

#### A TWO-SCALE MICRO-FACET REFLECTANCE MODEL COMBINING REFLECTION AND DIFFRACTION

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- Assume surface micro-geometry >> wavelength
  - Geometrical Optics
  - Micro-facet normal distribution D gives BRDF
  - [Torrance & Sparrow 1967][Cook & Torrance 1982][Walter et al. 2007][Heitz 2014]





- *If* micro-facets are specular: Dirac reflectance
- Integrate contributions from micro-facets:

$$\rho_{CT}(\boldsymbol{i}, \boldsymbol{o}) = \frac{F(\eta, \theta_d) D(\theta_h) G(\boldsymbol{i}, \boldsymbol{o})}{4 \cos \theta_i \cos \theta_o}$$



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Fresnel term. Gives color. Depends on index of refraction  $\eta$ 



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#### Micro-facets normal distribution

[Trowbridge & Reitz 1975][Walter et al. 2007] [Bagher et al. 2012][Löw et al. 2012][Burley 2012] [Brady et al. 2014]



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$$\rho_{CT}(i, o) = \frac{F(\eta, \theta_d) D(\theta_h) G(i, o)}{4 \cos \theta_i \cos \theta_o}$$
Shadowing-Masking [Smith 1967][Walter et al. [Heitz 2014]]

term

2007]



- Distance travelled by light depends on surface height
- Phase variation at the reflected wavefront
- Effect depends on wavelength





$$\rho_{\text{diffr.}}(i, o) = AF(i, o) \frac{\delta(\text{refl}(i), o)}{\cos \theta_o} + (1 - A)Q(i, o)S_{HS}(f)$$

Specular peak + diffraction lobe  $\rho_{\text{diffr.}}(i, o) = AF(i, o) \frac{\delta(\text{refl}(i), o)}{\cos \theta_o}$  $+(1-A)Q(i,o)S_{HS}(f)$ **Specular reflection** 

$$\rho_{\text{diffr.}}(i, o) = AF(i, o) \frac{S(\text{refl}(i), o)}{\cos \theta_o} + (1 - A)Q(i, o)S_{HS}(f)$$
  
Fresnel term  
Depends on index of refraction  $\eta$ 

Specular peak + diffraction lobe

$$\rho_{\text{diffr.}}(i, o) = AF(i, o) \frac{\delta(\text{refl}(i), o)}{\cos \theta_o} + (1 - A)Q(i, o)S_{HS}(f)$$

Diffraction lobe (depends on  $\lambda$ )



$$\rho_{\text{diffr.}}(i, o) = AF(i, o) \frac{\delta(\text{refl}(i), o)}{\cos \theta_o} + (1 - A)Q(i, o)S_{HS}(f)$$
  
Color term (not F)



$$\rho_{\text{diffr.}}(i, o) = AF(i, o) \frac{\delta(\text{refl}(i), o)}{\cos \theta_o} + (1 - A)Q(i, o)S_{HS}(f)$$
  
Repartition  
$$A = e^{-\left(2\pi \frac{\sigma_s}{\lambda}(\cos \theta_i + \cos \theta_o)\right)^2}$$



$$\rho_{\text{diffr.}}(i, o) = AF(i, o) \frac{\delta(\text{refl}(i), o)}{\cos \theta_o} + (1 - A)Q(i, o)S_{HS}(f)$$
  
Repartition  

$$A = e^{-\left(2\pi \frac{\sigma_S}{2} \left(\cos \theta_i + \cos \theta_o\right)\right)^2}$$
  
Surface roughness

**DIFFRACTION MODELS: PREVIOUS WORK** 



- Beckmann-Kirchhoff: infinite sum
  - [Beckmann & Spizzichino 1987][Vernold & Harvey 1998]
  - [He et al. 1991][Stam 1999]
- Harvey-Shack: Power Spectral Distribution (PSD)
  - [Harvey 1975][Krywonos 2006]
  - [Löw et al. 2012]
- K-correlation model: [Hoenders et al. 1979][Church & Takacs 1986]
- Periodic surfaces: [Toisoul and Gosh 2017][Werner et al. 2017]



- e.g. MERL database
- [Matusik et al. 2003]
- 100 materials measured



## **COMPARISON MODEL / MEASURES**

- Focus here on parameterisation / lobe shape
- Micro-facet model:
  - Explains peak shape [Ngan et al. 2005]
- Diffraction model:
  - Explains low energy behaviour [Löw et al. 2012]
- "Better fits using two lobes" [Everyone]



[Ngan et al. 2005]



[Löw et al. 2012] <sub>10</sub>

















#### WAVELENGTH DEPENDENCE: NICKEL





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#### WAVELENGTH DEPENDENCE: NICKEL





#### **OVERVIEW**

- Background
- Our contribution:
  - Two-scale representation
  - Combining the effects
- Comparisons with measured reflectance
- Conclusion and future directions



#### **OUR MAIN IDEA**



• Surface details *can* happen at all scales

#### **OUR MAIN IDEA**

Surface details *can* happen at all scales ightarrow





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#### **NEW REFLECTANCE MODEL**



- Separate details in two groups:
  - large ( $\gg\lambda$ ) and small ( $\approx\lambda$ )
- Return to micro-facet model definition:
  - Small facets: diffraction
  - Large facets: Cook-Torrance model
  - Combined effects: convolution

#### SINGLE MICRO-FACET CONTRIBUTION SINGLE MICRO-FACET CONTRIBUTION



# Before

# After

#### **ALL MICRO-FACETS TOGETHER**



$$\rho(i, o) = \int_{\Omega} \left| \frac{i \cdot m}{i \cdot n} \right| \rho_{\mu}(i, o, m) \left| \frac{o \cdot m}{o \cdot n} \right| G(i, o) D(m) \, \mathrm{d}\omega_{m}$$

• Dirac part of micro-facet reflectance: Cook-Torrance term

$$\rho(\mathbf{i}, \mathbf{o}) = A_{\text{spec}}(\theta_d)\rho_{CT}(\mathbf{i}, \mathbf{o}) + \rho_{CTD}(\mathbf{i}, \mathbf{o})$$

• Diffraction part: Combined Cook-Torrance-Diffraction (CTD)


$$\rho_{CTD}(i, o) = \int_{\Omega} \left| \frac{i \cdot m}{i \cdot n} \right| (1 - A) Q(i, o) S_{HS}(f) \left| \frac{o \cdot m}{o \cdot n} \right| G(i, o) D(m) \, \mathrm{d}\omega_m$$

- Large scale parameters:  $i, o, n, h, \theta_d$
- Small scale parameters:  $f, m, \theta_h$
- Small scale dependencies: *A*, *Q*
- Separation large scale / small scale?





$$\rho_{CTD}(\boldsymbol{i},\boldsymbol{o}) = \int_{\Omega} \left| \frac{\boldsymbol{i} \cdot \boldsymbol{m}}{\boldsymbol{i} \cdot \boldsymbol{n}} \right| (1 - A) Q(\boldsymbol{i},\boldsymbol{o}) S_{HS}(\boldsymbol{f}) \left| \begin{array}{c} \boldsymbol{o} \cdot \boldsymbol{m} \\ \boldsymbol{o} \cdot \boldsymbol{n} \end{array} \right| G(\boldsymbol{i},\boldsymbol{o}) D(\boldsymbol{m}) \, \mathrm{d}\omega_m$$

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$$\rho_{CTD}(i, o) = \int_{\Omega} \left| \begin{array}{c} i \cdot m \\ i \cdot n \end{array} \right| (1 - A) Q(i, o) S_{HS}(f) \left| \begin{array}{c} o \cdot m \\ o \cdot n \end{array} \right| G(i, o) D(m) d\omega_m$$

- Large scale parameters:  $i, o, n, h, \theta_d$
- Small scale parameters:  $f, m, \theta_h$
- Small scale dependencies: *A*, *Q*
- Separation large scale / small scale?





- A depends on  $(\cos \theta_i + \cos \theta_o)$
- Cosines relative to *m*, micro-facet normal
- $(\cos \theta_i + \cos \theta_o) = (i \cdot m + o \cdot m) = 2 \cos \theta_d (h \cdot m)$
- Separation large scale / small scale:

$$1 - A \approx (1 - A_{\text{spec}}(\theta_d))(\boldsymbol{h} \cdot \boldsymbol{m})^2$$





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- Diffraction parameter *f* :
  - $f = (2/\lambda) \sin \theta_h \cos \theta_d$
  - $f = (2/\lambda) \cos \theta_d (1 (\mathbf{h} \cdot \mathbf{m})^2)^{1/2}$
  - Product large/small scale



• Putting it into the integral:

$$u_{CTD}(i, o) \approx (1 - A_{\text{spec}}(\theta_d))G(i, o) \int_{\Omega} \left| \frac{i \cdot m}{i \cdot n} \right| \left| \frac{o \cdot m}{o \cdot n} \right| Q(i, o)S(h \cdot m)D(m) \, d\omega_m$$



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  - $f = (2/\lambda) \cos \theta_d (1 (h \cdot m)^2)^{1/2}$
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• Putting it into the integral:

$$\phi_{CTD}(i, o) \approx (1 - A_{\text{spec}}(\theta_d))G(i, o) \int_{\Omega} \left| \frac{i \cdot m}{i \cdot n} \right| \left| \frac{o \cdot m}{o \cdot n} \right| Q(i, o)S(h \cdot m)D(m) \, d\omega_m$$



$$\rho_{CTD}(i, o) \approx (1 - A_{\text{spec}}(\theta_d))G(i, o) \int_{\Omega} \left| \frac{i \cdot m}{i \cdot n} \right| \left| \frac{o \cdot m}{o \cdot n} \right| Q(i, o)S(h \cdot m)D(m) \, d\omega_n$$

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$$\rho_{CTD}(i, o) \approx (1 - A_{\text{spec}}(\theta_d))G(i, o) \int_{\Omega} \left| \frac{i \cdot m}{i \cdot n} \right| \left| \frac{o \cdot m}{o \cdot n} \right| Q(i, o)S(h \cdot m)D(m) d\omega_n$$









Slow variations: ≈ constant



$$\rho_{CTD}(i, o) \approx (1 - A_{\text{spec}}(\theta_d)) G(i, o) Q(i, o) \int_{\Omega} S(h \cdot m) D(m) \, \mathrm{d}\omega_m$$

- Spherical convolution
  - Compute using Spherical Harmonics
- Both functions have symmetry of revolution
  - Zonal Harmonics only



# $\rho_{CTD}(i, o) \approx (1 - A_{\text{spec}}(\theta_d))G(i, o)Q(i, o) \int S(h \cdot m)D(m) \, d\omega_m$

- Spherical convolution
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# SUMMARY



$$\rho(i, o) = A_{\text{spec}}(\theta_d) \rho_{CT}(i, o) + (1 - A_{\text{spec}}(\theta_d)) Q G (S * D)(\theta_h)$$

- Scaled Cook-Torrance lobe + Convolved Diffraction lobe
- Relative importance depends on  $\theta_d$
- Shadowing-Masking *G* applied to diffraction lobe
- Convolution:
  - Enlarges diffraction lobe
  - Depends on wavelength and  $\cos \theta_d$

# **PRACTICAL IMPLEMENTATION**



- $S_{\rm HS}$ , *D* are parametric functions:
  - Compute (S \* D) for all values of the parameters
  - Approximate:  $(S * D) \approx \hat{S}_{HS}$  (different parameters)
  - Table: input parameters  $\rightarrow$  parameters of  $\hat{S}_{HS}$
- Other precomputations:
  - *G* (shadowing-masking)
  - Renormalisation of  $S_{\rm HS}$  (for energy conservation)

### **OVERVIEW**

- Background
  - Cook-Torrance models
  - Diffraction models
- Our model:
  - Two-scale representation
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### **MEASURED NICKEL**





Diffraction + Micro-Facet = Full model

#### Difference



## **MEASURED GRAY-PLASTIC**





# **MEASURED YELLOW-MATTE-PLASTIC**



See Supplemental for full comparison

Difference



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# LOBE SHAPES (INCIDENT PLANE)







Smooth (diffraction only)

SGD (micro-facet only)<sub>29</sub>

# LOBE SHAPES (OUTSIDE INCIDENT)



### **MICRO-FACET + DIFFUSE LOBE?**





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# PARAMETERS





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# LIMITATIONS

- Single interface, single bounce
- Works better for smooth surfaces
  - Also hypotheses for convolution
- Worst case scenario:
  - Diffuse materials, fabrics, glossy metallic paints
- Computation time: + 10 %



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# CONCLUSION

- Two-scale micro-facet model for surface reflectance
- Combination Cook-Torrance and Combined CTD lobe
- Near specular peak: Cook-Torrance
- Low values: Combined CT-Diffraction
- Fits measured reflectance
- Code: Mitsuba plugin (see project page)





# **FUTURE DIRECTIONS**



- Low hanging fruits:
  - Simplified model
  - Anisotropic model (large scale only)
  - Multiple bounces
  - Multiple layers
- More difficult:
  - Anisotropic model (both scales)
  - More measured data (height fields, spectral reflectance)
  - More accurate convolution
  - Unified representation, large wavelength range

# THANK YOU





### NUMBER OF SPECTRUM SAMPLES





### Alum-bronze, RGB

### Alum-bronze, 30 samples

### NUMBER OF SPECTRUM SAMPLES





### Aluminium, RGB

### Aluminium, 30 samples 39

# **PRECOMPUTED TABLES**



- Shadowing-masking G: 390 kB
  - 2 parameters,  $\beta \tan \theta$  and p
- Renormalization: 8.9 MB
  - 3 parameters:  $\theta_i$ , *b* and *c*
  - Reduce parameter range?
- Convolution: 2.5 MB
  - 4 parameters: *β*, *p*, *c*, *u*
  - $u = (b/\lambda)\cos\theta_d$

### SHAPE-INVARIANCE



• D needs shape-invariance property [Heitz 2014]:

$$D(\theta) = \frac{\chi_{[0,\frac{\pi}{2}]}(\theta)}{\cos^4 \theta} P_{22} \left( \tan^2 \theta \right)$$

# **EXPONENTIAL POWER DISTRIBUTION**



• Extension of Gaussian distribution:

$$P_{22}(x) = \frac{p}{\pi\beta^2\Gamma(1/p)} e^{-\left(\frac{x}{\beta^2}\right)^p}$$

# **EXPONENTIAL POWER DISTRIBUTION**



• Extension of Gaussian distribution:



Normalisation factor

# **K-CORRELATION MODEL**



$$S_{HS}(f) = \frac{c-1}{2\pi} \frac{\sigma_s^2 b^2}{(1+b^2 f^2)^{\frac{c+1}{2}}}$$

# **K-CORRELATION MODEL**





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#### RENORMALISATION



# $\sigma_{\text{rel}}^2(\theta_i) = \int_{\|\boldsymbol{o}\| \le 1} S_{HS}(\boldsymbol{f}) \, \mathrm{d}f_x \, \mathrm{d}f_y$

- Effective surface roughness
- Divide  $S_{HS}$  by  $\sigma^2_{rel}$  so integral = 1
- Use  $\sigma_{rel}$  in place of  $\sigma_s$  for A [Harvey et al. 2012]
- Important for energy conservation

### NUMBER OF PARAMETERS



MODEL	NUMBER OF PARAMETERS	DIFFRACTION THEORY
Our Model	11	Hybrid Harvey-Shack
EPD + diffuse	11	None
Shifted Gamma Distribution	18	None
He et al. [1991]	11	Beckman - Kirchoff
Löw et al. [2012]	9	Inspired from Rayleigh-Rice





# $(1 - A_{\sigma_s}(i, o)) \approx (1 - A_{\sigma_s}(\theta_d))(h \cdot m)^2$





 $a = 2\pi\sigma_s/\lambda$ 

## **MERL APPARATUS EFFECT**





- Intensity decreases at grazing angles
- Modelled, compensated

## **HEMATITE: INDEX OF REFRACTION**





- Conductor below 650 nm
- Dielectric above