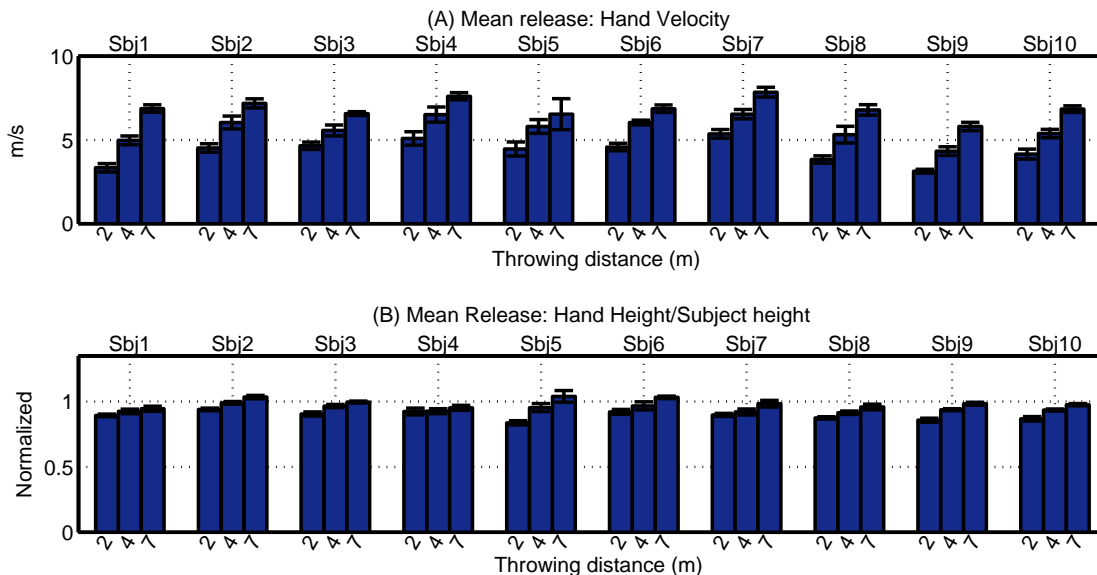


Figure 5: **Task space features per subject and throwing distance.** Both hand release velocity and release height increased with the throwing distance for all the subjects.

389 Next, k-means and hierarchical clustering were applied on the feature vectors in order to deter-
 390 mine if one sole task space control representation existed. We expected a unique control represen-
 391 tation if no strong separated groups can be found among the subject task vectors, in other words
 392 if a single cluster exists.

393 K-means was first applied while varying the number of clusters k . Figure 6 shows each cluster's
 394 silhouette. The average silhouette value \overline{Sil} is below 0.71 (Table 1) as k increases. At $k = 5$, it
 395 reaches a value above this threshold but clusters containing 1-2 subject vectors begin to be formed.
 396 Therefore, since the k-means analysis did not differentiate the subjects, a common representation
 397 of the control at the task space level can be obtained from the averaging of the task space features.

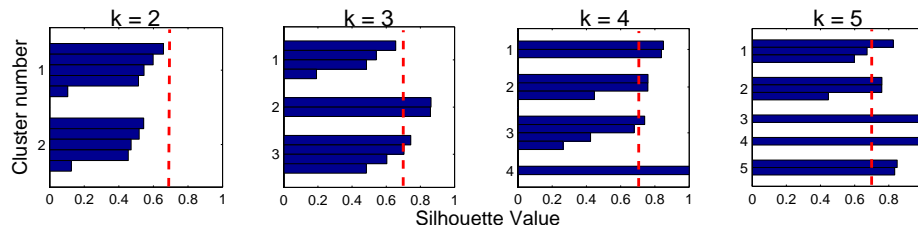


Figure 6: **Task space cluster separation quality using k-means.** Strongly separated clusters ($\overline{Sil} \geq 0.71$) containing a similar number of subjects cannot be found. A single cluster exists across subjects.

398 Hierarchical clustering was then applied in order to verify if the results of the k-means clustering
 399 was consistent with the natural division of the data. The hierarchical clustering was only used to

400 consider qualitative and visual informations about data division. Figure 7 features the resulting
 401 cluster tree. In this tree, no visually significant divisions are found. This is shown by the fact
 402 that heights of the links at each level are not qualitatively different from the heights of the links
 403 below them, indicating a high closeness across groups. Furthermore, with this procedure we can
 404 see that as the number of clusters increases, groups containing very few subject vectors begin to be
 405 formed. Consequently, we concluded that all subjects were presenting similar changes in the task
 406 space features with regard to the task constraints (throwing distance). In other words, subjects
 407 increased significantly the hand release velocity and height as the throwing distance increased.
 408 The average task features $\mathbf{f}_{T,All}$ were computed by averaging the subject task feature vectors and
 409 it is shown in Figure 8. Velocity increments of about 1.3 m/s and hand-height/subject-height
 410 increments of 0.05 are seen as the distance increases by 2 to 3 m . Moreover, the range is roughly
 411 proportional to the square of the release speed, as can also be evidenced through the equations
 412 of projectile motion. Thus, our results are consistent with other studies that indicate an increase
 413 in height and speed with throwing distance, and the existence of a proportionality relationship
 414 between speed and range [23]. This is a straightforward result that may be mostly induced by the
 415 motion constraints (distance to throw, motion type) and the difference of strategy between subjects
 416 may appear in the amount of changes from one distance to one other as it can be observed in
 417 Figure 5. However, the averaging of the task space features as a unique representation of the
 418 control in the task space makes sense since the trends featured in Figure 8 respects the same
 419 pattern as the one seen for all the subjects.

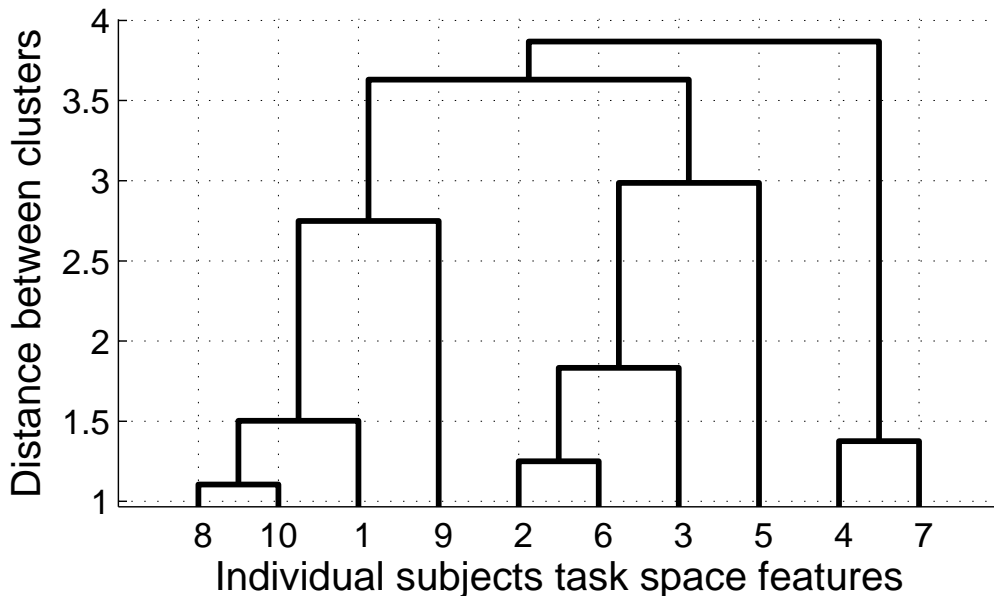


Figure 7: **Task space cluster separation using hierarchical clustering.** In this case the clusters are not well separated.

420 5.3 Joint Space Control Representation

421 The subject joint features $\mathbf{f}_{Q,j}$ were then used to determine if a common joint space control rep-
 422 resentation $\mathbf{f}_{Q,All}$ existed, as described in section 4.3.1. The features showed a high repeatability
 423 across subjects at each throwing distance, regardless of the inexperience and small differences in
 424 style of our throwers. These kinematic similarities were quantified as correlations among subjects,
 425 and are shown in Table 2. As the throw is performed, the motion is repeatable in the forward
 426 direction. Thus, high correlations are seen in the joint trajectories, especially in shoulder ($\mathbf{q}_2(t)$)
 427 and elbow ($\mathbf{q}_3(t)$) flexion/extension. Lower but still significant correlation is seen in the shoulder
 428 internal/external rotation ($\mathbf{q}_1(t)$) and wrist flexion/extension ($\mathbf{q}_4(t)$). The differences in inter-
 429 nal/external rotation ($\mathbf{q}_1(t)$) could be due to each subject's throwing style. While the differences

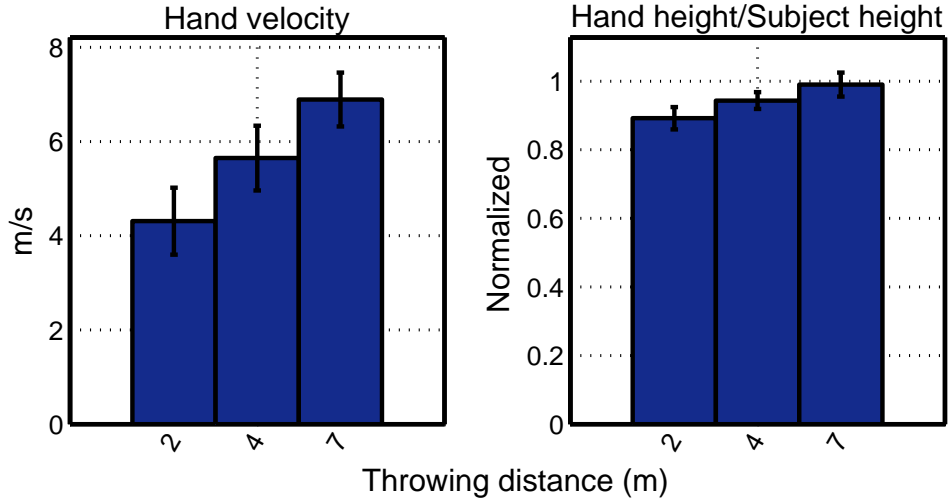


Figure 8: **Representative task space representation for all subjects ($\mathbf{f}_{T,All}$)**. These are the mean and standard deviation values of the features shown in figure 5. The global increase of both features with regard to the throwing distance is straightforward.

Table 2: **Mean inter-subject cross-correlation coefficient per throwing distance.**

Mean inter-subject cross-correlation coefficient per throwing distance			
DoF	$corr_{2m}$	$corr_{4m}$	$corr_{7m}$
\mathbf{q}_1	0.6681 ± 0.2396	0.7493 ± 0.1579	0.6335 ± 0.2048
\mathbf{q}_2	0.9684 ± 0.0259	0.9494 ± 0.0485	0.9403 ± 0.0529
\mathbf{q}_3	0.9723 ± 0.0207	0.9526 ± 0.0316	0.9329 ± 0.0394
\mathbf{q}_4	0.6025 ± 0.2520	0.6077 ± 0.2634	0.6210 ± 0.1922
$\hat{\mathbf{q}}_1$	0.7349 ± 0.1012	0.6290 ± 0.1392	0.4830 ± 0.1317
$\hat{\mathbf{q}}_2$	0.6878 ± 0.1257	0.6374 ± 0.1671	0.5588 ± 0.1063
$\hat{\mathbf{q}}_3$	0.8170 ± 0.1036	0.8502 ± 0.0880	0.7954 ± 0.0840
$\hat{\mathbf{q}}_4$	0.7217 ± 0.1309	0.6540 ± 0.2192	0.7277 ± 0.1226

430 in wrist flexion/extension ($\mathbf{q}_4(t)$) could be linked to the fact that the most distal segments have
 431 larger contributions to accuracy over speed [32, 55].

432 In terms of articular velocities the motion is less repeatable. Nevertheless, as seen in the
 433 previous section, different velocity control strategies in the joint space can result in a common
 434 velocity feature in the task space accross subjects. These differences may be linked to individual
 435 differences in the throwing strategy and can not be used as a common feature of the control
 436 representation in the joint space.

437 Finally, a representative joint space control strategy was computed by making averages across
 438 subjects and throwing distances for joint trajectory included as a feature. This control representa-
 439 tion is featured in Figure 9 and in Table 3. Similar kinematic trends are shared across throw types.
 440 For instance, as the throw progresses, the shoulder is internally rotated and flexed, while the elbow
 441 is extended and the wrist is gradually flexed. Lastly, these similarities were also reflected in the
 442 inter-subject and inter-distance correlation, which resulted in very high correlation coefficients for
 443 all DoF, as shown in Table 3.

444 5.4 Activation Space Control Representation

445 The synergy extraction method described in section 4.3.2 was applied on each of the subject's
 446 EMG dataset while varying the number of synergies. The objective was to identify a model with
 447 less synergies than the number of recorded muscles or actuators ($N < M$), for each subject, that
 448 would guarantee a good reconstruction of the original EMG signals. Figure 10 depicts the quality

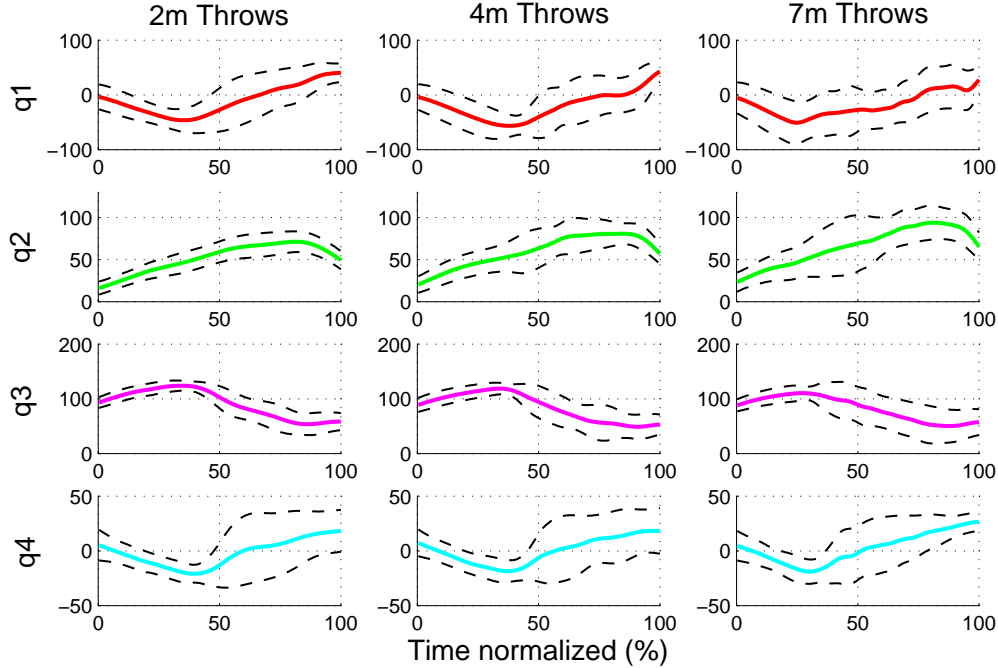


Figure 9: **Representative joint space strategy for all subjects ($\mathbf{f}_{Q,All}$)**. Computed by averaging joint features across subjects for each DoF. Solid lines are the average values, dashed lines the standard deviation values. The joint space analysis focused on the following degrees of freedom of the throwing arm: shoulder internal/external rotation, and shoulder, elbow, and wrist flexion/extension ($\mathbf{q}_1(t)$, $\mathbf{q}_2(t)$, $\mathbf{q}_3(t)$ and $\mathbf{q}_4(t)$ respectively). Angles are given in degrees.

Table 3: **Mean inter-subject and inter-distance cross-correlation coefficients**. The representative joint space strategy shown in Figure 9 was correlated across throwing distances.

Mean inter-distance cross-correlation coefficients	
DoF	$corr_{2m,4m,7m}$
\mathbf{q}_1	0.9171 ± 0.0213
\mathbf{q}_2	0.9989 ± 0.0005
\mathbf{q}_3	0.9991 ± 0.0004
\mathbf{q}_4	0.9315 ± 0.0467

449 of reconstruction r^2 for each subject and synergy model. The sharpest change in slope of this curve
 450 occurred at $N = 2$ for 8 subjects and at $N = 3$ for 2 subjects. Thus, we opted for the 2-synergy
 451 model which allowed an average quality reconstruction of 0.7382 across subjects.

452 5.4.1 Synergy Model (W)

453 Method I was applied in order to determine a common representation of the control in the activation
 454 space. First, the 2-synergy models were extracted for each subject. Then, a pool containing the
 455 individual synergies \mathbf{w}_i of all subjects was constructed, without specifying if the synergies belonged
 456 to the same subject. Thus, the pool contained 20 synergies (2 synergies per subject). Finally, k-
 457 means clustering was applied on this pool while varying the number of clusters k . We expected a
 458 common control representation to exist when $k = N$, or when the number of clusters is equal to
 459 the number of synergies extracted per each subject. Figure 11 shows that indeed, the best cluster
 460 separation is achieved at $k = N$ or $k = 2$, where the average silhouette value for both clusters is
 461 equal to 0.7181. If a higher number of clusters or separations is found, the average silhouette values
 462 decrease and clusters containing very few synergies are formed. This evidences that 2 clusters are
 463 sufficient to classify the synergies.

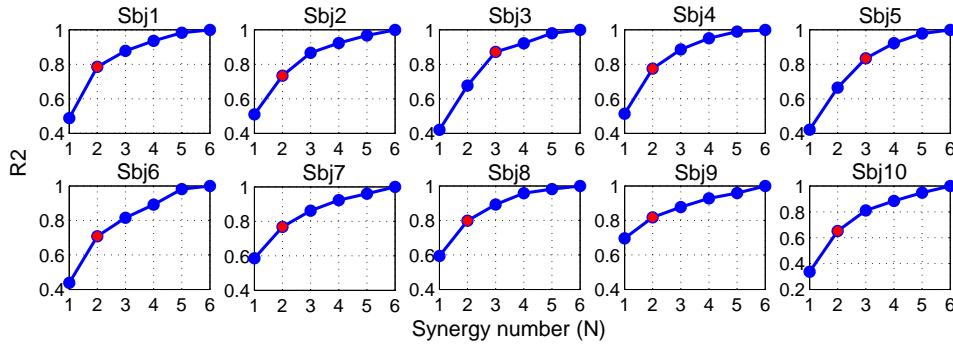


Figure 10: **Activation reconstruction quality across synergy models per subject.** The NMF algorithm was applied on each of the subject’s EMG data set while varying the number of synergies (N). The resulting curve depicts the r^2 values for each model.

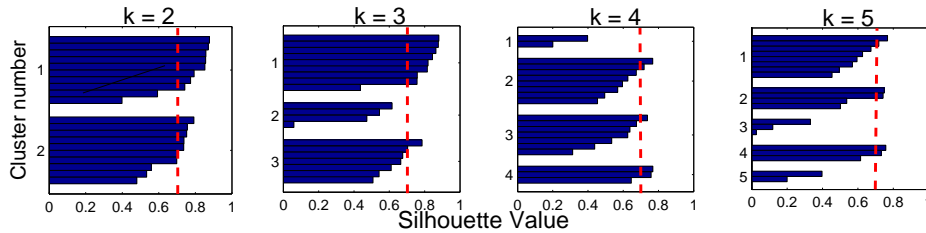


Figure 11: **Cluster separation quality using k-means.** Strongly separated clusters ($\overline{Sil} > 0.71$) are found at $k = N$ or $k = 2$. A common activation space control representation exists across subjects.

464 To further verify if the natural divisions of the data corresponded to 2 groups of distinctive
 465 synergies, hierarchical clustering was applied. This resulted in the cluster tree in Figure 12. In
 466 this tree we can see how the 20 synergies in the pool are partitioned into 2 clusters as well. This
 467 is shown by the fact that the link separating the synergy data into two branches is inconsistent
 468 with the links below it. It indicates a higher closeness among the synergies within each group than
 469 across each group.

470 Interestingly, the individual synergies within each cluster in the tree matched those in the
 471 clusters computed via k-means. Thus, a mean activation control representation $W_{I,All}$ for all
 472 subjects was extracted from the centroids of the 2-cluster model obtained via k-means (Figure 6
 473 top). Each of these centroids or mean synergies contains the relative action levels of groups of
 474 muscles. Finally, we wanted to demonstrate how well the synergy $W_{I,All}$ represented all of the
 475 subjects’ individual synergies. In order to do this, the normalized dot product between the synergy
 476 $W_{I,All}$ (centroid) and each of the subjects’ 2-synergy models W (cluster points) was computed.
 477 The results showed that a high similarity exists between these models, with a mean normalized
 478 dot product of 0.9495 ± 0.0485 for w_1 , and 0.9170 ± 0.0537 for w_2 .

479 Method II was then applied to identify the representative synergy model directly from a pool
 480 containing the EMG signals of all subjects. Thus, this pool contained 6 signals (one per muscle),
 481 and each signal contained 180 concatenated activations corresponding to each of the subjects’ trials
 482 (10 subjects, 3 throwing distances, 6 trials per distance). As in the individual subject synergy
 483 extractions, the number of synergies was chosen as the number corresponding to the sharpest
 484 change in the r^2 curve. This change occurred again at $N = 2$ synergies, where the quality of
 485 reconstruction was of 0.6526. This slight decrease in quality of reconstruction with respect to the
 486 individual extractions is expected since method II attempts to reconstruct a higher number of trials
 487 performed by different subjects simultaneously.

488 The resulting representative synergy $W_{II,All}$ is depicted in Figure 13 (bottom). Again, each
 489 synergy contains the relative activation levels of a group of muscles throughout the motion. The
 490 first synergy w_1 can be seen as the agonist synergy, and the second synergy w_2 can be seen

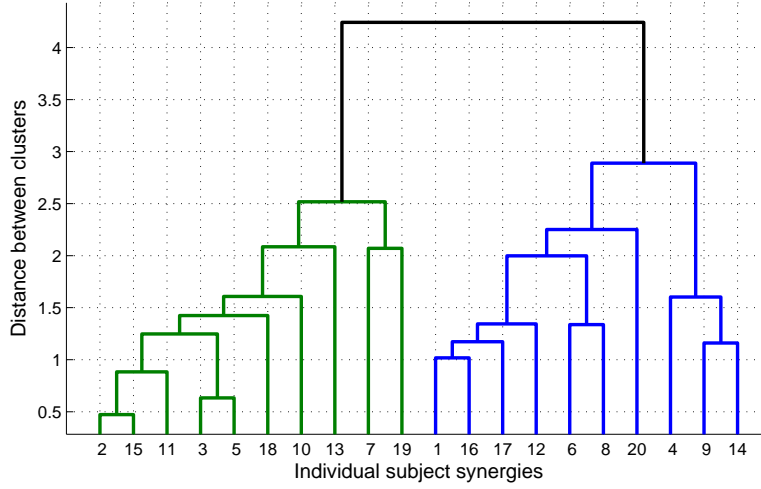


Figure 12: **Synergy clusters using hierarchical clustering.** Natural data divisions are found when the height of a link strongly differs to the height of the links below it. Thus, in this case the clusters are well separated.

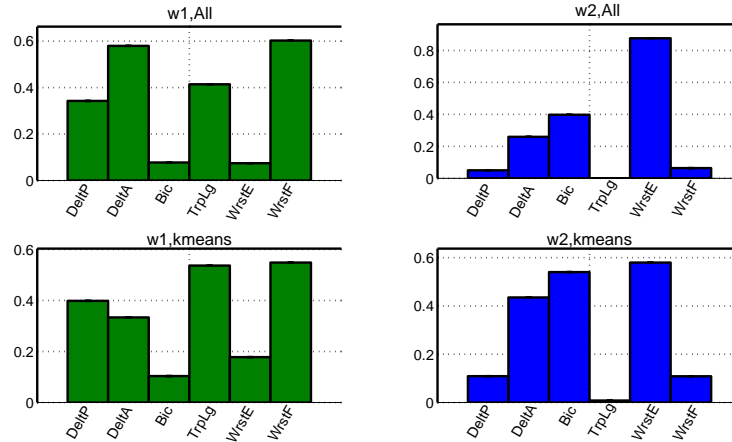


Figure 13: **Representative activation space strategy (synergies) $W_{I,All}$ and $W_{II,All}$ for all subjects.** A common control representation in the activation space was identified for the time-invariant part of the synergies (W) through method I and method II.

491 as the antagonist synergy to the motion. Therefore, \mathbf{w}_1 contains a high activation of muscles
 492 corresponding to shoulder flexion, internal rotation (deltoid anterior), elbow extension (triceps
 493 longs) and wrist flexion (wrist flexor group). While \mathbf{w}_2 contains a high activation of muscles
 494 corresponding to elbow flexion (biceps), wrist extension (wrist extensor group), and a very low
 495 activation of the shoulder muscles (deltoid anterior and posterior).

496 Finally, the representative synergy vectors (W) computed with both methods are similar, as
 497 shown by their normalized dot products (0.9248 for \mathbf{w}_1 and 0.9524 for \mathbf{w}_2). Consequently, a com-
 498 mon grouping and relative activation of muscles was found for different task space conditions and
 499 subjects during a throwing motion. This emphasizes the consistency of the results obtained by
 500 both methods to find a proper activation space control representation of the motion. However, in
 501 order to define a common control representation for throwing in the activation space, it is also nec-
 502 essary to identify a representative pattern for the time-varying part of the synergies (combination
 503 coefficients). The following section presents the results of this analysis.

Table 4: Synergy coefficients mean inter-subject cross-correlation coefficient per throwing distance.

Mean inter-subject cross-correlation coefficient per throwing distance			
Synergy Coeff	$corr_{2m}$	$corr_{4m}$	$corr_{7m}$
c_1	0.9129 ± 0.0534	0.9390 ± 0.0320	0.9264 ± 0.0382
c_2	0.8761 ± 0.0727	0.8615 ± 0.0769	0.8702 ± 0.0650

504 **5.4.2 Combination Coefficients (C)**

505 Method II also resulted in a set of time-varying coefficients which encoded the triggering times
 506 and intensity for each subject and their repetitions $C_{II}(t)$. The average coefficients computed per
 507 subject and throwing distance are featured in Figure 14 and Figure 15. Repeatable trends can be
 508 seen among and across subjects. For instance, the first coefficient c_1 is generally bell shaped (as
 509 the velocity profile in ballistic movements), while the second coefficient c_2 is more irregular, it has
 510 a lower amplitude, and it tends to decrease as the throw is performed. A considerable inter-subject
 511 repeatability at each throwing distance is also demonstrated by high correlation coefficients, as
 512 featured in Table 4.

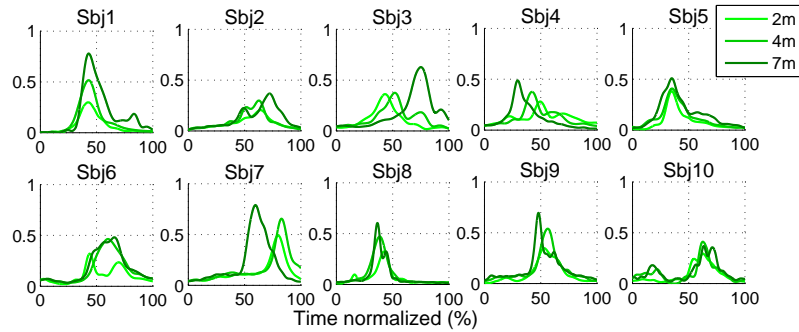


Figure 14: Average combination coefficient c_1 per subject and throwing distance.

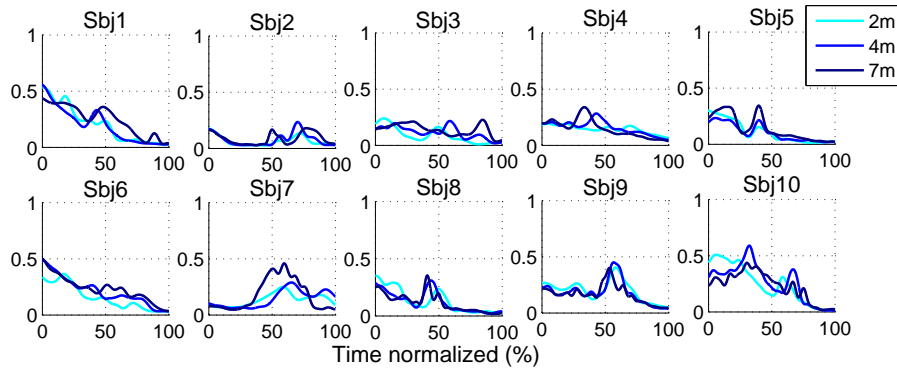


Figure 15: Average combination coefficient c_2 per subject and throwing distance.

513 The high intra and inter-subject repeatability outlines the existence of similar patterns of activation
 514 for each synergy across subjects and throwing distances. Therefore, an activation space control
 515 representation $\bar{C}_{II,AU}(t)$ was computed by performing averages across subjects at each
 516 throwing distance. The mean coefficients per distance are depicted in Figure 16. A high inter-
 517 distance correlation is seen for both coefficients (Table 5). Thus, these coefficients not only preserve
 518 the main trends in each of the subjects' averages, but also emphasize the similarities in terms of
 519 shape across throwing distances.

520 Besides a repeatability in terms of shape, the combination coefficients exhibit discrepancies

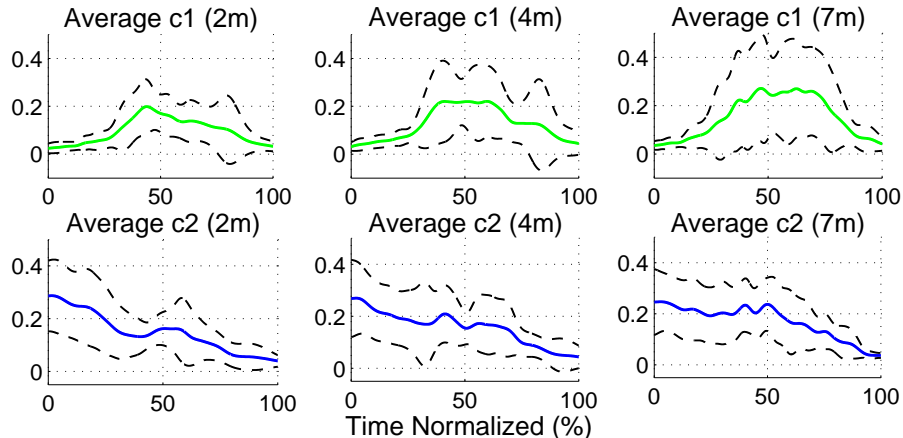


Figure 16: **Representative activation space strategy (combination coefficients) $\bar{C}_{II,AU}(t)$ for all subjects.** A common control strategy in the activation space was identified for the time-variant part of the synergies through method II.

Table 5: **Synergy coefficients mean inter-distance cross-correlation coefficients**

Mean inter-distance cross-correlation coefficients	
Synergy Coeff.	$corr_{2m,4m,7m}$
c_1	0.9895 ± 0.0035
c_2	0.9833 ± 0.0089

521 across subjects. Figure 17 shows the coefficients c_1 and c_2 from Figure 14 and Figure 15 in one
522 same plot. Globally, at the beginning of the throw c_2 (antagonist synergy) is activated, then the
523 amplitude of this synergy is diminished, until c_1 (agonist synergy) is activated. At this moment c_2
524 is activated again, and the most significant co-activation occurs among the synergies. The same
525 behavior is seen on the representative activation space strategy in Figure 16. This is consistent with
526 the fact that ballistic movements exhibit concurrent agonist and antagonist muscle activation [56].
527 During these motions a first activation is needed to accelerate the limb toward the target (c_1),
528 followed by a second activation to decelerate and stop the movement (c_2). This sequence of bursts
529 (from antagonist to agonist, and from agonist to antagonist) are characteristic of the antagonist
530 activity in the upper extremity while throwing. Such “triad” burst sequences have been previously
531 identified in EMG analysis of throwing (at the wrist and elbow muscles) [32], and in badminton
532 smash strokes [55].

533 Individual differences in combination coefficients triggering can be seen between subjects, espe-
534 cially for c_1 . This indicates that even if it is possible to find a common representation of the control
535 in the activation space for time-invariant features W , the combination coefficients C encapsulate
536 individual strategies and differences between subjects.

537 Another characteristic that was analyzed was the change in energy across throw types. Figure 18
538 shows the average energy at each throwing distance per subject, as described in Equation 10. The
539 results show that the energy changes in the coefficients are linked to changes in the task space
540 features: Like the task space features, the energy in the coefficients increases with the throwing
541 distance. For c_1 (agonist synergy), this increment is always gradually incrementing, and it is
542 statistically relevant for 6/10 of the subjects. This increment in the actuation signals (or synergies),
543 is consistent with the increment in torque magnitudes, observed during the synthesis of throwing
544 motions to different ranges [23].

545 This link between the task space and the activation space is fundamental in order to specify
546 muscle-based controllers available to synthesize motions from task space goals. Indeed, such a
547 controller will define a control law to actuate muscle in order to achieve task space goals and the
548 results of the current study is helpful to design these control laws [2].

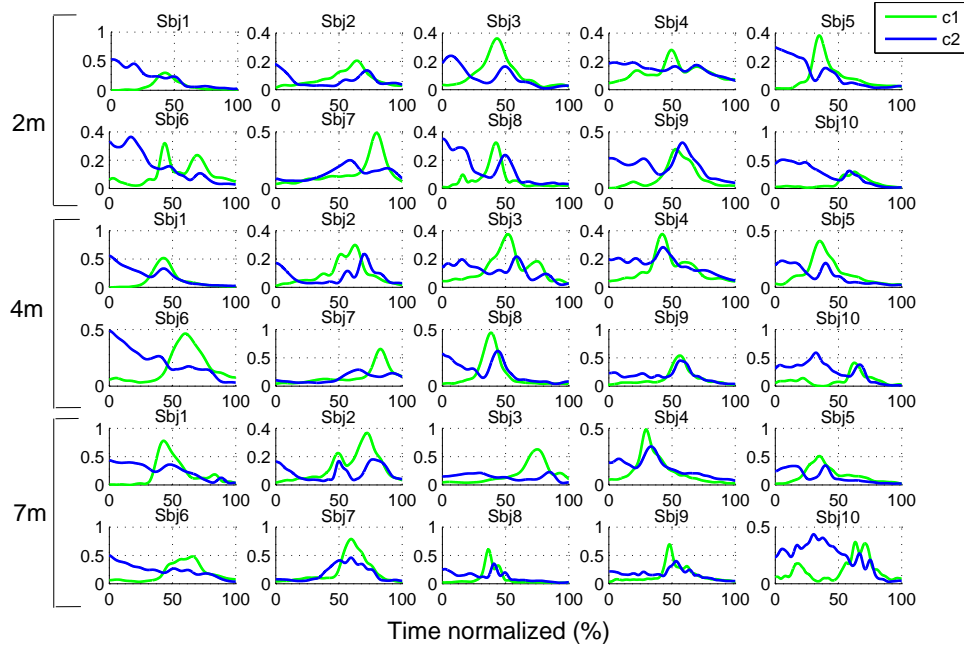


Figure 17: **Triggering order and co-activation per subject and throwing distance.** A repeatable triggering tendency is seen across subjects: 1) c_2 triggering, 2) c_1 triggering, and 3) c_2 triggering. This sequence is consistent with the expected triggering in ballistic motions.

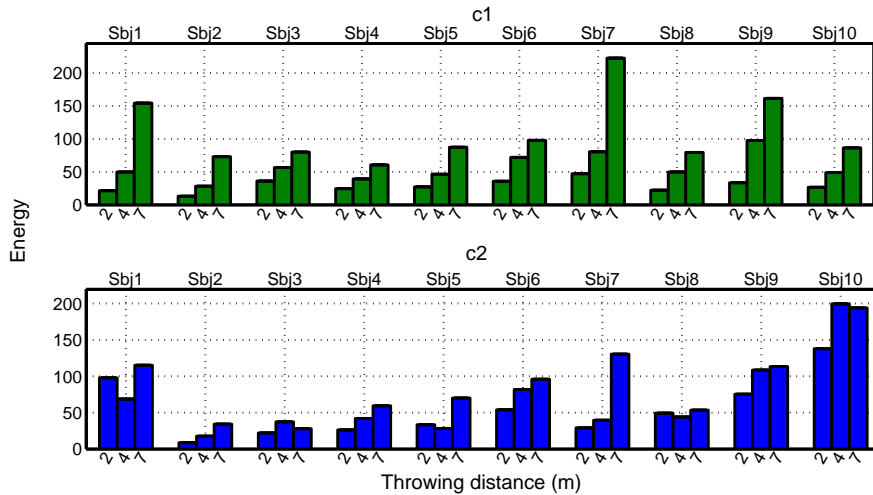


Figure 18: **Combination coefficient energy per subject and throwing distance.** The energy in c_1 (agonist synergy) gradually increases with throwing distance.

549 5.4.3 Activation Reconstruction

550 Finally, we show the quality of EMG reconstruction using the representative synergy model ($W_{II,AU}$,
 551 $C_{II,AU}(t)$) found through method II. We finally get an overall reconstruction quality of $r^2 = 0.6526$
 552 for the 180 concatenated muscle activations. This is reflected through different degrees of qual-
 553 ity reconstruction among the subject trials. In Figure 19 and Figure 20 are shown examples of the
 554 activation reconstruction of a 7m trial for different subjects. In the first case, the triggering
 555 order and shape of the reconstructed activations follow closely the recorded ones. In the second
 556 example, the original activations contain many small oscillations, which are not well reconstructed.

557 These oscillations may be noise artifacts and were therefore excluded from the reconstruction by
 558 the reconstruction quality criteria r^2 , as it has been explained in the methods section. More-
 559 over, considering the number trials that are being reconstructed simultaneously, such differences
 560 in reconstruction accuracy were expected.

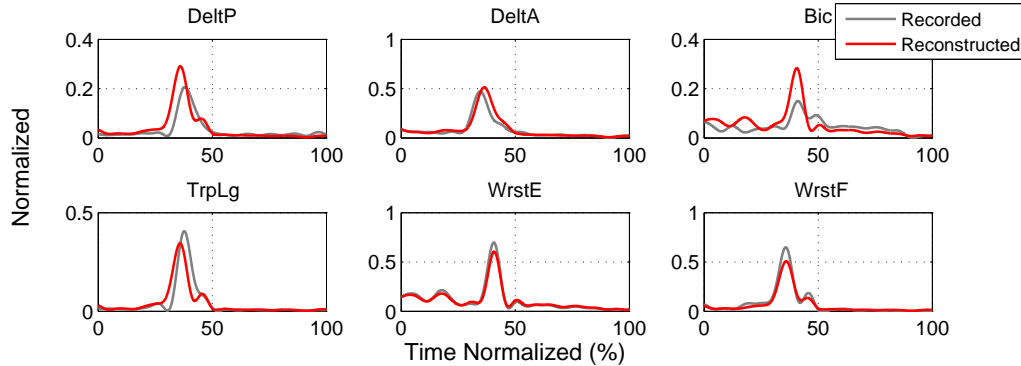


Figure 19: **Example 1: Activation reconstruction using $W_{II,AU}$ and $C_{II,AU}(t)$.**

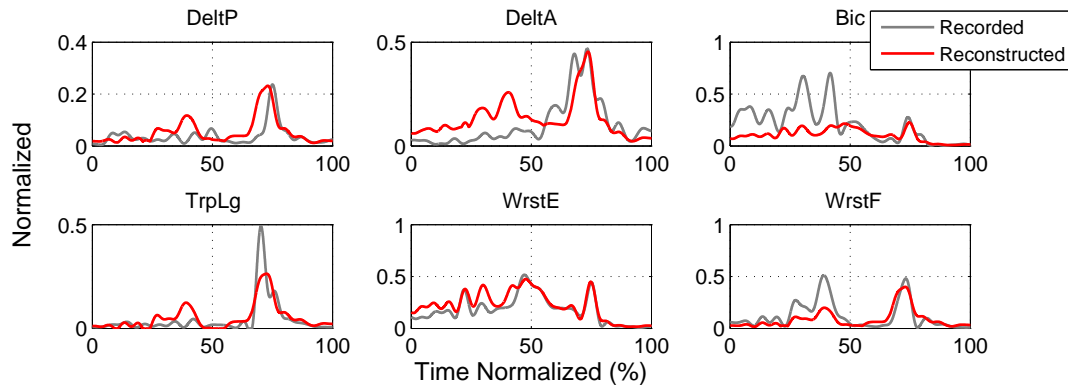


Figure 20: **Example 2: Activation reconstruction using $W_{II,AU}$ and $C_{II,AU}(t)$.**

561 In addition, we can see (Figure 21) the reconstruction quality per muscle with regard to the
 562 number of synergies extracted on the global set (method II). We can see that the results are quite
 563 consistent from one muscle to one other. Indeed, Most muscle respect the rule that the biggest
 564 change of slope of r^2 appears after 2 synergies. However, the Biceps exhibits a relatively low
 565 reconstruction level with 2 synergies and seem to have its highest change in slope at 3 synergies.
 566 This result can be explained by the relatively low level of activation of the Biceps during the task,
 567 that may be less well captured by the synergy extraction than more activated muscles like the
 568 Triceps Long. In a more general manner, muscles that stabilizes the motion may be less well
 569 captured by the low order synergies than the muscles producing the motion.

570 5.5 Summary

571 The previous results show the existence of a common control representation (for a subset of muscles
 572 and DoF) in various throwing tasks, and subjects with no particular training on throwing motions
 573 or throwing sports. This representation was described through a set of features in the task, joint,
 574 and activation spaces. The control representation identified in the task space consisted in increasing
 575 the hand release height and velocity to reach longer distance targets. These endpoint features were
 576 achieved through a common set of joint trends, but with different velocity trends across subjects.

577 In the activation space a lower dimensional control representation and its link with changes in
 578 the task space features was identified. This control strategy consisted in using only 2 synergies

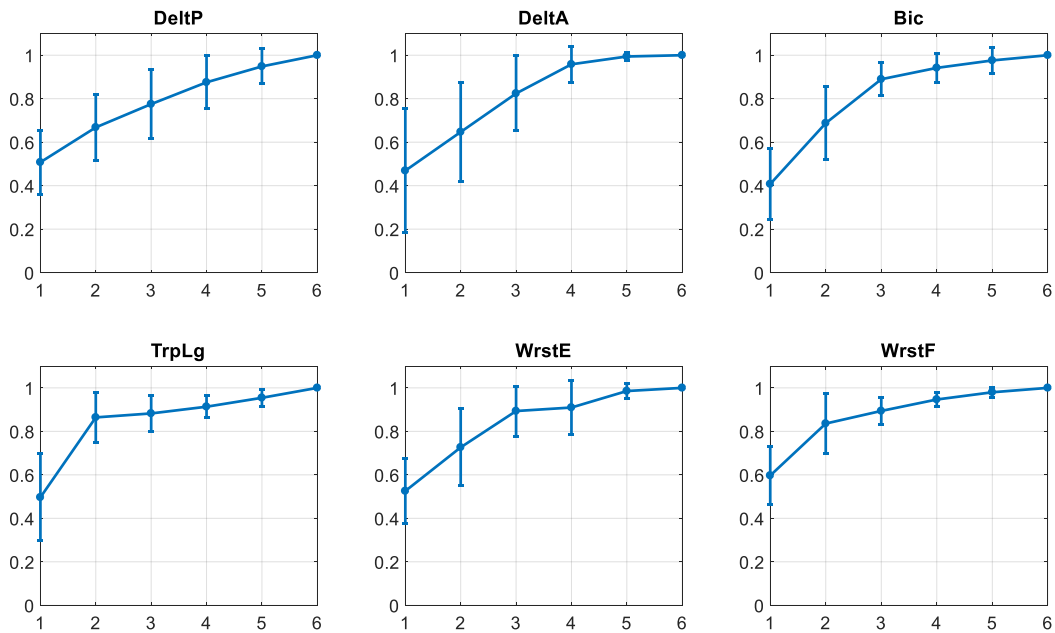


Figure 21: **Activation reconstruction quality per muscle** . Results per muscle shows discrepancies in the reproducibility of the results. Some muscles have relatively consistent patterns to reconstruct with 2 synergies (Triceps Long, Wrist Flexors Group) whereas some other may be better reconstructed thanks to higher synergy levels (Biceps).

579 (an agonist and an antagonist synergy), to represent the activation of 6 muscles of the right arm.
 580 These synergies were triggered with the order and concurrency expected from ballistic movements,
 581 and their triggering intensity was linked to the desired launch distance, the increments in velocity
 582 and height of release. Therefore, at the actuation level we were able to extract a reduced control
 583 representation (muscle synergies) linked to task conditions, for a highly redundant, nonlinear, and
 584 dynamic motion. Such a method, by providing a compact representation, has a interest to depict
 585 the individual and common control features in the way the motion is generated by each subject and
 586 seems useful to better understand the control strategies used. This does not prove the existence of
 587 a motor control mechanism that would be muscle synergies. However, the results are compatible
 588 with the notion of muscle synergies organized by the nervous system to implement such control
 589 strategies.

590 Moreover, the direct extraction of a single synergy model from an experiment involving such
 591 a complex motion, and a variety of human morphologies, skills, and task conditions, is also a
 592 contribution. The results obtained by both methods of synergy extraction showed encouraging
 593 results, since their consistency and robustness was clearly established through their comparison.
 594 The accuracy of this synergy model is supported by studies [57] that evidence a higher performance
 595 of matrix factorization algorithms in experimental protocols that incorporate unconstrained tasks,
 596 a variety of conditions, and motor variability (synergy extraction from EMG time-series data, and
 597 not averages).

598 It is worth noting that these results span a limited set of degrees of freedom and muscles,
 599 and that the extracted synergies for this task can change depending on the number and choice
 600 of muscles [58]. They also highlight generic but basic mechanisms needed to control an overhead
 601 throwing motion to a specific distance. To analyze the accuracy, efficiency, or performance of the
 602 throw, studies with additional features at key moments, and their relationship to successful target
 603 hits are needed. These features could include: task space features, such as release angle, joint space
 604 features, such as velocities and accelerations at release, and activation space features that include
 605 more muscles and quantify subtle difference in the way in which the synergies are triggered across
 606 different throws. With more features, we could expect to find more links across task, joint, and
 607 activation spaces.

608 Future contributions could include repeating this analysis on professional throwers (such as
609 football players or pitchers). We expect a higher repeatability at all levels for trained subjects.
610 Also, future analysis could include throws to larger distances and the usage of balls of different
611 masses and sizes.

612 Finally, the synergies obtained in the current studies will be applied and validated in the domain of
613 muscle-based character animation. For instance, the relationships between the 2 control variables
614 (or synergies) and well defined task space goals (desired release speed and height) will be exploited
615 to control highly redundant characters. A previous study [8, 24] has already tested synergies on
616 a subject-scaled character. It would be interesting to test the generic synergies presented in this
617 paper on a variety of morphologies. Ultimately, this application could also entail the construction
618 of a synergy database for animation. A database containing synergies and their relationships with
619 task space goals, for a richer variety of motions (reaching, writing or other arm gestures), degrees
620 of freedom, and muscles, which could also serve as a basis to synthesize motions in physics-based
621 animation.

622 6 Conclusion

623 It seems that motion control can be encapsulated through lower dimensional control representation
624 of each task we perform, to achieve fast, efficient, and coordinated movements. Synergies encode
625 a variety of muscle information in a reduced set of temporal and spatial signals, and are thus a
626 good candidate to represent the control in a compact way. Many studies have extracted muscle
627 synergies from EMG signals in both upper-body and lower-body motions. Our study has found
628 common control features among subjects in the task, joint and activation space, especially though
629 the extraction of muscle synergies from a set of EMG signals, for a dynamic and acyclic motion.
630 A motion which was performed by unexperienced subjects while following general guidelines that
631 allowed a free throwing motion.

632 We first described the throwing task and experiments from which the control strategy was
633 extracted. Next, we characterized the motion through a set of control features in the task, joint, and
634 activation spaces, and detailed the methods to extract them. Finally, the results showed that with
635 this set of features: 1) a common control representation exists across subjects, 2) this representation
636 significantly reduces the redundancy in the activation space through the encapsulation of the co-
637 activated muscles in a low-dimensional representation (2 synergies encode the actions of 6 muscles),
638 3) links exist between the task and activation space features, which were revealed by varying the
639 throwing distance, 4) and finally both methods of synergy extraction were able to provide consistent
640 and similar results and are therefore legitimate these methods of extraction.

641 Lastly, since the identified control representation comprises the use of less control signals than
642 actuators and DoF, it would be useful for synthesizing motions with overactuated or muscle-based
643 characters at a reduced computational cost.

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