

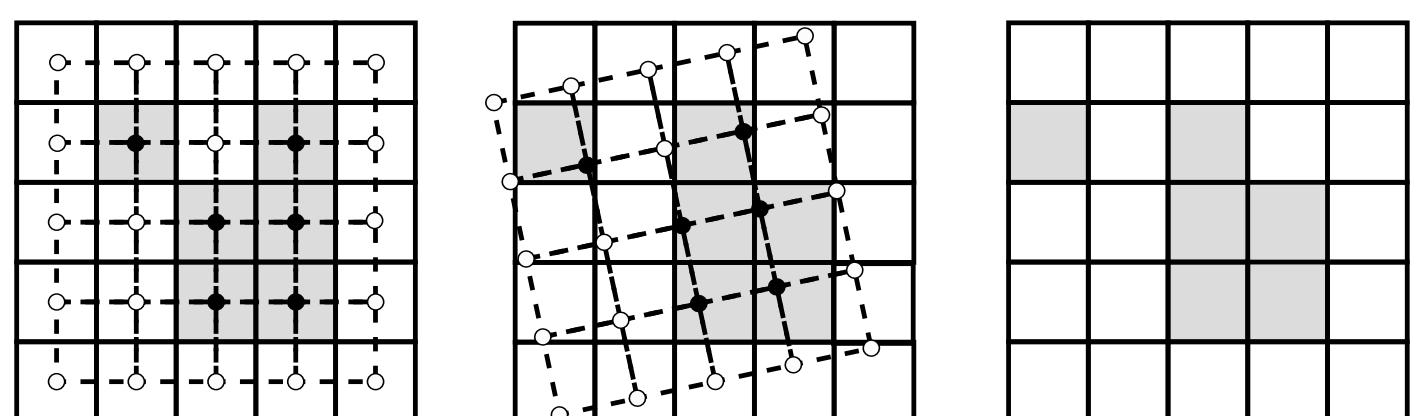
1. MOTIVATION

Rigid motions on \mathbb{R}^2 are defined by a mapping:

$$\begin{cases} \mathfrak{T} : \mathbb{R}^2 \rightarrow \mathbb{R}^2 \\ x \mapsto Rx + t \end{cases}$$

where R is rotation matrix and $t \in \mathbb{R}^2$ is translation vector.

Digital rigid motion is obtained by $\mathcal{T}_{point} = \mathcal{D} \circ \mathfrak{T}|_{\mathbb{Z}^2} \subset \mathbb{Z}^2$ where \mathcal{D} is a digitization operator. Topology and geometry are not preserved by \mathcal{T}_{point} due to operator \mathcal{D} .



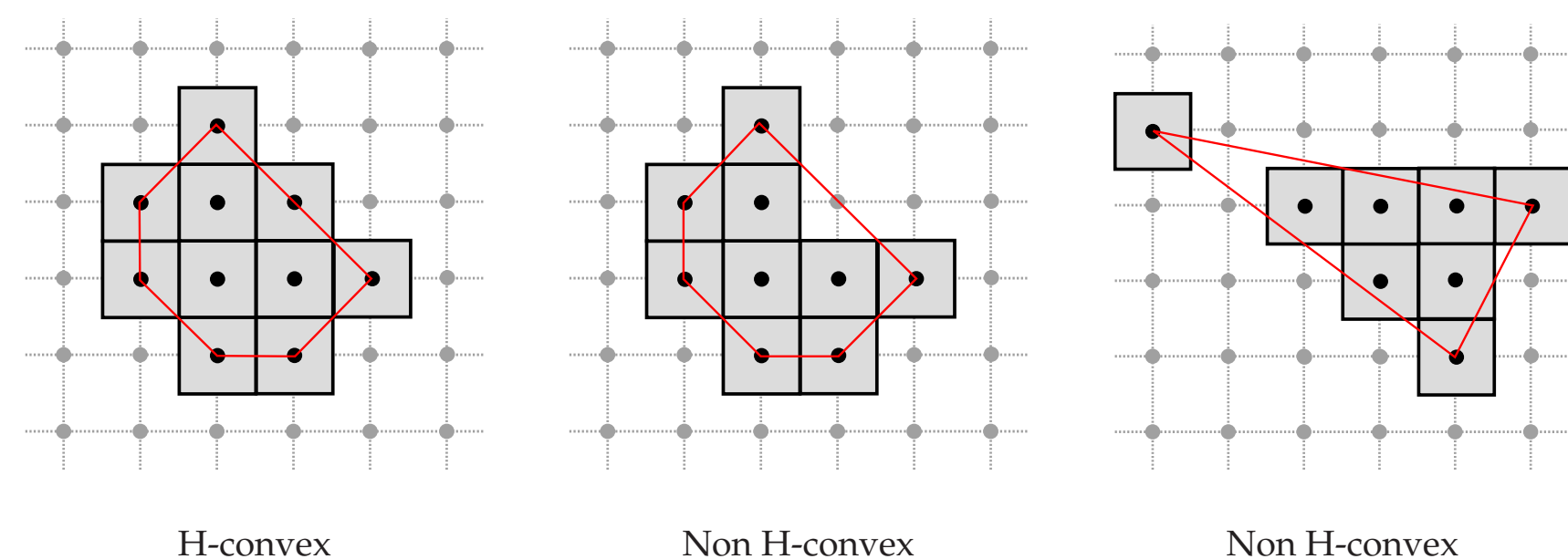
Objectives and key results:

- Propose a method for rigid motions of convex digital objects which preserves the connectivity and convexity
 - Using the **half-plane representation**
- Characterize digital objects that guarantee connectivity and convexity preservation under rigid motions
 - Using the notion of **quasi-r-regularity**
- Extend the proposed method to non-convex objects
 - Using the **concavity tree representation**

2. H-CONVEX DIGITAL OBJECT

Definition 1 ([1]) A digital (connected) object X is H-convex if $X = Conv(X) \cap \mathbb{Z}^2$, where $Conv(X)$ is the convex hull of X .

Property 2 ([2]) Let X and Y be two digital objects. If X and Y are H-convex and $X \cap Y$ is connected, then $X \cap Y$ is H-convex.



Let X be an H-convex digital object and $Conv(X)$ be its convex hull, then

$$Conv(X) = \bigcap_{H \in \mathcal{R}(Conv(X))} H,$$

where $\mathcal{R}(P)$ is the minimal set of closed half-planes constituting a convex polygon P and H is a closed half-plane of the form

$$H = \{(x, y) \in \mathbb{R}^2 \mid ax + by + c \leq 0, a, b, c \in \mathbb{Z}, \gcd(a, b) = 1\}.$$

Property 3 Let $X \subset \mathbb{Z}^2$ be an H-convex digital object, X can be represented as

$$X = \left(\bigcap_{H \in \mathcal{R}(Conv(X))} H \right) \cap \mathbb{Z}^2 = \bigcap_{H \in \mathcal{R}(Conv(X))} (H \cap \mathbb{Z}^2)$$

where each $H \cap \mathbb{Z}^2$ is called a digital half-plane.

3. RIGID MOTION OF H-CONVEX OBJECTS

Rational rigid motion on a digital half-plane is defined as

$$\mathcal{T}_{conv}(H \cap \mathbb{Z}^2) = \mathfrak{T}(H) \cap \mathbb{Z}^2,$$

where \mathfrak{T} is rational rigid motion with R and t have rational elements, and $\mathfrak{T}(H)$ is the transformed half-plane

$$\mathfrak{T}(H) = \{(x, y) \in \mathbb{R}^2 \mid \alpha x + \beta y + \gamma \leq 0, \alpha, \beta, \gamma \in \mathbb{Q}\}.$$

Definition 4 Rational rigid motion on an H-convex digital object X is defined as

$$\mathcal{T}_{conv}(X) = \mathfrak{T}(P) \cap \mathbb{Z}^2 = P' \cap \mathbb{Z}^2,$$

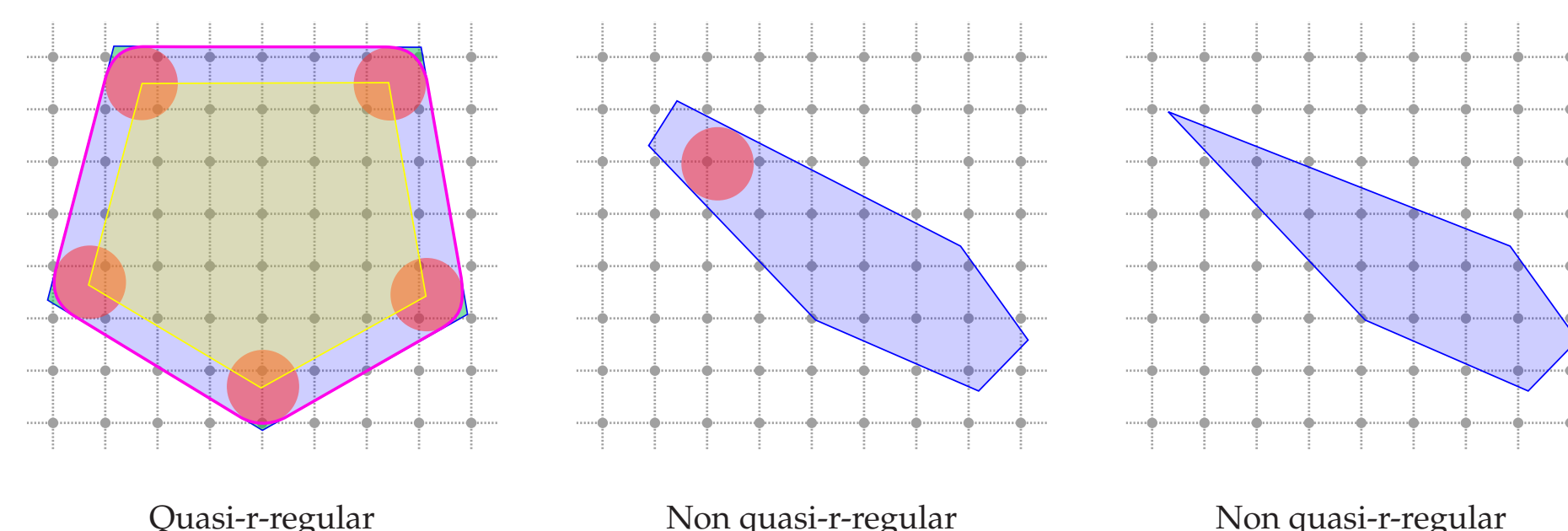
where $P = \left(\bigcap_{H \in \mathcal{R}(Conv(X))} H \right)$ is the convex polygon of X and $P' = \left(\bigcap_{H \in \mathcal{R}(Conv(X))} \mathfrak{T}(H) \right)$ is the transformed convex polygon of P by \mathfrak{T} .

Property 5 Let X be an H-convex digital object. Let consider the two convex polygons $P = \left(\bigcap_{H \in \mathcal{R}(Conv(X))} \mathfrak{T}(H) \right)$ and $P' = Conv(\mathcal{T}_{conv}(X))$. Then, $P' \subseteq P$.

4. QUASI- r -REGULAR CONVEX POLYGON

Definition 6 A convex polygon P is quasi- r -regular if

- $\forall H \in \mathcal{R}(P), \exists B_r \subset P$ such that H is tangent to B_r , and
- $\forall v \in V(P), \theta(v) \geq \frac{\pi}{2}$ where $V(P)$ is the set of vertices of P and $\theta(v)$ is the angle at v .



Property 7 Let P be a convex polygon. If $P \cap \mathbb{Z}^2 \neq \emptyset$ and P is quasi- $(\frac{\sqrt{2}}{2})$ -regular, then $P \cap \mathbb{Z}^2$ is 4-connected.

The H-convexity is preserved under rigid motions if the convex hull of the considered H-convex digital object is quasi- $(\frac{\sqrt{2}}{2})$ -regular.

Proposition 8 Let X be an H-convex digital object. If $Conv(X)$ is quasi- $(\frac{\sqrt{2}}{2})$ -regular, then $\mathcal{T}_{conv}(X)$ is 4-connected and thus H-convex.

REFERENCES

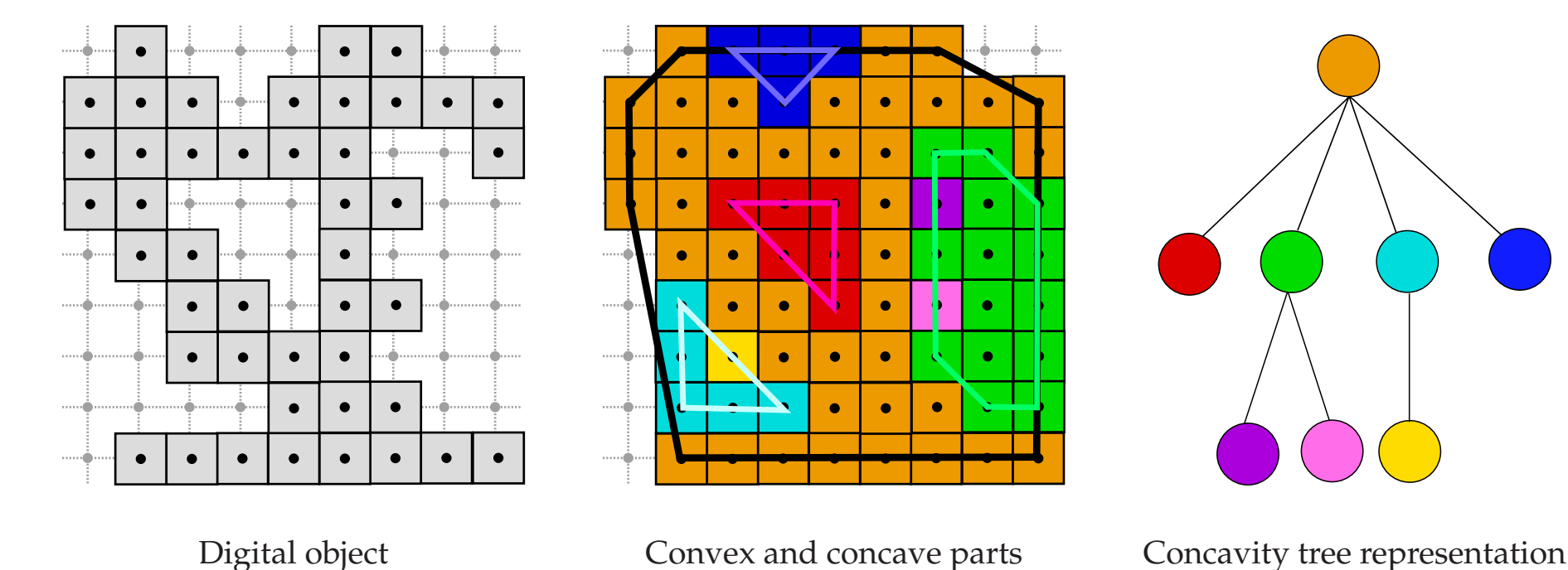
- C. E. Kim. On the cellular convexity of complexes. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 3(6):617–625, 1981.
- G. Cristescu and L. Lupsa. *Non-Connected Convexities and Applications*. Dordrecht: Kluwer Academic Publishers, 2002.
- J. Sklansky. Measuring concavity on a rectangular mosaic. *IEEE Transactions on Computers*, C-21(12):1355–1364, 1972.
- L. J. Latecki, C. Conrad, and A. Gross. Preserving topology by a digitization process. *Journal of Mathematical Imaging and Vision*, 8(2):131–159, 1998.

4. RIGID MOTION OF NON-CONVEX OBJECTS

A non-convex object X can be decomposed into a set of digitized convex hulls of hierarchical concavity parts.

$$X = \left(Conv(X) \cap \mathbb{Z}^2 \right) \setminus \left(\bigcup_{X' \in \mathcal{C}((Conv(X) \cap \mathbb{Z}^2) \setminus X)} X' \right)$$

This decomposition can be represented by a concavity tree [3] in which the root corresponds to X , and each node corresponds to a concavity part of its parent.



Rigid motion of non-convex digital object X is defined by the recursive process as

$$\mathcal{T}_{conv}(X) = \mathcal{T}_{conv} \left(Conv(X) \cap \mathbb{Z}^2 \right) \setminus \left(\bigcup_{X' \in \mathcal{C}((Conv(X) \cap \mathbb{Z}^2) \setminus X)} \mathcal{T}_{conv}(X') \right)$$

5. EXPERIMENTAL RESULTS

	Original digital object	Point-by-point rigid motion	Convex-preserving rigid motion
Convex objects			
Non-convex objects			

6. CONCLUSION & PERSPECTIVES

- Method for convex-preserving rigid motions of digital objects.
- Condition of quasi- r -regularity for guaranteeing convexity and topology during rigid motions of convex digital objects.
- Extension to non-convex digital objects using concavity tree.
- Strategy to guarantee the quasi- r -regularity of digital objects.
- Extension to higher dimensions.