High order Padé schemes for ODEs
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Introduction

We consider the following linear test equation

\[ y'(t) = i \omega y(t), \quad y(0) = y_0, \quad t \in [t_0, T] \]  

(1)

which has the exact solution after one step ( \( h = t_{n+1} - t_n \))

\[ y_n = e^{i \omega h} y_{n-1}. \]

Numerical solution is obtained by approximating

\[ \phi_h \approx R(h, \omega). \]

- The scheme is \( R(h, \omega) \) such that the scheme is
  - stable if \( |R(i \omega h)| \leq 1 \),
  - non dissipative if \( |R(i \omega h)| = 1 \),
  - non dispersive if \( \phi(i \omega h) = i \omega h - \text{arg}(R(i \omega h)) = 0 \).

\( \forall \omega \in \mathbb{C}, R(\omega) \) is called the stability function of the numerical scheme.

Main goals

Find \( R(\omega) \) such that the scheme is unconditionally stable, less dissipative and less dispersive.

Diagonal Padé schemes

The \((r + s)^{th}\) order Padé approximation of \( e^\omega \) is \( R_{r,s}(\omega) = \frac{P_{r,s}(\omega)}{Q_{r,s}(\omega)} \), with

\[ P_{r,s}(\omega) = \sum_{k=0}^r \omega^k \frac{s!}{k!} (\omega - \lambda)^k \quad \text{and} \quad Q_{r,s}(\omega) = \sum_{k=0}^s \omega^k \frac{r!}{k!} (\omega - \lambda)^k. \]

✓ Diagonal Padé schemes correspond to the case where \( r = s = m \).

Furthermore, dispersion error decreases with higher order

\[ \phi_h = \frac{e^{i \omega h} - 1}{i \omega h}. \]

Fig 1: Dispersion error of diagonal Padé schemes.

Undesirable effects of time schemes

- Unstable scheme: the numerical solution grows more rapidly than the physical solution.
- Dissipative scheme: numerical solution has amplitude error.
- Dispersion scheme: numerical solution has phase error.

Application to Maxwell’s equations

We consider

\[ \begin{align*}
\frac{\partial E}{\partial t} - \nabla \times H + \sigma E &= F_E, \\
\frac{\partial H}{\partial t} + \nabla \times E &= F_B,
\end{align*} \]

(2)

\( F_E \) and \( F_B \) are respectively the electric and magnetic fields (source term), \( \sigma \) is the conductivity of the medium, \( \mu \) is the permeability of the medium and \( \epsilon \) is the permittivity.

Efficiency in term of computational time

We performed numerical tests with the C++ code Montjoly.  

- 10th order FEM on regular mesh \([-4, 4]^2\).

We compared computational times of different schemes after imposing 0.1% of errors.

Further research and perspectives

- Development of high order implicit time schemes for ODEs, application to Maxwell’s equations (conference ICOSAHOM2016 Rio June, 27th – July, 14th).  
- CFD optimization for explicit time schemes (Runge-Kutta and Runge-Kutta-Nyström).
- Development of locally implicit time schemes.
- Modeling of seismic-electric waves.
- Teaching: 64 hours (helping class and TD).

References

