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# Dipole recovery from sparse measurements of its field on a cylindrical geometry

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**Abstract.** We consider dipole recovery issues from sparse magnetic data, with the use of best quadratic rational approximation techniques, together with geometrical and algebraic properties of the poles of the approximants.

Paleomagnetic studies revealed evidences for an ancient Moon dynamo field [1], some of which come from the remanent magnetization presence in rocks found on the Moon’s surface. Planetary scientists who study those rocks (available at NASA, from Apollo missions) want to recover their magnetic moment by taking sparse measurements of their magnetic field. In contrast to other paleomagnetic studies, where the sample was sliced or remagnetized, here the sample must be protected from any kind of damage. For this nondestructive inspection purpose, a special magnetometer has been constructed by scientists at CEREGE\* which they call “lunometer”. This device encloses the rock sample in a nonmagnetic cubic box and isolates it from external electromagnetic fields with a mu-metal shield. The issue of recovering the remanent magnetization of a rock from the measurements provided by the lunometer is the inverse problem that we address.

The main geometry is based on the data acquisition process. The lunometer provides measurements of the magnetic field  $B$  on the sections of three orthogonal cylinders (in Figure 1, the colored planes cross perpendicularly those cylinders  $j = 1, 2, 3$ ). More specifically, data on each cylinder are given on three sections  $i = 1, 2, 3$  (in Figure 1, the colored circles represents those sections and each color corresponds to measurements of a component of the field  $B$ ).

The underlying magnetic phenomenon is modeled by Maxwell equations in the magnetostatic and macroscopic framework [2]. We preliminary assume that the sample contains a unique pointwise dipolar unknown magnetic source located at  $X_d$ , with moment  $M_d$ . The expression of the magnetic field at  $X \neq X_d$  (where  $\mu_0$  is the permeability of the free space) has the form:

$$-\frac{4\pi}{\mu_0}B(X) = \frac{|X - X_d|^2 M_d - 3[M_d \cdot (X - X_d)](X - X_d)}{|X - X_d|^5} \quad (1)$$

From measurements of the magnetic field  $B$  at sensors positions, we want to recover the moment  $M_d$  and the location  $X_d$  of the dipole. Our strategy is to decompose this inverse problem into two subproblems and search firstly for the location  $X_d$  of the dipole (which is a nonlinear problem) and secondly for its moment  $M_d$  (which becomes a linear problem when the location is known).

Let  $B_i^j$  denote the available data of a component of  $B$  (on a given section  $i$  of the cylinder  $j$ ). Then,  $(B_i^j)^2$  can be seen as the trace on the circle of a rational function of the complex variable in the corresponding plane [3]. This function has one pole inside the disk and one outside. From the values of this function, we use the best quadratic rational algorithm called RARL2<sup>‡</sup> which provides us with an approximation  $\xi_i^j$  of the pole inside. Vieta’s formula allows us to estimate the pole outside, hence to fully recover the denominator

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‡<https://project.inria.fr/rarl2/>

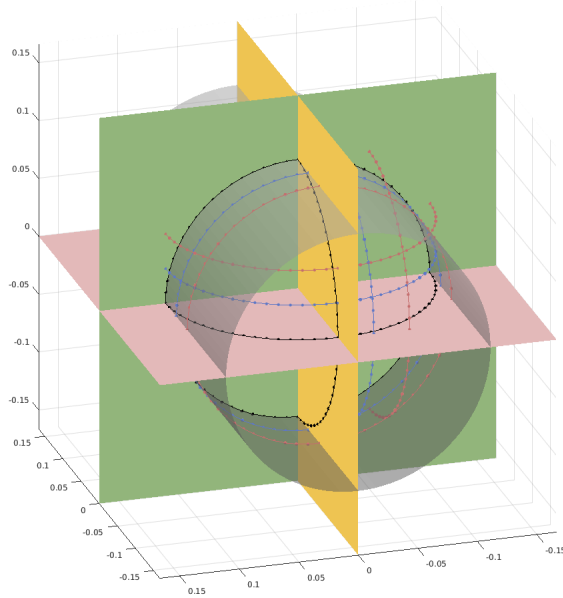


Figure 1: Available data geometry.

of the rational function. We establish relations between the denominator and the unknown actual dipole position  $X_d$ . Because of the available data geometry, we obtain nine such relations.

Observe that a variety of factors can affect the moment recovery process. Indeed, the data is subject to measurement errors and the condition number of the problem depends on the actual dipole location/orientation. Geometrical methods could work as supplementary to the algebraic methods. Moreover, combining information from different sections of the orthogonal cylinders increases the robustness of our estimation scheme.

Finally, once the dipole location  $X_d$  is estimated, we can estimate its moment from expression (1): we solve the linear system obtained for the unknown moment  $M_d$  by using mean squares minimization. During the talk, these techniques will be explained and numerical results will be discussed.

## References

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- [2] J. D. Jackson, *Classical electrodynamics*, third edition, John Wiley & Sons, 2001.
- [3] L. Baratchart, J. Leblond and J. P. Marmorat, *Inverse source problem in 3D ball from best meromorphic approximation on 2D slices*, Electronic Transactions on Numerical Analysis, 25, pp. 41-53, 2006.

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\*\*<http://maglune.cerege.fr/>