Design of Limb for Parallel Mechanism Based on Screw Theory

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Abstract. Based on the reciprocal relationship of twist and wrench in screw theory, the mathematical model for limb of parallel manipulator is established in this paper. According to the motion modes of mobile platform (translation or rotation), we concluded the geometric conditions which the prismatic joint or revolute joint must meet with by analyzing the constraint screw on the platform, which provides the background for development of parallel mechanism.

Keywords: parallel mechanism, screw theory, limb, geometric conditions

1. Introduction

In recent year, since parallel mechanism can offer higher stiffness and larger load capability than those of serial mechanism, it has become a hot research topic in international robotics area. However, it is very difficult to design because of the complexity of kinematics and dynamics, the diversity of limb and the coupling of architecture.

It is the most important task to meet with the DOF of the required motion for designing the parallel mechanism. In fact, DOF is the outward feature. The key is the design of constraint to implement the DOF of the required motion. The DOF of motion is objective in the limb. However, the constraint is designed in the limb by designer. And there are strict requirements to the geometric conditions which the prismatic joint or revolute joint must meet with in the limb.

Based on the reciprocal relationship, in this paper, we concluded the geometric conditions which the prismatic joint or revolute joint must meet with in the limb by analyzing the constraint screw on the platform. According to the limb, we can design the parallel mechanism which is satisfied the required movement. It is a common method to[1] the basic design of the parallel mechanism.

2 Structural synthesis of the constraint screw on the platform

Each limb should provide a constraint screw for the moving platform in the parallel mechanism. According to the constraint characteristics provided to the platform, screw can be divided into twist and wrench. An arbitrary motion screw in space can include six motions at most, three translations along the X, Y, Z axes and three rotations around the X, Y, Z axes.

Twist should provide constraint for movement in space. It can be defined as $_r = (s_r; r_r \times s_r)$. In the formula, s_r stands for a unit vector along axis of twist and r_r for a point on the axis direction of twist. Wrench should provide constraint for rotation in space. It can be defined as $_r = (0 \quad 0 \quad 0; l_r \quad m_r \quad n_r)$ and here we have $l_r^2 + m_r^2 + n_r^2 = 1$.

Twist and wrench are decided by the structural conditions of limb in parallel mechanism. So the type of constraint depends on the structural conditions of limb. According to the type of constraint, we can obtain the characteristic of limb structural. According to the difference of constraint in the limb, they can be divided into unconstrained limb　single constrained limb　double constrained limb　three-constrained limb　four-constrained limb　five-constrained limb and six-constrained limb. However, the five-constrained limb and six-constrained limb belong to planar limb which have no requirement in the space.

This paper analyzes the limb types including: the limb providing only one twist　the limb with two twists　the limb with three twists; the limb with only one wrench　the limb with two wrenches　the limb with three wrenches; the limb with one twist and one wrench　the limb with two twists and one wrench　the limb with one twist and two wrenches.

2.1 The limb with only one twist

The basic expression of screw is $_r = (s_r; r_r \times s_r)$. It is known from the reciprocal of screw that the limb should consist of the five independent screws which are reciprocal with the twist

When the joint is revolute $= (s; r \times s)$

$$_r \circ \quad = s_r \bullet (r \times s) + s \bullet (r_r \times s_r) = s_r \bullet [(r - r_r) \times s] = 0$$

According to the geometric feature of vector, it should be known that there is a common perpendicular among s_r　$r - r_r$ and S, that is to say, the axis direction of the revolute joint and the axis direction of twist must be in the same plane.

When the joint is prismatic $= (0; s)$

$$_r \circ \quad = s_r \bullet s = 0$$

According to the geometric feature of vector, it should be known that s_r and s must be perpendicular each other, that is to say, the moving direction of the prismatic joint and the axis direction of twist must be perpendicular each other.

When the structural conditions of limb meet with the above requirements, the limb should provide only one twist. According to the requirements above, we should put up the limb-RRPRR shown as **Fig 1**. First, we construct two parallel revolute joints, which make sure the direction of twist. Second, we establish the moving direction of the prismatic joint and the axis direction of twist must be vertical each other. Last, the axis direction of the last two revolute joints must intersect at o point which is the point of action of s_r.
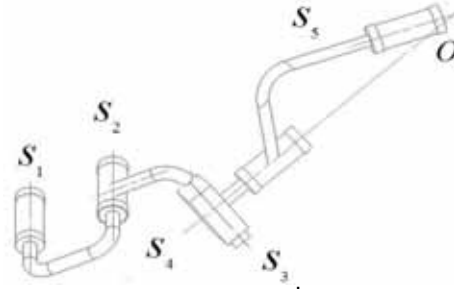

Fig. 1. Structur of Limb RRPRR-1F ($s_r // s_1 // s_2 \perp s_3$, s_4 and s_5 intersect at o, s_r acting on o)

2.2 The limb with two twists

The basic expression of screw is
$$\begin{cases} {}_{r1} = (s_{r1}; r_{r1} \times s_{r1}) \\ {}_{r2} = (s_{r2}; r_{r2} \times s_{r2}) \end{cases}$$
. It is known from the reciprocal of screw that the limb should consist of the four independent screws which are reciprocal with the two twists.

When the joint is revolute $= (s; r \times s)$

$$\begin{cases} {}_{r1} \circ = s_{r1} \bullet (r \times s) + s \bullet (r_{r1} \times s_{r1}) = s_{r1} \bullet [(r - r_{r1}) \times s] = 0 \\ {}_{r2} \circ = s_{r2} \bullet (r \times s) + s \bullet (r_{r2} \times s_{r2}) = s_{r2} \bullet [(r - r_{r2}) \times s] = 0 \end{cases}$$

According to the geometric feature of vector, it is known that there is a common perpendicular among s_{r1} $r - r_{r1}$ and S; a common perpendicular among s_{r2} $r - r_{r2}$ and S, that is to say, the axis direction of the revolute joint must pass the point which is the point of intersection with the two twists or be parallel the plane which makes sure by the two twists.

When the joint is prismatic $= (0; s)$

$$\begin{cases} {}_{r1} \circ = s_{r1} \bullet s = 0 \\ {}_{r2} \circ = s_{r2} \bullet s = 0 \end{cases}$$

According to the geometric feature of vector, it should be known that s_{r1} and s must be vertical each other; s_{r2} and s must be vertical each other, that is to say, the moving direction of the prismatic joint must be parallel the cross-produce of the two twists. And there is the only prismatic joint in the limb.

When the structural conditions of limb meet with the above requirements, the limb should provide two twists. According to the requirements above, we should put up the limb-RRPR shown as **Fig 2**.
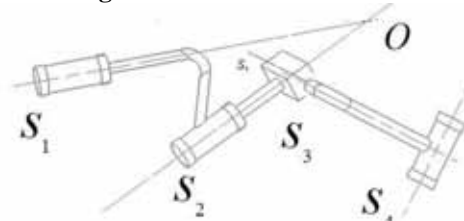

Fig. 2. Structur of Limb RRPR-2F ($s_1 \perp s_{r1} \times s_{r2} s_2 \perp s_{r1} \times s_{r2}$, o is the point of action with s_{r1} and s_{r2})

2.3 The limb with three twists

The basic expression of screw is

$$\begin{cases} _{r1} = (s_{r1}; r_{r1} \times s_{r1}) \\ _{r2} = (s_{r2}; r_{r2} \times s_{r2}) \\ _{r3} = (s_{r3}; r_{r3} \times s_{r3}) \end{cases}$$

The limb should provide three independence twists which intersected at a point for platform. The limb constrained the three directions moving of the platform, that is to say, the platform just can be round the point to revolve. The limb should consist of three revolute joints, and that, there is only one type of the limb-RRR. According to the requirements above, we should put up the limb-RRR shown as **Fig. 3**.
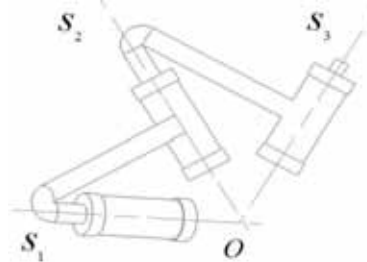

Fig. 3. Structur of Limb RRR-3F (s_1, s_2, and s_3 intersect at o)

2.4 The limb with only one wrench

The basic expression of screw is $_r = (0; s_r)$. It is known from the reciprocal of screw that the limb should consist of the five independent screws which are reciprocal with the wrench.

When the joint is revolute $= (s; r \times s)$

$$_r \circ \quad = s \bullet s_r = 0$$

According to the geometric feature of vector, it should be known that s_r and s must be vertical each other, that is to say, the axis direction of the revolute joint and the axis direction of wrench must be vertical each other.

When the joint is prismatic $= (0; s)$

$$_r \circ \quad = 0$$

The equation is satisfied under any conditions, that is to say, the moving direction of the prismatic joint is independent of the axis direction of wrench.

When the structural conditions of limb meet with the requirements above, the limb should provide only one wrench. According to the requirements above, we should put up the limb-RRPRP show as **Fig. 4**.
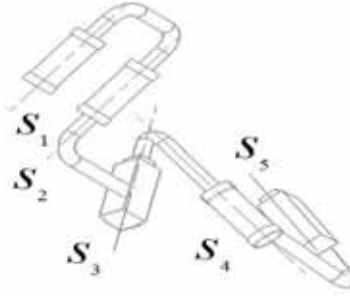

Fig. 4. Structur of Limb RRPRP-1M $s_1//s_2, s_r//s_2 \times s_4$

2.5 The limb with two wrenches

The basic expression of screw is
$$\begin{cases} _{r1} = (0; s_{r1}) \\ _{r2} = (0; s_{r2}) \end{cases}$$
. It is known from the reciprocal of screw that the limb should consist of the four independent screws which are reciprocal with the two wrenches.

When the joint is revolute $= (s; r \times s)$

$$\begin{cases} _{r1} \circ \quad = s \bullet s_{r1} = 0 \\ _{r2} \circ \quad = s \bullet s_{r2} = 0 \end{cases}$$

According to the geometric feature of vector, it should be known that s_{r1} and s must be vertical each other; s_{r2} and s must be vertical each other, that is to say, the axis direction of the revolute joint must be parallel the cross-produce of the two wrenches.

When the joint is prismatic $\quad = (0; s)$

$$\begin{cases} {}_{r1} \circ \quad = 0 \\ {}_{r2} \circ \quad = 0 \end{cases}$$

The equation is satisfied under any conditions, that is to say, the moving direction of the prismatic joint is independent of the axis direction of the wrenches.

When the structural conditions of limb meet with the requirements above, the limb should provide only one wrench. According to the requirements above, we should put up the limb-RRPR shown as **Fig. 5**.
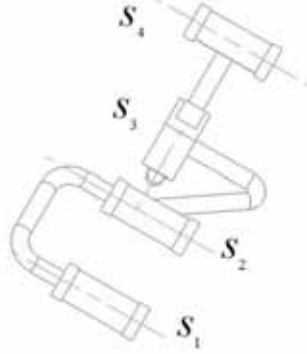

Fig. 5. Structur of Limb RRPR-2M $(s_{r1} \times s_{r2} // s_1 // s_2 // s_3)$

2.6 The limb with three wrenches

The basic expression of screw is

$$\begin{cases} {}_{r1} = (0; s_{r1}) \\ {}_{r2} = (0; s_{r2}) \\ {}_{r3} = (0; s_{r3}) \end{cases}$$

The limb should provide three independence wrenches which are independent each other. The limb constrained the three directions rotation of the platform, that is to say, the platform just can move along the X, Y, Z axis. The limb should consist of three independent prismatic joints, and that, there is only one type of the limb-PPP shown as **Fig. 6**.
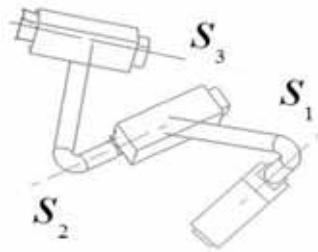
Fig. 6. Structur of Limb PPP-3M (s_1, s_2 and s_3 are independent each other)

2.7 The limb with one twist and one wrench

The basic expression of screw is
$$\begin{cases} {}_{r1} = (s_{r1}; r_{r1} \times s_{r1}) \\ {}_{r2} = (0; s_{r2}) \end{cases}.$$
It is known from the reciprocal of screw that the limb should consist of the four independent screws which are reciprocal with the one twist and one wrench.

When the joint is revolute $= (s; r \times s)$
$$\begin{cases} {}_{r1} \circ = s_{r1} \bullet (r \times s) + s \bullet (r_{r1} \times s_{r1}) = s_{r1} \bullet [(r - r_{r1}) \times s] = 0 \\ {}_{r2} \circ = s \bullet s_{r2} = 0 \end{cases}$$

According to the geometric feature of vector, it should be known that there is a common perpendicular among s_{r1} $r - r_{r1}$ and S; s_{r2} and S must be vertical each other that is to say, the axis direction of the revolute joint must be located in the normal plane, which contained the twist, of the wrench.

When the joint is prismatic $= (0; s)$
$$\begin{cases} {}_{r1} \circ = s \bullet s_{r1} = 0 \\ {}_{r2} \circ = 0 \end{cases}$$

According to the geometric feature of vector, it should be known that s_{r1} and S must be vertical each other, that is to say, the moving direction of the prismatic joint and the axis direction of twist must be vertical each other.

When the structural conditions of limb meet with the above requirements, the limb should provide one twist and one wrench. According to the requirements above, we should put up the limb-RPRR shown as **Fig. 7**.

Fig. 7. Structur of Limb RPRR-1F1M($s_2 \perp s_{r1}, s_1 \perp s_{r2}$, $s_3 \perp s_{r2}, s_4 \perp s_{r2}, s_1, s_3, s_4$ and s_{r2} are in the same plane

2.8 The limb with two twists and one wrench

The basic expression of screw is

$$\begin{cases} _{r1} = (s_{r1}; r_{r1} \times s_{r1}) \\ _{r2} = (s_{r2}; r_{r2} \times s_{r2}) \\ _{r3} = (0; s_{r3}) \end{cases}$$

It is known from the reciprocal of screw that the limb should consist of the three independent screws which are reciprocal with the screws.

When the joint is revolute $= (s; r \times s)$

$$\begin{cases} _{r1}{}^\circ = s_{r1} \bullet (r \times s) + s \bullet (r_{r1} \times s_{r1}) = s_{r1} \bullet [(r - r_{r1}) \times s] = 0 \\ _{r2}{}^\circ = s_{r2} \bullet (r \times s) + s \bullet (r_{r2} \times s_{r2}) = s_{r2} \bullet [(r - r_{r2}) \times s] = 0 \\ _{r3}{}^\circ = s \bullet s_{r3} = 0 \end{cases}$$

According to the geometric feature of vector, it should be known that there is a common perpendicular among s_{r1} $r - r_{r1}$ and S; a common perpendicular among s_{r2} $r - r_{r2}$ and S; s_{r3} and S must be vertical each other, that is to say, the axis direction of revolute joint, which must be vertical the axis direction of the wrench, must pass the point which is the point of intersection with the two twists or be parallel the plane which make sure by the two twists.

When the joint is prismatic $= (0; s)$

$$\begin{cases} _{r1}{}^\circ = s \bullet s_{r1} = 0 \\ _{r2}{}^\circ = s \bullet s_{r2} = 0 \\ _{r3}{}^\circ = 0 \end{cases}$$

According to the geometric feature of vector, it should be known that s_{r1} and S must be vertical each other; s_{r2} and S must be vertical each other; s_{r3} is independent of S, that is to say, the axis moving direction of the prismatic joint,

which is independent of the axis direction of the wrench, must be parallel the cross-produce of the two twists.

When the structural conditions of limb meet with the above requirements, this limb should provide two twists and one wrench. According to the requirements above, we should put up the limb-RRP shown as **Fig. 8**.
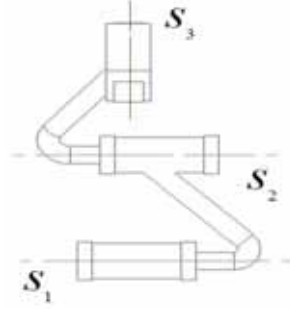

Fig. 8. Structur of Limb RRP-2F1M $s_1 \perp s_{r3}, s_2 \perp s_{r3}, s_3 \perp s_{r1}, s_3 \perp s_{r2}, s_1 \perp s_{r1} \times s_{r2},$
$s_2 \perp s_{r1} \times s_{r2})$

2.9 The limb with one twist and two wrenches

The basic expression of screw is

$$\begin{cases} {}_{r1} = (s_{r1}; r \times s_{r1}) \\ {}_{r2} = (0; s_{r2}) \\ {}_{r3} = (0; s_{r3}) \end{cases}$$

It is known from the reciprocal of screw that the limb should consist of the three independent screws which are reciprocal with the screws.

When the joint is revolute $= (s; r \times s)$

$$\begin{cases} {}_{r1}{}^{\circ} = s_{r1} \bullet (r \times s) + s \bullet (r_{r1} \times s_{r1}) = s_{r1} \bullet [(r - r_{r1}) \times s] = 0 \\ {}_{r2}{}^{\circ} = s \bullet s_{r2} = 0 \\ {}_{r3}{}^{\circ} = s \bullet s_{r3} = 0 \end{cases}$$

According to the geometric feature of vector, it should be known that there is a common perpendicular among s_{r1} $r - r_{r1}$ and S; s_{r2} and S must be vertical each other; s_{r3} and S must be vertical each other, that is to say, the axis direction of revolute joint, which is in the same plane with the twist, must be parallel the cross-produce of the two wrenches.

When the joint is prismatic $= (0; s)$

$$\begin{cases} {}_{r1}{}^{\circ} = s \bullet s_{r1} = 0 \\ {}_{r2}{}^{\circ} = 0 \end{cases}$$

$$_{r3} \circ \quad = 0$$

According to the geometric feature of vector, it should be known that s_{r1} and s must be vertical each other, that is to say, the axis moving direction of the prismatic joint must be vertical the axis direction of the twist.

When the structural conditions of limb meet with the above requirements, the limb should provide one twist and two wrenches. According to the requirements above, we should put up the limb-RRP shown as **Fig. 9**.
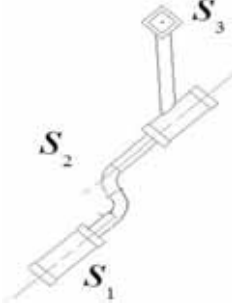

Fig. 9. Structur of Limb RRP-1F2M (s_1// s_2//s_{r2}×s_{r3}, s_3⊥s_{r1} s_1, s_2 and s_{r1} are in the same plane)

3. Example of the application

Since it is regular for the geometric conditions of the joint in the limb, which connects fixed platform with mobile platform, the limb, which meets with the geometric conditions, can be used to establish the parallel mechanism which be satisfied with the mode of motion. Now take the limb **Fig. 7** as an example to establish the 3-2T1R parallel mechanism as shown in **Fig. 10**. The limb in **Fig.7** provides one twist along the Z-axis, which constrain the moving along the Z-axis, and one wrench along the X-axis, which constrain the rotation around the X-axis. Thanks to the wrenches which provided by the three limbs are dependent in the XY plane, it should constrain the rotation around the X-axis and Y-axis   that is to say   the platform just can revolve around Z-axis and move in the XY plane.
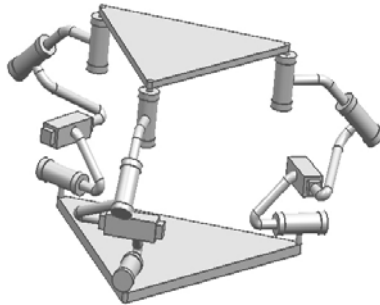
Fig. 10. 3-2T1R parallel mechanism

4. Conclusions

According to analyze the type of limb and the type of restraint in screw theory, it is given a general design method of limb in the parallel mechanism. And that, it obtained the geometric conditions which the prismatic joint or revolute joint of over-constrained parallel mechanism must be meet with. The method analyzed from basic concept of the reciprocal produce in screw theory. It should make sure be general and pragmatic. It is a common reference value to the basic design of the parallel mechanism.

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