



**HAL**  
open science

# Comparison of damage localization in mechanical systems based on Stochastic Subspace Identification method

Guillaume Gautier, Md Delwar Hossain Bhuyan, Michael Döhler, Laurent Mevel

► **To cite this version:**

Guillaume Gautier, Md Delwar Hossain Bhuyan, Michael Döhler, Laurent Mevel. Comparison of damage localization in mechanical systems based on Stochastic Subspace Identification method. EGU General Assembly, Apr 2017, Vienna, Austria. 2017. hal-01545346

**HAL Id: hal-01545346**

**<https://inria.hal.science/hal-01545346>**

Submitted on 22 Jun 2017

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Comparison of damage localization in mechanical systems based on Stochastic Subspace Identification method

Guillaume Gautier<sup>1,2</sup> & Md Delwar Hossain Bhuyan<sup>2</sup> & Michael Döhler<sup>2</sup> & Laurent Mevel<sup>2</sup>

<sup>1</sup> CEA-Tech Pays-de-la-Loire, Technocampus Océan, 5 rue de l'Halbrane, 44340 Bouguenais, France (e-mail: guillaume.gautier@ifsttar.fr),  
<sup>2</sup> Inria/IFSTTAR, I4S, Campus de Beaulieu, 35042 Rennes, France (e-mails: {md-delwar-hossain.bhuyan, michael.doebler, laurent.mével}@inria.fr).

## Context

- Vibration-based damage localization on structures relates to the monitoring of the changes in the modal parameters
- Based on finite Element model of the structure and output-only measurement data in the reference and damaged states
- Comparison of Stochastic Dynamic Damage Locating Vector (SDDLTV) approach and Subspace Fitting (SF) method, for damage localization

## Covariance computation

- System matrices are subject to uncertainties due to unknown excitation, noise and finite data length
- Estimation from subspace identification, based on covariance-driven Hankel matrix  $\mathcal{H}$
- Let  $f$  be a function of  $\mathcal{H}$ , then  $\text{cov}(f(\mathcal{H})) \approx \mathcal{J}_{f,\mathcal{H}} \hat{\Sigma}_{\mathcal{H}} \mathcal{J}_{f,\mathcal{H}}^T$  with sensitivity  $\mathcal{J}_{f,\mathcal{H}} = \partial f(\mathcal{H}) / \partial \text{vec}(\mathcal{H})$

## SDDLTV

$$\text{Transfer matrix} \begin{cases} G(s) = R(s)D_c \\ R(s) = C_c(sI - A_c)^{-1} \begin{bmatrix} C_c A_c \\ C_c \end{bmatrix}^+ \begin{bmatrix} I \\ 0 \end{bmatrix} \end{cases}$$

### Damage localization procedure

- From data: changes in the transfer matrix between both reference and damaged state:  $\partial R(s)^T = \hat{R}(s)^T - R(s)^T$  load vector  $v(s)$  in the null space of  $\partial R(s)^T$
- From FE model: apply load vector  $v(s)$  to model  
Stress computation:  $\hat{S}(s) = \mathcal{L}_{\text{model}}(s)v(s)$
- Damaged element indicated by stress = 0 (or close to 0)

### Uncertainty propagation and covariance computation

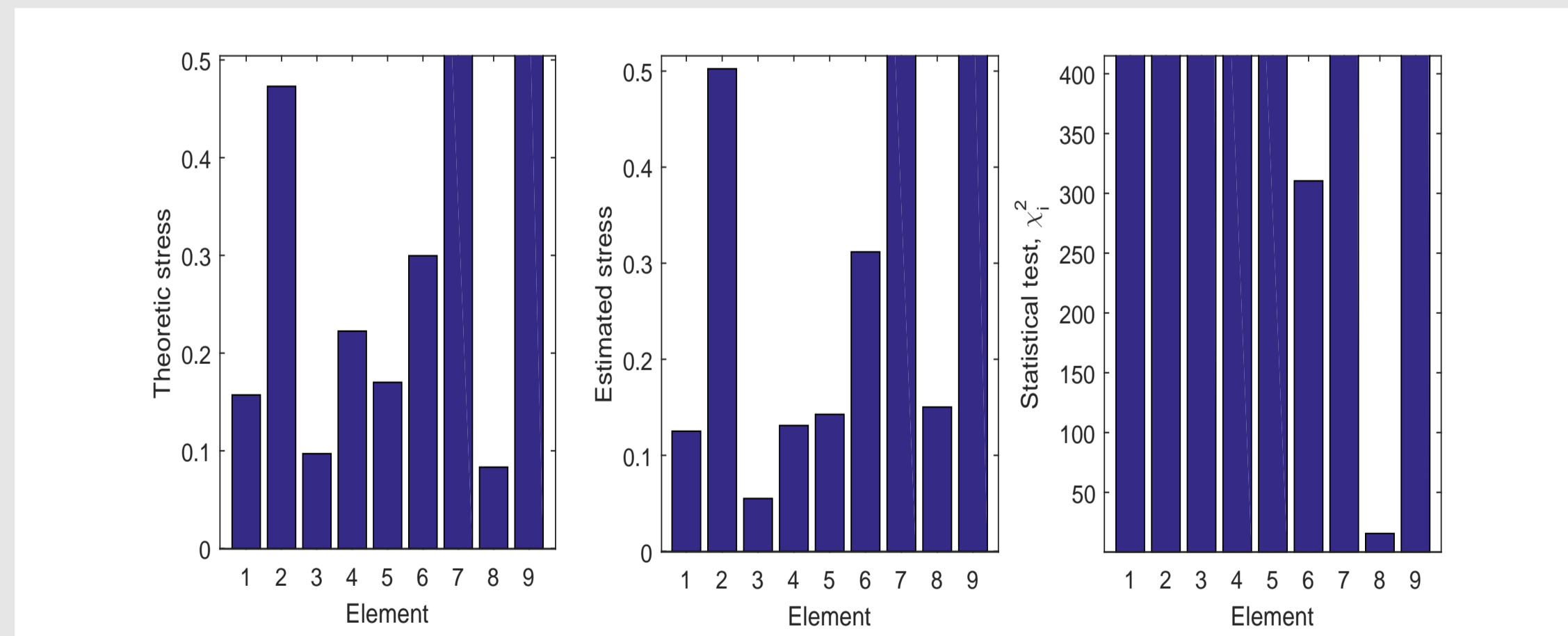
- Computed stress for damage localization is afflicted with uncertainties  
– Sensitivity-based uncertainty propagation from measurement data ( $\mathcal{H}$ ) to computed stress

$$\text{Reference state: } \mathcal{H}^{\text{ref}} \xrightarrow{\mathcal{J}_{A,\mathcal{H}}} A, C \xrightarrow{\mathcal{J}_{R,A}} R \quad \delta R \xrightarrow{\mathcal{J}_{v,R}} v(s) \xrightarrow{\mathcal{J}_{S(s)}} S(s)$$

$$\text{Damaged state: } \mathcal{H}^{\text{dam}} \xrightarrow{\mathcal{J}_{A,\hat{\mathcal{H}}}} \hat{A}, \hat{C} \xrightarrow{\mathcal{J}_{R,\hat{A}}} \hat{R}$$

$$\text{cov}(S(s)) = \mathcal{J}_{S(s)}(\text{cov}(\text{vec}(\hat{R}^T)_{\text{re}})) + (\text{cov}(\text{vec}(\hat{R}^T)_{\text{re}})) \mathcal{J}_{S(s)}^T$$

- $\chi^2$  test  $S_i^T \text{cov}(S_i)^{-1} S_i$  for each element  $i$



## Mathematical model

$$\text{Mechanical model} \quad M\ddot{v}(t) + C\dot{v}(t) + K v(t) = u(t)$$

$$\text{State space model} \quad \begin{cases} \dot{x}(t) = Ax(t) + B_c e(t) \\ y(t) = C_c x(t) + D_c e(t) \end{cases}$$

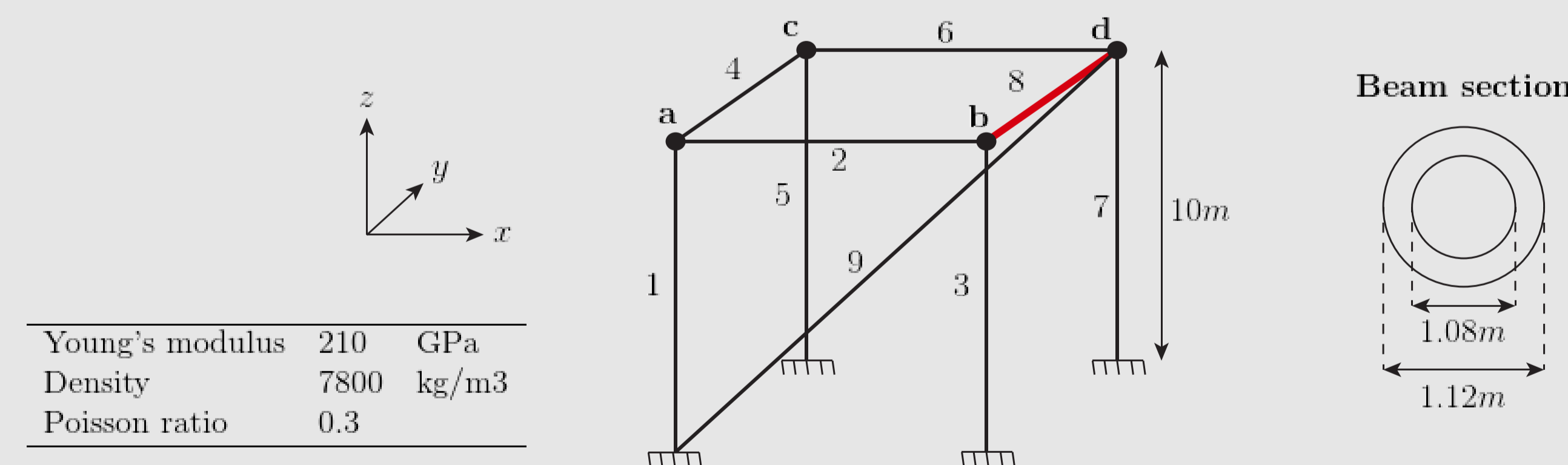
## Subspace Identification

$$\mathcal{Y}^- = \begin{bmatrix} y_q & y_{q+1} & \dots & y_{N+q-1} \\ y_{q-1} & y_q & \dots & y_{N+q-2} \\ \vdots & \vdots & \ddots & \vdots \\ y_1 & y_2 & \dots & y_N \end{bmatrix}, \quad \mathcal{Y}^+ = \begin{bmatrix} y_{q+1} & y_{q+2} & \dots & y_{N+q} \\ y_{q+2} & y_{q+3} & \dots & y_{N+q+1} \\ \vdots & \vdots & \ddots & \vdots \\ y_{q+p+1} & y_{q+p+2} & \dots & y_{N+p+q} \end{bmatrix}$$

$$\hat{\mathcal{H}} = \frac{1}{N} \mathcal{Y}^+ (\mathcal{Y}^-)^T = \mathbf{U} \Delta \mathbf{V}^T = [\mathbf{U}_1 \ \mathbf{U}_0] \begin{bmatrix} \Delta_1 & 0 \\ 0 & \Delta_0 \end{bmatrix} \mathbf{V}^T \approx \mathbf{U}_1 \Delta_1 \mathbf{V}_1^T, \quad \hat{\mathcal{O}} = \mathbf{U}_1$$

## Application

- Number of degree of freedom: 24
- Damping ratio = 2%, noise = 5%
- Output sensors:  $x$  and  $y$  directions at node a and d
- Number of sample: 200000
- Damaged state: decreasing 25% Young modulus at element 8



## SF

$$\text{Observability matrix} \quad \mathcal{O} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^p \end{bmatrix}$$

### Damage localization procedure

- FE model updating procedure to identify structure parameters:  
 $\theta = \text{argmin} \|\mathbf{r}\|_2^2$   
 $\mathbf{r} = [\mathbf{I}_{2n} \otimes (\mathbf{I}_{(p+1)r} - \hat{\mathcal{O}} \hat{\mathcal{O}}^+)] \text{vec} \{ \mathcal{O}^h(\theta^h) \}$   
 $\theta_k = \theta_{k-1} - \mathcal{J}_{\mathbf{r},\theta}^+ \mathbf{r}_k$
- $\theta$  related to damage (element stiffness reduction)

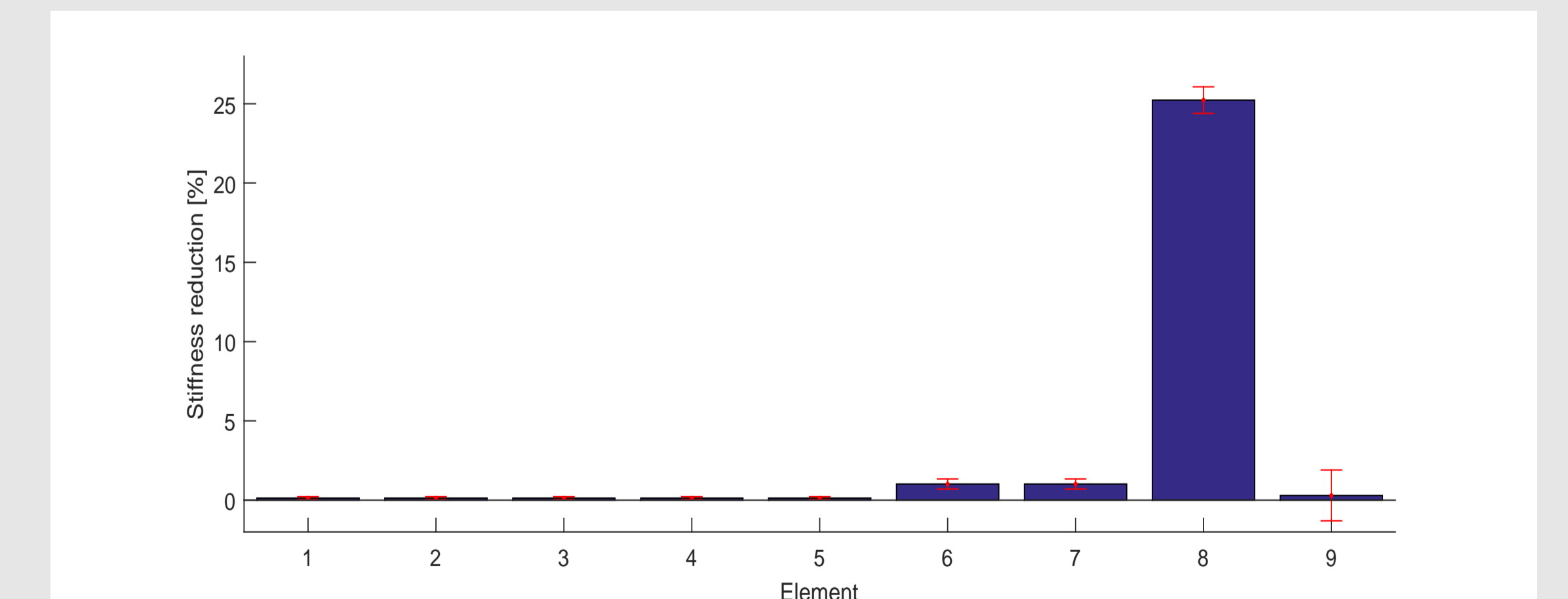
### Uncertainty propagation and covariance computation

- Updating parameters for damage localization is afflicted with uncertainties  
– Sensitivity-based uncertainty propagation from measurement data ( $\mathcal{H}$ ) to updated parameters, through each iteration step of the updating minimization problem

$$\text{Reference state: } \mathcal{H}^{\text{ref}} \xrightarrow{\mathcal{J}_{\theta^{\text{ref}},\mathcal{H}}} \theta^{\text{ref}}$$

$$\text{Damaged state: } \mathcal{H}^{\text{dam}} \xrightarrow{\mathcal{J}_{\theta^{\text{dam}},\mathcal{H}}} \theta^{\text{dam}}$$

- Damaged if  $\theta_i^{\text{dam}} \pm \sigma_{\theta_i^{\text{dam}}} - \theta_i^{\text{ref}} > \sigma_{\theta_i^{\text{ref}}}$  for each element  $i$



## References

- [1] M. Döhler, L. Marin, D. Bernal, L. Mevel, *Statistical decision making for damage localization with stochastic load vectors*. Mechanical Systems and Signal Processing, 39(1), 426-440, 2013.
- [2] M. D. H. Bhuyan, M. Döhler, L. Mevel, *Statistical Damage Localization with Stochastic Load Vectors Using Multiple Mode Sets*. EWSHM-8th European Workshop on Structural Health Monitoring, 2016.
- [3] G. Gautier, J.-M. Mencik, R. Serra, *A finite element-based subspace fitting approach for structure identification and damage localization*. Mechanical Systems and Signal Processing, 58, 143-159, 2015.
- [4] G. Gautier, L. Mevel, J.-M. Mencik, R. Serra, M. Döhler, *Variance analysis for model updating with a finite element based subspace fitting approach*. Mechanical Systems and Signal Processing, 91, 142-156, 2017.