A reconstruction of the Mathematical Tables Project's table of natural logarithms (4 volumes, 1941)

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"[f]or a few brief years, [the Mathematical Tables Project] was the largest computing organization in the world, and it prepared the way for the modern computing era."
D. Grier, 1997 [29]

"Blanch, more than any other individual, represents that transition from hand calculation to computing machines."
D. Grier, 1997 [29]

"[Gertrude Blanch] was virtually the backbone of the project, the hardest and most conscientious worker, and the one most responsible for the amount and high quality of the project's output."
H. E. Salzer, 1989 [66]

1 The Mathematical Tables Project

The present table was published by the Mathematical Tables Project, a project of the Works Progress Administration (WPA, renamed Works Projects Administration), a New Deal agency established by President Roosevelt to alleviate unemployment through public works. The purpose of the Mathematical Tables Project was to compute tables of higher mathematical functions. Because the Mathematical Tables Project was part of the WPA, much of the computation was done by hand. This project was in operation since January 1938 and its administrative director was Arnold Lowan.[1] The mathematical leader of the Project was Gertrude Blanch[2] [74, 29, 30, 31, 32, 33, 28].

Prior to the Mathematical Tables Project, the British association for the advancement of science had started publishing volumes of tables in 1931. Between 1931 and 1946, 11 volumes were published, and a final one in 1952 [20], [33, p. 174]. The British group appears with hindsight to have been driven less by the production of general fundamental tables than the Mathematical Tables Project. Instead, it was more aimed at organizing earlier tables. These twelve volumes are the following ones:

[1]Arnold Noah Lowan (1898–1962) [11, 5] was born Leibovici in Iasi (Romania). He graduated from the Bucharest Polytechnical Institute of Chemical Engineering in 1924, and the same year moved to the United States. He obtained a Master of Science from New York University in 1929 and a PhD from Columbia University in 1934. He was a fellow at the Institute for Advanced Study, Princeton (1928–1931), lecturer of mathematics at Brooklyn College, New York (1935–1940).

From 1938 to 1949, he was the director of the computation laboratory at the National Bureau of Standards, where he was directing the publication of a number of mathematical tables. From 1950 to 1952, he was a consultant at the US Naval Ordnance Laboratory and from 1955 to 1962, he was professor of mathematics at Yeshiva University, New York.

[2]Gertrude Blanch (1897–1996) was born in Poland and moved to the United States around 1907. After having graduated from high school in 1914, she first worked as a clerk for 14 years, honing her skills and knowledge of accounting, inventory, planning, risk calculations, and so on. In 1928, she fulfilled her dream to become a mathematician and matriculated to New York University. She received a BSc in Mathematics from NYU in 1932 and a PhD in mathematics from Cornell University in 1935. Around the end of 1937, while attending a continuing education class on relativity taught by Arnold Lowan, Lowan offered her the job of technical director of the Mathematical Tables Project, which she joined in February 1938. Within that project, she designed algorithms that were executed by teams of human computers. Blanch also worked regularly with the Manhattan Project, both during and after the war. In the mid-1950s, she was hired by the Air Force and continued to work on numerical analysis, in particular on Mathieu functions.

I. *Circular and hyperbolic functions, exponential and sine and cosine integrals, factorial and allied functions, hermitian probability functions* (1931)

II. *Emden functions, being solutions of Emden's equation together with certain associated functions* (1932)

III. *Minimum decompositions into fifth powers* (1933)

IV. *Cycles of reduced ideals in quadratic fields* (1934)

V. *Factor table giving the complete decomposition of all numbers less than 100,000 (1935)* [56]

VI. *Bessel functions. Part I. Functions of orders zero and unity.* (1937)

VII. *The probability integral* (1939)

VIII. *Number-divisor tables* (1940)

IX. *Table of powers, giving integral powers of integers* (1940)

X. *Bessel functions. Part II. Functions of positive integer order.* (1952)

Part-volume A. *Legendre polynomials* (1946)

Part-volume B. *The Airy integral, giving tables of solutions of the differential equation* $y'' = xy$ (1946)

On the other hand, the Mathematical Tables Project computed a number of large tables mostly *ab initio*. Moreover, the purpose of the Project was not so much to complete the computations quickly, but to keep the (human) computers busy, and at the same time to conduct some useful work. At one point the Mathematical Tables Project employed 450 human computers, sometimes aided by mechanical calculating machines, a group which was reminiscent of the one set up for the famed French *Tables du cadastre* [61].

The main tables published between 1939 and 1949 by the Mathematical Tables Project are the following ones:[3]

- *Table of the first ten powers of the integers from 1 to 1000*, 1939
- *Tables of the exponential function* e^x, 1939 (reconstructed in [65])
- *Tables of circular and hyperbolic sines and cosines for Radian arguments*, 1939
- *Tables of sines and cosines for Radian arguments*, 1940
- *Tables of sine, cosine and exponential integrals*, 1940 (2 volumes)
- *Table of natural logarithms*, 1941 (4 volumes)
- *Tables of the moment of inertia and section modulus of ordinary angles, channels, and bulb angles with certain plate combinations*, 1941
- *Miscellaneous physical tables*, 1941

[3]Numbers such as MT1, MT2, etc. were given to each volume, but only at a later time. They served for a proper identification of each volume. However, the numbers given in the National Bureau of Standards's publication list [54] and by Grier [30] do not completely coincide. It was possibly only after 1948 that a set of 28 "main tables" was presented, with numbers from MT1 to MT28. The list given here is that given by Grier.

- *Table of sine and cosine integrals for arguments from 10 to 100*, 1942
- *Tables of probability functions*, 1942 (2 volumes)
- *Table of arc tan x*, 1942
- *Table of reciprocals of the integers from 100,100 through 200,009*, 1943
- *Table of the Bessel functions $J_0(z)$ and $J_1(z)$ for complex arguments*, 1943
- *Table of circular and hyperbolic tangents and cotangents for radian arguments*, 1943
- *Tables of Lagrangian interpolation coefficients*, 1944
- *Table of arc sin x*, 1945
- *Tables of associated Legendre functions*, 1945
- *Tables of fractional powers*, 1946
- *Tables of spherical Bessel functions*, 1947 (2 volumes)
- *Tables of Bessel functions of fractional orders*, 1948 & 1949 (2 volumes)
- *Tables of Bessel functions $Y_0(x)$, $Y_1(x)$, $K_0(x)$, $K_1(x)$, $0 \leq x \leq 1$*, 1949

Many other smaller or more specialized tables were also published by the Mathematical Tables Project. Lists of published tables are given in the appendices of each of the published volumes. The announcement published in 1941 [25] also lists the tables published so far, those for which computation had been completed or was in progress, and those which were considered for calculation. Archibald's survey gives the status of computations by the end of 1942 [9].

The WPA was terminated in 1943, but the Mathematical Tables Project continued to operate in New York until 1948. That year, a number of members of the Mathematical Tables Project moved to Washington, DC to become the Computation Laboratory of the National Bureau of Standards, now the National Institute of Standards and Technology. But Blanch moved to Los Angeles to lead the computing office of the Institute for Numerical Analysis at UCLA, anf Lowan joined the faculty at Yeshiva University in New York. Other tables continued to be computed, of which a detailed list is given by Fletcher *et al.* [26, pp. 718–720].

The greatest legacy of the Project is the *Handbook of Mathematical Functions* [1], published in 1964, and edited by Milton Abramowitz (1915–1958) and Irene A. Stegun (1919–2008), two veterans of the Project. But more broadly, the Project developed "the numerical methods of scientific computation [and demonstrated] that computation could solve practical and important problems" [29].

2 Tables of natural logarithms

In this section, we review the main tables of natural logarithms. Although sometimes called *Napierian logarithms*, the first table of natural logarithms was not published by Napier, but by Speidell a few years later [70, 60, 45].

An extensive table of natural logarithms was first computed by Isaac Wolfram, a Dutch lieutenant of artillery[4] and published by Schulze in 1778 [68] (figure 1). Wolfram's table represents six years of work [17, p. 166]. The original table omits half-a-dozen logarithms which Wolfram could not compute because of an illness. These logarithms were supplied in the Berlin Ephemeris for 1783 [69, p. 191], [24, p. 602].

As highlighted by Archibald [12, p. 193], Wolfram made use of two or three different methods for the computation of the natural logarithms. A first way is to take an accurate value of the decimal logarithm and to multiply it by $\ln 10$. Two other ways are to make use of the formulæ

$$\ln(1+x) = \sum_{m=1}^{\infty}(-1)^{m+1}\frac{x^m}{m} \tag{1}$$

$$\ln\frac{1}{1-x} = \sum_{m=1}^{\infty}\frac{x^m}{m} \tag{2}$$

and to choose appropriate values for x. For instance, with $x = 1/23400$, we have

$$1+x = \frac{23401}{23400} = \frac{7\cdot 3343}{2^3\cdot 3^2\cdot 5\cdot 13}$$

and the computation of $\ln(1+x)$, combined with the values of the logarithms of 2, 3, 5, 7, and 13, provides the logarithm of 3343. With $x = \frac{1}{2651\cdot 10^3}$, we have

$$1-x = \frac{2651\cdot 10^3}{2651\cdot 10^3 - 1} = \frac{11\cdot 241\cdot 10^3}{13\cdot 61\cdot 3343}$$

and we have another way to compute $\ln 3343$. This method was in fact anticipating the work of Edward Sang [67] who proceeded much in the same way.

Apart from Wolfram's table which was reprinted in 1794 by Vega [79] (figure 2), in 1795 by Callet (first hundred primes, to 20 places) [18], and in 1922 by Peters & Stein [55], we mention the large tables of Thiele (1908, 48 decimals) [72], Vietz (1825, 81 decimals) [78], Warmus (1954, 108 decimals) [14], Uhler (1942, 137 decimals) [76], and Uhler (1943, 155 decimals) [77]. Uhler (1940) [75] gave a few basic natural logarithms to about 330 places. The present table published in 1941 gave the logarithms to 16 places, and finally, Spenceley, Spenceley & Epperson (1952) [71] gave them to 23 places.

Mention should also be made of Kulik who had prepared a table of natural logarithms to 48 places from 1 to 11000, based on Wolfram's table [38, 36, 37, 7, 10, 12, 13], but although this table may have been printed, no copy has been located anywhere.

A number of extensive smaller tables of natural logarithms have also been published, in particular by Barlow (1814) [15] (reprinted in Rees's Cyclopædia (1819) [4]) and Dase (1850) [22].

For an extensive list of existing tables of natural logarithms by about 1960, see the survey published by Fletcher in 1962 [26, p. 249]

[4]For more on Wolfram, see [27, 8, 12, 13]. Wolfram had also been working on factor tables [39, 40].

3 The Project's table of natural logarithms (1941)

3.1 Description

The Project's table of natural logarithms spans four volumes [43]. The first two volumes give the natural logarithms of the integers from 1 to 49999 and from 50000 to 99999 to sixteen decimal places. The last two volumes give the natural logarithms of the decimal numbers from 0 to 10, by steps of 0.0001, also to sixteen decimal places. The latter two volumes can be (and were) derived from the first two by subtracting $\ln(10000) = 4 \ln 10$, so that for instance $\ln 0.2805 = \ln 2805 - 4 \ln 10$.

The technical staff involved in the preparation of these volumes was made of Gertrude Blanch (1897–1996), Frederick G. King, Milton Abramowitz (1915–1958), Jack Laderman, William Kaufman, Matilda Persily, and Jacob Miller for volumes 1 and 2, in that order. In addition, volumes 3 and 4 list William Horenstein, Ida Rhodes (1900–1986) and Herbert E. Salzer (1915–2006), but this time the names are listed alphabetically.

3.2 Algorithms

The starting point of the Project's table was Wolfram's table of natural logarithms published by Schulze in 1778 [68]. This table gave the natural logarithms of the first 2200 integers and of primes and certain composite numbers up to 10009, all to 48 decimal places. The only omissions were the logarithms of 9769, 9781, 9787, 9771, 9783, and 9907. The Project used Vega's reprint published in 1794 [79], where the omissions had been filled.

The logarithms of composite numbers not found in Wolfram's table were obtained by the formula $\ln AB = \ln A + \ln B$. Logarithms of primes greater than 10009 were computed as follows: using

$$\operatorname{argth}(x) = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \cdots \tag{3}$$

and

$$\operatorname{argth}(x) = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right) \tag{4}$$

we have

$$\operatorname{argth}\left(\frac{1}{2n^2-1} \right) = \frac{1}{2} \ln \left(\frac{1 + \frac{1}{2n^2-1}}{1 - \frac{1}{2n^2-1}} \right) = \frac{1}{2} \ln \left(\frac{n^2}{n^2-1} \right) \tag{5}$$

$$= \ln n - \frac{\ln(n^2-1)}{2} = \ln n - \frac{\ln(n-1) + \ln(n+1)}{2} \tag{6}$$

From that it follows that

$$\ln n = \frac{\ln(n-1)+\ln(n+1)}{2} + \operatorname{argth}\left(\frac{1}{2n^2-1}\right) \tag{7}$$

$$= \frac{\ln(n-1)+\ln(n+1)}{2} + \frac{1}{2n^2-1} + \frac{1}{3\cdot(2n^2-1)^3} + \frac{1}{5\cdot(2n^2-1)^5} + \cdots \tag{8}$$

Hence, $\ln n$, where n is prime, was obtained from the logarithms of two even numbers, as well as a series. This series converges very rapidly, and when $n > 10009$, only the first term was used, all the others being very small.

These logarithms were computed to 20 places and differencing tests were applied to ensure that the 20th place was correct.[5] These 20-place values were then rounded to 16 places and further subjected to differencing tests. The 20-place worksheet values were also used to derive the last two volumes, by subtraction of $4\ln 10$, and the values were then rounded to 16 places.

The first 20 decimal places of all of Wolfram's logarithms were compared with those found here, and several errors were found in Wolfram's table. Most, if not all, of these errors were actually typos, rather than genuine computation errors.[6]

In order to ensure the correctness of the table, the 20-place values were added by groups of ten, and comparisons were made between the two volumes, as the difference of two corresponding sums had to be $40\ln 10$. Moreover, the values from 10000 to 20000 and from 30000 to 100000 were compared with Thompson's *Logarithmetica Britannica* [73], by adding the values in groups of ten and multiplying them by $\ln 10$. Not a single error was found in either table. (The logarithms of integers from 20000 to 30000 had not yet been published by Thompson, whose work was only completed in 1952.)

In addition to the description of the construction of the table, Lowan also describes interpolation methods. For direct interpolations of logarithms, one might make use of Everett's formula, which is (with the original notations)

$$u_p = pu_1 + qu_0 + \frac{1}{6}p(p^2-1)d^2u_1 + \frac{1}{6}q(q^2-1)d^2u_0$$

where p is the decimal part of the number, $q = 1-p$, $u_0 = \ln x_0$, $u_1 = \ln(x_0+1)$, $u_p = \ln(x_0+p)$ and d^2u_0 and d^2u_1 are the second central differences corresponding to u_0 and u_1 respectively. In order to use this formula, it is therefore necessary to compute the second central differences from the values in the table, since the original table does not give them directly. We do not go into the details of Everett's formula here, and we direct the reader to our introduction to the reconstruction of Thompson's table of logarithms [59].

On the other hand, it can often be dispensed with Everett's formula and in most cases, finding the logarithm of a decimal number can be done by using the development of $\ln(1+x)$ and dividing the decimal number by its integral part. For instance, in order

[5] No details on these tests are given, but we can expect occasional errors of one unit in the 20th place.

[6] A table of erroneous logarithms is given in the original introduction, but we do not reproduce it here. The faulty values are the logarithms of 829, 1099, 1409, 1937, 1938, 2093, 3571, 4757, 6343, 7853, 8023, 8837, and 9623.

to compute $\ln 1.23456$, one can write

$$\begin{aligned}\ln 1.23456 &= \ln\left(\frac{12345.6}{10000}\right) \\ &= \ln\left(\frac{12345.6}{12345}\right) + \ln 12345 - \ln 10000 \\ &= \ln(1.0000486\ldots) + \ln 12345 - \ln 10000\end{aligned}$$

and use the tabulated values of $\ln 10000$ and $\ln 12345$, and the series development for $\ln 1.0000486\ldots$

For inverse interpolation, the simplest method is by the linear interpolation formula

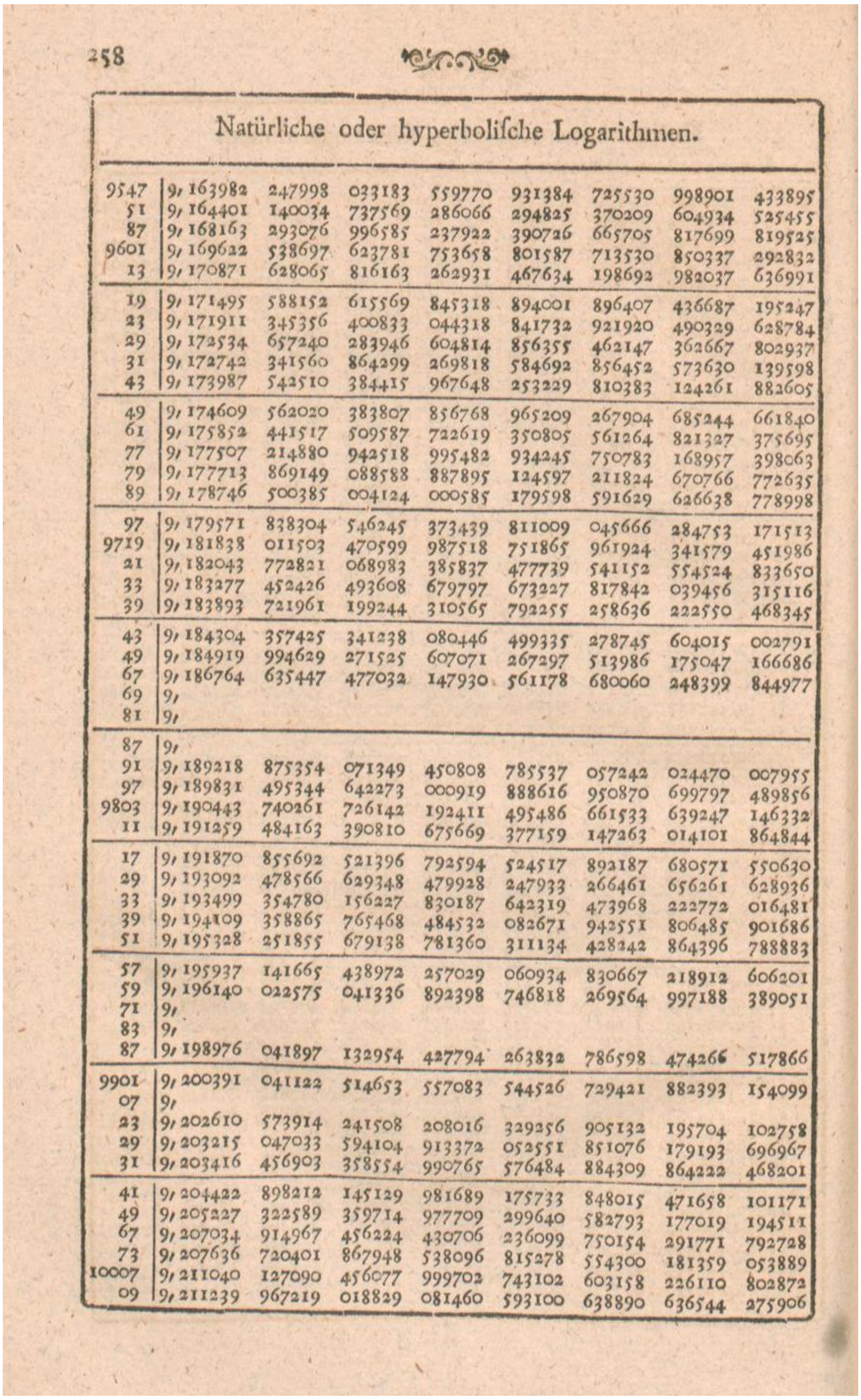
$$x = x_0 + \frac{\ln x - \ln x_0}{\ln(x_0+1) - \ln x_0}.$$

By linear inverse interpolation, it is possible to obtain ten significant digits in the argument. The introduction to the original volumes gives hints for more accurate methods of inverse interpolations, and sometimes it may be useful to make use of the table of exponentials [42]. But in any case, one should be aware that the number of meaningful figures in the argument is about the same as in the logarithms, so that for instance if a logarithm is given to 10 decimal places, this will often mean that there is an uncertainty of possibly $0.5 \cdot 10^{-10}$ and this uncertainly will be transferred to the argument. It is therefore usually useless to try to obtain more significant places in the arguments than there are significant places in the logarithms, except if the logarithm is considered to be exact, in which case different methods may be more appropriate anyway.

3.3 Notes on the layout

The layout of the tables is in general quite straightforward. There is just one idiosyncrasy. In volume 3 of the original tables, the logarithms up to 0.9999 are negative and the signs are given every five values. But when the integer part changes, the original table gives the absolute value of the logarithm, and not its real value. One has to assume that the sign should be taken from the previous full value given. This may be confusing. For instance, $\ln 0.0004$ is given as $7.82404\ldots$, but it is really $-7.82404\ldots$. We have respected this peculiarity. There is only one such case where the original value is given with its sign, namely for $\ln 0.0498$, but for reasons of consistency, we have also used the absolute value in our reconstruction.

Interestingly, the original volume 4 that we had in hands uses two different types of paper, one light, the other darker, and a note printed at the beginning of the volume reads "Because of a paper shortage caused by the war, it was necessary to use two types of paper for this volume."



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Natürliche oder hyperbolische Logarithmen.

9547	9, 163982	247998	033183	559770	931384	725530	998901	433895
51	9, 164401	140034	737569	286066	294825	370209	604934	525455
87	9, 168163	293076	996585	237922	390726	665705	817699	819525
9601	9, 169622	538697	623781	753658	801587	713530	850337	292832
13	9, 170871	628065	816163	262931	467634	198692	982037	636991
19	9, 171495	588152	615569	845318	894001	896407	436687	195247
23	9, 171911	345356	400833	044318	841732	921920	490329	628784
29	9, 172534	657240	283946	604814	856355	462147	362667	802937
31	9, 172742	341560	864299	269818	584692	856452	573630	139598
43	9, 173987	542510	384415	967648	253229	810383	124261	882605
49	9, 174609	562020	383807	856768	965209	267904	685244	661840
61	9, 175852	441517	509587	722619	350805	561264	821327	375695
77	9, 177507	214880	942518	995482	934245	750783	168957	398063
79	9, 177713	869149	088588	887895	124597	211824	670766	772635
89	9, 178746	500385	004124	000585	179598	591629	626638	778998
97	9, 179571	838304	546245	373439	811009	045666	284753	171513
9719	9, 181838	011503	470599	987518	751865	961924	341579	451986
21	9, 182043	772821	068983	385837	477739	541152	554524	833650
33	9, 183277	452426	493608	679797	673227	817842	039456	315116
39	9, 183893	721961	199244	310565	792255	258636	222550	468345
43	9, 184304	357425	341238	080446	499335	278745	604015	002791
49	9, 184919	994629	271525	607071	267297	513986	175047	166686
67	9, 186764	635447	477032	147930	561178	680060	248399	844977
69	9,							
81	9,							
87	9,							
91	9, 189218	875354	071349	450808	785537	057242	024470	007955
97	9, 189831	495344	642273	000919	888616	950870	699797	489856
9803	9, 190443	740261	726142	192411	495486	661533	639247	146332
11	9, 191259	484163	390810	675669	377159	147263	014101	864844
17	9, 191870	855692	521396	792594	524517	892187	680571	550630
29	9, 193092	478566	629348	479928	247933	266461	656261	628936
33	9, 193499	354780	156227	830187	642319	473968	222772	016481
39	9, 194109	358865	765468	484532	082671	942551	806485	901686
51	9, 195328	251855	679138	781360	311134	428242	864396	788883
57	9, 195937	141665	438972	257029	060934	830667	218912	606201
59	9, 196140	022575	041336	892398	746818	269564	997188	389051
71	9,							
83	9,							
87	9, 198976	041897	132954	427794	263832	786598	474266	517866
9901	9, 200391	041122	514653	557083	544526	729421	882393	154099
07	9,							
23	9, 202610	573914	241508	208016	329256	905132	195704	102758
29	9, 203215	047033	594104	913372	052551	851076	179193	696967
31	9, 203416	456903	358554	990765	576484	884309	864222	468201
41	9, 204422	898212	145129	981689	175733	848015	471658	101171
49	9, 205227	322589	359714	977709	299640	582793	177019	194511
67	9, 207034	914967	456224	430706	236099	750154	291771	792728
73	9, 207636	720401	867948	538096	815278	554300	181359	053889
10007	9, 211040	127090	456077	999702	743102	603158	226110	802872
09	9, 211239	967219	018829	081460	593100	638890	636544	275906

Figure 1: An excerpt of Wolfram's table as published by Schulze [68], with the six missing values added later.

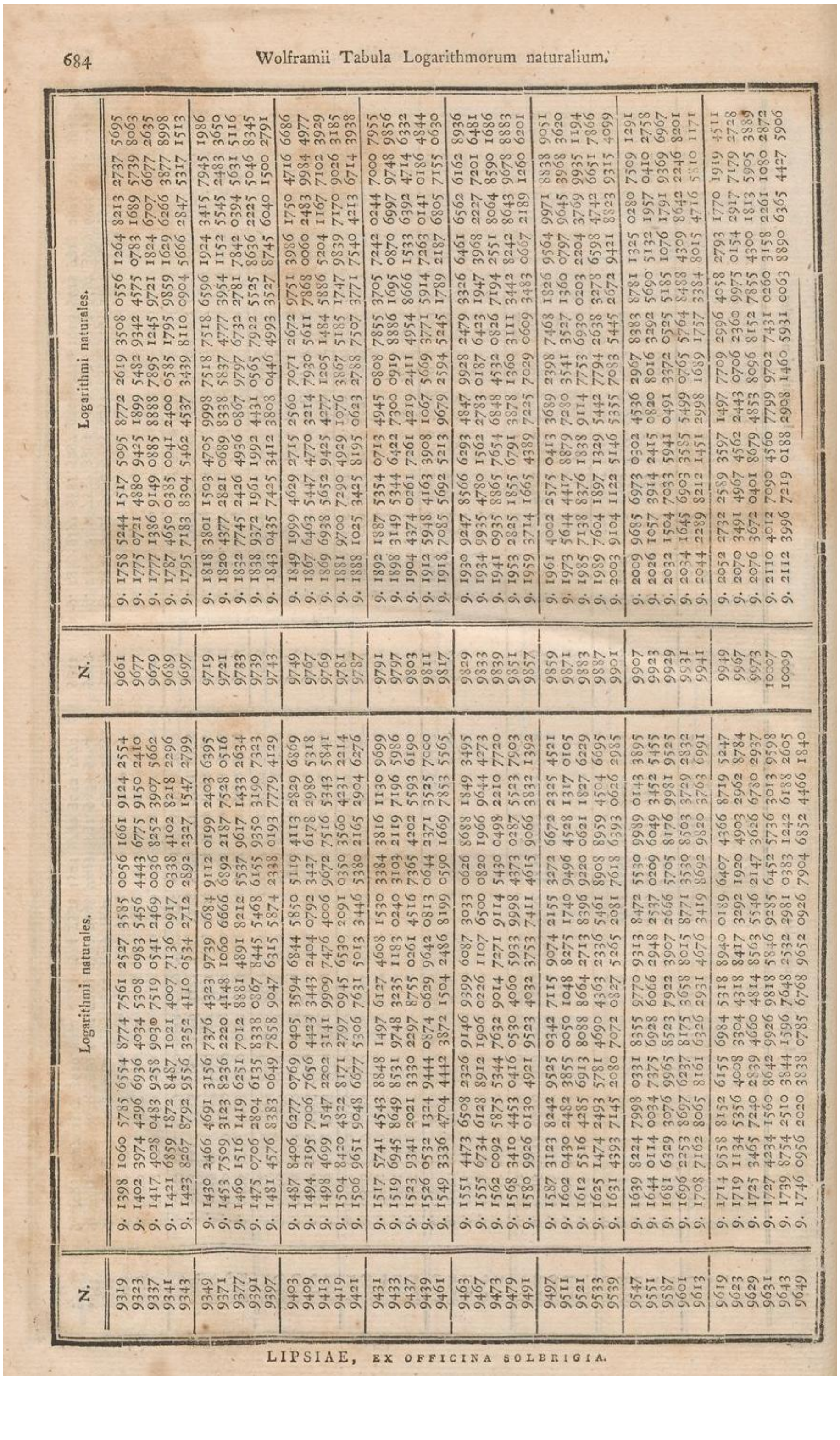

Figure 2: An excerpt of Wolfram's table as published by Vega [79].

2

x	$\log_e x$	x	$\log_e x$
0	$-\infty$	50	3.91202 30054 281461
1	0.00000 00000 000000	51	.93182 56327 243258
2	.69314 71805 599453	52	.95124 37185 814274
3	1.09861 22886 681097	53	.97029 19135 521218
4	.38629 43611 198906	54	.98898 40465 642744
5	1.60943 79124 341004	55	4.00733 31852 324709
6	.79175 94692 280550	56	.02535 16907 351492
7	.94591 01490 553133	57	.04305 12678 345502
8	2.07944 15416 798359	58	.06044 30105 464193
9	.19722 45773 362194	59	.07753 74439 057195
10	2.30258 50929 940457	60	4.09434 45622 221007
11	.39789 52727 983705	61	.11087 38641 733112
12	.48490 66497 880003	62	.12713 43850 450916
13	.56494 93574 615367	63	.14313 47263 915327
14	.63905 73296 152586	64	.15888 30833 596719
15	2.70805 02011 022101	65	4.17438 72698 956371
16	.77258 87222 397812	66	.18965 47420 264255
17	.83321 33440 562161	67	.20469 26193 909661
18	.89037 17578 961647	68	.21950 77051 761067
19	.94443 89791 664405	69	.23410 65045 972594
20	2.99573 22735 539910	70	4.24849 52420 493590
21	3.04452 24377 234230	71	.26267 98770 413154
22	.09104 24533 583159	72	.27666 61190 160553
23	.13549 42159 291497	73	.29045 94411 483911
24	.17805 38303 479456	74	.30406 50932 041698
25	3.21887 58248 682007	75	4.31748 81135 363104
26	.25809 65380 214820	76	.33073 33402 863311
27	.29583 68660 043291	77	.34380 54218 536838
28	.33220 45101 752039	78	.35670 88266 895917
29	.36729 58299 864740	79	.36944 78524 670215
30	3.40119 73816 621554	80	4.38202 66346 738816
31	.43398 72044 851462	81	.39444 91546 724388
32	.46573 59027 997265	82	.40671 92472 642531
33	.49650 75614 664802	83	.41884 06077 965979
34	.52636 05246 161614	84	.43081 67988 433136
35	3.55534 80614 894137	85	4.44265 12564 903165
36	.58351 89384 561100	86	.45434 72962 535077
37	.61091 79126 442244	87	.46590 81186 545837
38	.63758 61597 263858	88	.47733 68144 782065
39	.66356 16461 296464	89	.48863 63697 321398
40	3.68887 94541 139363	90	4.49980 96703 302651
41	.71357 20667 043078	91	.51085 95065 168500
42	.73766 96182 833683	92	.52178 85770 490403
43	.76120 01156 935624	93	.53259 94931 532559
44	.78418 96339 182612	94	.54329 47822 700039
45	3.80666 24897 703198	95	4.55387 68916 005408
46	.82864 13964 890950	96	.56434 81914 678362
47	.85014 76017 100586	97	.57471 09785 033828
48	.87120 10109 078909	98	.58496 74786 705719
49	.89182 02981 106266	99	.59511 98501 345899
50		100	

Figure 3: An excerpt of the Project's table of natural logarithms (volume 1) [43].

2

x	$\log_e x$	x	$\log_e x$
5.0000	1.60943 79124 341004	5.0050	1.61043 74127 671839
.0001	.60945 79122 341030	.0051	.61045 73925 875660
.0002	.60947 79116 341217	.0052	.61047 73720 087628
.0003	.60949 79106 341724	.0053	.61049 73510 307904
.0004	.60951 79092 342710	.0054	.61051 73296 536645
5.0005	1.60953 79074 344337	5.0055	1.61053 73078 774013
.0006	.60955 79052 346763	.0056	.61055 72857 020167
.0007	.60957 79026 350149	.0057	.61057 72631 275265
.0008	.60959 78996 354655	.0058	.61059 72401 539468
.0009	.60961 78962 360441	.0059	.61061 72167 812935
5.0010	1.60963 78924 367666	5.0060	1.61063 71930 095825
.0011	.60965 78882 376491	.0061	.61065 71688 388297
.0012	.60967 78836 387075	.0062	.61067 71442 690512
.0013	.60969 78786 399579	.0063	.61069 71193 002629
.0014	.60971 78732 414162	.0064	.61071 70939 324806
5.0015	1.60973 78674 430984	5.0065	1.61073 70681 657204
.0016	.60975 78612 450204	.0066	.61075 70419 999982
.0017	.60977 78546 471984	.0067	.61077 70154 353299
.0018	.60979 78476 496482	.0068	.61079 69884 717314
.0019	.60981 78402 523858	.0069	.61081 69611 092187
5.0020	1.60983 78324 554273	5.0070	1.61083 69333 478077
.0021	.60985 78242 587886	.0071	.61085 69051 875144
.0022	.60987 78156 624857	.0072	.61087 68766 283547
.0023	.60989 78066 665345	.0073	.61089 68476 703444
.0024	.60991 77972 709511	.0074	.61091 68183 134997
5.0025	1.60993 77874 757514	5.0075	1.61093 67885 578363
.0026	.60995 77772 809514	.0076	.61095 67584 033702
.0027	.60997 77666 865671	.0077	.61097 67278 501173
.0028	.60999 77556 926145	.0078	.61099 66968 980936
.0029	.61001 77442 991094	.0079	.61101 66655 473150
5.0030	1.61003 77325 060680	5.0080	1.61103 66337 977974
.0031	.61005 77203 135061	.0081	.61105 66016 495567
.0032	.61007 77077 214398	.0082	.61107 65691 026089
.0033	.61009 76947 298850	.0083	.61109 65361 569699
.0034	.61011 76813 388576	.0084	.61111 65028 126556
5.0035	1.61013 76675 483737	5.0085	1.61113 64690 696819
.0036	.61015 76533 584492	.0086	.61115 64349 280647
.0037	.61017 76387 691001	.0087	.61117 64003 878200
.0038	.61019 76237 803424	.0088	.61119 63654 489636
.0039	.61021 76083 921919	.0089	.61121 63301 115116
5.0040	1.61023 75926 046647	5.0090	1.61123 62943 754797
.0041	.61025 75764 177768	.0091	.61125 62582 408840
.0042	.61027 75598 315440	.0092	.61127 62217 077403
.0043	.61029 75428 459824	.0093	.61129 61847 760646
.0044	.61031 75254 611079	.0094	.61131 61474 458727
5.0045	1.61033 75076 769365	5.0095	1.61133 61097 171806
.0046	.61035 74894 934841	.0096	.61135 60715 900042
.0047	.61037 74709 107667	.0097	.61137 60330 643594
.0048	.61039 74519 288002	.0098	.61139 59941 402620
.0049	.61041 74325 476006	.0099	.61141 59548 177281
5.0050		5.0100	

Figure 4: An excerpt of the Project's table of natural logarithms (volume 4) [43].

MATHEMATICAL TABLES

The tables listed below (with the exception of MT15) were prepared by the Project for the Computation of Mathematical Tables conducted by the Federal Works Agency, Work Projects Administration for the city of New York, under the sponsorship of and made available through the National Bureau of Standards. They are of special interest to physicists, engineers, chemists, biologists, mathematicians, computers, and others engaged in scientific and technical work.

The tables have been arranged in the following groups: Those obtainable from : (1) the Superintendent of Documents, Government Printing Office, (2) Columbia University Press, and (3) those available elsewhere.

(1) TABLES OBTAINABLE FROM THE SUPERINTENDENT OF DOCUMENTS

MT1. Table of the first ten powers of the integers from 1 to 1,000.
MT2. Tables of the exponential function e^x. $3.00.
MT3. Tables of circular and hyperbolic sines and cosines for radian arguments. $2.50.
MT4. Tables of sines and cosines for radian arguments. $2.00.
MT5. Tables of sine, cosine, and exponential integrals, volume I. $2.75.
MT6. Tables of sine, cosine, and exponential integrals, volume II. $2.00.
MT7. Table of natural logarithms, volume I. $3.00.
MT8. Tables of probability functions, volume I. $2.00.
MT9. Table of natural logarithms, volume II. $3.00.
MT10. Table of natural logarithms, volume III. $3.00.
MT11. Tables of the moments of inertia and section moduli of ordinary angles, channels, and bulb angles with certain plate combinations. $2.00.
MT12. Table of natural logarithms, volume IV. $3.00.
MT13. Table of sine and cosine integrals for arguments from 10 to 100. $2.00.
MT14. Tables of probability functions, volume II. $2.25.
MT15. The hypergeometric and Legendre functions with applications to integral equations of potential theory. Chester Snow, National Bureau of Standards.
MT16. Table of arc tan x. $2.00.
MT17. Miscellaneous physical tables: Planck's radiation functions, and electronic function. $1.50.
MT18. Table of the zeros of the Legendre polynomials of order 1 — 16 and the weight coefficients for Gauss' mechanical quadrature formula. A. N. Lowan, N. Davids, and A. Levenson. 25c.
MT19. On the function $H(m,a,x) = \text{EXP}\,(-ix)\,F\,(m+1-ia, 2m+2; ix)$. With table of the confluent hypergeometric function and its first derivative. A. N. Lowan and W. Horenstein. 25c.
MT20. Table of integrals $\int_0^x J_0(t)dt$ and $\int_0^x Y_0(t)dt$. Arnold N. Lowan and Milton Abramowitz. 25c.
MT21. Table of $Ji_0(x) = \int_x^\infty \frac{J_0(t)}{t}dt$ and related functions. Arnold N. Lowan, G. Blanch, and M. Abramowitz. 25c.
MT22. Table of coefficients in numerical integration formulae. A. N. Lowan and Herbert Salzer.
MT23. Table of Fourier coefficients.....Arnold N. Lowan and Jack Laderman
Reprinted from Journal of Mathematics and Physics, September 1943. 11 p.
MT24. Coefficients for numerical differentiation with central differences. Herbert E. Salzer
Reprinted from Journal of Mathematics and Physics, September 1943. 21 p. 25c.
MT25. Seven-point Lagrantian integration formulas....G. Blanch and I. Rhodes
Reprinted from Journal of Mathematics and Physics, December 1943. 4 p. 25c.
MT26. A short table of the first five zeros of the transcendental equation $J_0(x)Y_0(kx) - J_0(kx)Y_0(x) = 0$..................A. N. Lowan and A. Hillman
Reprinted from Journal of Mathematics and Physics, December 1943. 2 p. 25c.

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Figure 5: The list of mathematical tables available from the National Bureau of Standards in 1948 (1/3) [54].

MT27. Table of coefficients for inverse interpolation with central differences.
Herbert E. Salzer

Reprinted from Journal of Mathematics and Physics, December 1943. 15 p. 25c.

MT28. Table of $f_n(x) = \frac{n!}{(x/2)^n} J_n(x)$............The Mathematical Tables Project

Reprinted from Journal of Mathematics and Physics, February 1944. 16 p. 25c.

MT29. Table of coefficients for inverse interpolation with advancing differences.
Herbert E. Salzer

Reprinted from Journal of Mathematics and Physics, May 1944. 28 p. 25c.

MT30. A new formula for inverse interpolation....................H. E. Salzer

Reprinted from Bulletin of the American Mathematical Society, August 1944. 4 p. 25c.

MT31. Coefficients for interpolation within a square grid in the complex plane.
A. N. Lowan and H. E. Salzer

Reprinted from Journal of Mathematics and Physics, August 1944. 11 p. 25c.

MT32. Table of coefficients for differences in terms of the derivatives..H. E. Salzer

Reprinted from Journal of Mathematics and Physics, November 1944. 4 p. 25c.

MT33. Table of coefficients for numerical integration without differences..
A. N. Lowan and H. E. Salzer

Reprinted from Journal of Mathematics and Physics, February 1945. 21 p. 25c.

MT34. Inverse intepolation for eight-, nine-, ten-, and eleven-point direct interpolation..H. E. Salzer

Reprinted from Journal of Mathematics and Physics, May 1945. 4 p. 25c.

MT35. Table of coefficients for double quadrature without differences, for integrating second order differential equations......................H. E. Salzer

Reprinted from Journal of Mathematics and Physics, November 1945. 6 p. 25c.

MT36. Formulas for direct and inverse interpolation of a complex function tabulated along equidistant circular arcs..........................H. E. Salzer

Reprinted from Journal of Mathematics and Physics, November 1945. 3 p. 25c.

Coordinate conversion tables.

Published as Technical Manual TM 4-238 of the War Department. March 25, 1943. 338 p., 5½ by 8½ in. 40c.

Hydraulic tables (2d ed.).

Published by the Corps of Engineers, War Department. (1944) 565 p. Blue imitation leather flexible cover, 4½ by 6¾ in. $1.50.

(2) TABLES OBTAINABLE FROM THE COLUMBIA UNIVERSITY PRESS

The following four tables can be obtained from the Columbia University Press, Morningside Heights, New York 27, N. Y.

Table of reciprocals of the integers from 100,000 through 200,009.

(1943) 201 p. Buckram cover. $4.00.

Table of Bessel functions $J_0(z)$ and $J_1(z)$ for complex arguments.

(1943) 403 p. Buckram cover. $5.00.

Table of circular and hyperbolic tangents and cotangents for radian arguments.

(1943) 410 p. Buckram cover. $5.00.

Tables of Lagrangian interpolation coefficients.

(1944) 392 p. Buckram cover. $5.00.

Table of arc sin x.

(1945) 121 p. Buckram cover. $3.50.

Tables of associated Legendre functions.

(1945) 302 p. Buckram cover. $5.00.

(3) TABLES AVAILABLE ELSEWHERE

The eight tables listed below can be consulted in libraries maintaining a file of mathematical and technical journals. No reprints of them are obtainable from the Bureau.

On the computation of second differences of the $Si(x)$, $Ei(x)$, and $Ci(x)$ functions.
Arnold N. Lowan

Bulletin of the American Mathematical Society, vol. 45, No. 8, pp. 583-588 (August 1939).

On the distribution of errors in the nth tabular differences.
Arnold N. Lowan and Jack Laderman

Annals of Statistics, vol. X, No. 4, pp. 360-364 (December 1939).

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Figure 6: The list of mathematical tables available from the National Bureau of Standards in 1948 (2/3) [54].

Note on the computation of the differences of the $Si(x)$, $Ci(x)$, $Ei(x)$ and $-Ei(-x)$ functions..Milton Abramowitz
Bulletin of the American Mathematical Society, vol. 46, No. 4, pp. 332-333 (April 1940).

Errors in Hayashi's table of Bessel functions for complex arguments.
Arnold N. Lowan and Gertrude Blanch
Bulletin of the American Mathematical Society, vol. 47, No. 4, pp. 291-293 (April 1941).

Tables of stellar functions for "point-source" models.
Published under the title "The Internal Temperature-Density Distribution of the Sun" in the Astrophysical Journal (Yerkes Observatory, Williams Bay, Wis.) vol. 94, pp. 37–45 (July 1941). By G. Blanch, A. N. Lowan, R. E. Marshak, and H. A. Bethe.

On the inversion of the q-series associated with Jacobian elliptic functions.
A. N. Lowan, G. Blanch, and W. Horenstein
Bulletin of the American Mathematical Society, vol. 48, No. 10, pp. 737-738 (October 1942).

A table of coefficients for numerical differentiation.
Arnold N. Lowan, Herbert E. Salzer, and Abraham Hillman
Bulletin of the American Mathematical Society, vol. 48, No. 12, pp. 920-924 (December 1942).

Roots of sin $z = z$..............................A. P. Hillman and H. E. Salzer
Gives the first 10 nonzero roots of sin $z = z$ in the first quadrant to six decimal places.
Roots of sin $z = z$, where $z = x + iy$. Philosophical Magazine, Series 7, vol. XXXIV, p. 575 (August 1943).

Figure 7: The list of mathematical tables available from the National Bureau of Standards in 1948 (3/3) [54].

BRITISH ASSOCIATION MATHEMATICAL TABLES

Volume I Circular and Hyperbolic Functions, Exponential, Sine and Cosine Integrals, Factorial Function and Allied Functions, Hermitian Probability Functions. First edition, 1931. Second edition, 1946. Third edition, 1951.

II Emden Functions, being Solutions of Emden's Equation together with Certain Associated Functions. 1932

III Minimum Decompositions into Fifth Powers.
Prepared by L. E. Dickson. 1933

IV Cycles of Reduced Ideals in Quadratic Fields.
Prepared by E. L. Ince. 1934. Reprinted 1966

V Factor Table, giving the Complete Decomposition of all Numbers less than 100,000.
Prepared independently by J. Peters, A. Lodge and E. J. Ternouth, E. Gifford. 1935

VI Bessel Functions. Part I, Functions of Orders Zero and Unity. 1937. Reprinted 1950, 1958

VII The Probability Integral.
Initiated and in part prepared by W. F. Sheppard. 1939. Reprinted 1966

VIII Number-divisor Tables.
Designed and in part prepared by J. W. L. Glaisher. 1940. Reprinted 1966

IX Table of Powers, giving Integral Powers of Integers.
Initiated by J. W. L. Glaisher. Extended by W. G. Bickley, C. E. Gwyther, J. C. P. Miller, E. J. Ternouth. 1940. Reprinted 1950

X Bessel Functions. Part II, Functions of Positive Integer Order 2 to 20.
Prepared by W. G. Bickley, L. J. Comrie, J. C. P. Miller, D. H. Sadler and A. J. Thompson. 1952. Reprinted 1960

PART-VOLUME A Legendre Polynomials.
Prepared by L. J. Comrie. 1946

B The Airy Integral, giving Tables of Solutions of the Differential Equation $y''=xy$
Prepared by J. C. P. Miller. 1946
(Auxiliary tables I and II are included with Part-Volume B.)

AUXILIARY TABLES

Number I Coefficients in the Modified Everett Interpolation Formula. 1946

II Table for Interpolation with Reduced Derivatives. Coefficients for Function and for First Derivative. 1946

Note. In July 1948 the Royal Society assumed responsibility for the work on mathematical tabulation formerly undertaken by the British Association.

Figure 8: The list of mathematical tables from the British association for the advancement of science (excerpt from the 1968 edition of volume 4).

References

The following list covers the most important references[7] related to the Mathematical Tables Project's table. Not all items of this list are mentioned in the text, and the sources which have not been seen are marked so. We have added notes about the contents of the articles in certain cases.

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[6] Raymond Clare Archibald. Errors found by Duarte in Wolfram's table of natural logarithms. *Scripta Mathematica*, 4:293, 1936.

[7] Raymond Clare Archibald. Kulik's table of natural logarithms. *Scripta Mathematica*, 4:340, 1936. [this query is repeated in *Mitteilungen zur Geschichte der Medizin der Naturwissenschaften und der Technik*, volume 37, 1938, page 89]

[8] Raymond Clare Archibald. Wolfram's table. *Scripta Mathematica*, 4(1):99–100, 1936.

[9] Raymond Clare Archibald. The New York Mathematics Tables Project. *Science*, 96 (New Series)(2491):294–296, 1942.

[10] Raymond Clare Archibald. A volume of tables by Kulik. *Mathematical Tables and other Aids to Computation*, 2(13):59–60, 1946.

[11] Raymond Clare Archibald. *Mathematical table makers. Portraits, paintings, busts, monument. Bio-bibliographical notes.* New York: Scripta Mathematica, 1948. [contains a photograph of Lowan]

[7] **Note on the titles of the works:** Original titles come with many idiosyncrasies and features (line splitting, size, fonts, etc.) which can often not be reproduced in a list of references. It has therefore seemed pointless to capitalize works according to conventions which not only have no relation with the original work, but also do not restore the title entirely. In the following list of references, most title words (except in German) will therefore be left uncapitalized. The names of the authors have also been homogenized and initials expanded, as much as possible.

The reader should keep in mind that this list is not meant as a facsimile of the original works. The original style information could no doubt have been added as a note, but we have not done it here.

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Note

The four volumes of reconstructed tables are given in four separate documents.