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Modeling Energy Performance of Manufacturing Systems using Gi/M/1 Queues

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Abstract. Energy expended in a discrete manufacturing system can be saved by turning idle machines off. Utilizing this idea, previous research has contributed preliminary models in which mainly M/M/1 machines are considered. To generalize existing approaches, this paper proposes a new energy model based on a Gi/M/1 queueing network. To start, a simulation model is built with Gi/M/1 machines. The proposed model is then built by fitting each GED (Generalized Erlang Distribution) to the observed first two moments of simulated interarrival times of all machines. Consumed energy is calculated separately by the proposed and simulation models, and, in the comparison between two estimations, the proposed method shows at most 4% different energy estimation from simulated values, suggesting that the proposed approach is promising for the energy analysis about a Gi/M/1 queueing network.

Keywords: Energy Aware Model, Gi/M/1, Queueing, Distribution Fitting

1 Introduction

1.1 Motivation and Previous Research

Energy demand by the U.S. industrial sector accounted for more than 30% of the total U.S. energy demand, and electricity consumption of the sector took up about 10% of the total demand in 2010 [1]. The American electricity price for industrial consumption has continuously increased since 1990, and there remains an acute need to monitor industrial/manufacturing facilities to find a way of saving energy [2]. In the light of this observation, one noteworthy research contribution implies that 30–85% of energy in machining is spent at a constant rate [3]. Since it suggests that the significant amount of energy is wasted for machine idling, research works have been conducted to save energy by turning idle systems off especially in the DPM (Dynamic Power Management) field [4]. Thus DPM is grounded in theory, but there are inherent limitations in their applicability to manufacturing: First, each microprocessor state in DPM does not correspond well to that of a machine [4], [5]. Second, while electrical signals in DPM can be freely created and discarded among processors, physical parts in manufacturing cannot. Hence there has been difficulty in applying DPM theories

directly to manufacturing. On the other hand in manufacturing research, power and energy analysis has relied on the approaches which focus on a short span of time in machining. The traditional method has limitations in observing long run properties of energy consumption by machines [6]. One analysis, to address the problem, takes the idle state into consideration, but it still depends on coarse textbook tables for important parameters [7]. In more recent literature, the focus is on energy optimization of a unit machining process, but the methods are still unable to analyze energy consumption of yearlong machining [8], [9]. Queueing theory models, in consideration of described problems, can be alternatives to previous research since they can include working and idling states in the long run [10], [11]. However queueing based models in [10], [11] also have limitations in that they assume only M/M/1 systems or the restricted machine states. As a consequence, we need to improve existing queueing models in order to take more general types of manufacturing systems into accounts.

1.2 Contribution and Organization

The main contribution of this paper is the development of an extended energy aware model of machining. A machine network, in this paper, consists of multiple machines, and each machine has renewal arrivals and exponential processing times (Gi/M/1) to allow more generality. Dealing with renewal interarrivals, we assume that the first two moments of the arrival distribution are known, and the distribution fitting is performed to generate the arrival flow following the two moments. For machine states, total five states are included: setup, tool change, cutting, nominal power idling, and low power idling. Thus this research aims to provide a more general energy analysis than previous models described earlier in [10], [11].

The rest of this paper is organized as follows: After the machine states and the energy control policy are defined, a brief experiment is introduced to measure energy/power parameters of a milling machine in Section 2. Then in Section 3, the distribution fitting method with two moments is described. Section 4 introduces simulation experiments, and examines the comparison between the result by the proposed method and that by the simulation experiments. The conclusion and future research directions are detailed in Section 5.

2 Energy Performance Model

2.1 Machine States, Power Consumption Levels, and Energy Policy

Generally, machine states are defined as working, nominal power idling, and low power idling states [10], but this research regards the working state as the combined state of setup, tool change, and cutting. Thus we define five machine states, and power consumption level *W* of each state is as follows:

- Setup: Generic machine state of waking-up with W_S
- Tool Change: State for tool (cutter) changing with W_T

- Cutting: State of air or material cutting with W_C
- Nominal Power Idling: Idle state whose duration is less than τ with W_N
- Low Power Idling: Idle state whose duration is greater than or equal to τ with W_L

For the energy policy, machines are assumed to enter the low power idling state if the current idle duration is greater than the time threshold τ . Otherwise machines stay in the nominal power idling state during idling [10], [11].

2.2 Power Data and Experiment

Power data was collected in the experiment with a Haas VF3 milling machine. Six cuts were made on the steel block in 6 x 4 inches size, and depth of cut and contact surface (full/half) in each pass varied. Power and processing times were collected as in Figure 1 and Table 1, and data values of Table 1 are averaged during each machine state.

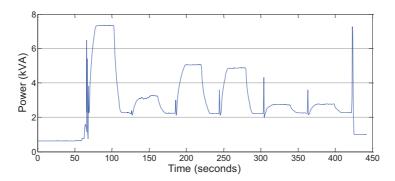


Fig. 1. Power Data (Haas VF3 Milling Machine)

Table 1. Averaged Power Data (kVA)

| Pass No. | Machine State | | | | | | | | | |
|----------|---------------|------|----------------|---------------|--------------------|--|--|--|--|--|
| Pass No. | Off | Idle | Cutting (time) | Setup (time) | Tool Change (time) | | | | | |
| 1 | 0.63 | 0.99 | 5.24 (57 secs) | 1.89 (5 secs) | 3.25 (1 sec) | | | | | |
| 2 | - | - | 2.78 (58 secs) | - | - | | | | | |
| 3 | - | - | 3.81 (59 secs) | - | - | | | | | |
| 4 | - | - | 3.72 (57 secs) | - | - | | | | | |
| 5 | - | - | 2.54 (57 secs) | - | - | | | | | |
| 6 | - | - | 2.56 (58 secs) | - | - | | | | | |

2.3 Theoretical Model of Consumed Energy Amount

The consumed energy for a unit time under the energy policy is written as [10], [11],

$$E[Energy] = \begin{cases} W_S P_S + W_T P_T + W_C P_C + W_N P_N + W_L P_L \\ where P_X is probability in state X \end{cases}$$
 (1)

3 Distribution Fitting with Two Moments

 $P(X > \tau)$ and $P(X \le \tau)$ of the arrival process are important parameters in our energy analysis since the two parameters are used in calculating $P_N: P(X < \tau)P(idle)$ and $P_L: P(X \ge \tau)P(idle)$ in (1) [11]. Thus we need to have the probability distribution function from which the two probabilities can be calculated, and the distribution has to be fitted to the first two moments. For this approach, any single machine in a network is decomposed as an independent system, and GED (Generalized Erlang Distribution) is fitted to each machine's arrival flow [12], [13], [14], [15]. Using GED for fitting has two advantages: First, it allows us to consider relatively regular interarrivals with less coefficient of variation ($C_X < 1$). Second, GED is easier to treat than other probability distributions as seen in its probability density distribution:

$$f(t) = p\mu^{k-1} \frac{t^{k-2}}{(k-2)!} e^{-\mu t} + (1-p)\mu^k \frac{t^{k-1}}{(k-1)!} e^{-\mu t} \text{ for } t \ge 0$$
 (2)

$$p = \frac{kC_X^2 - \sqrt{k(1 + C_X^2) - k^2 C_X^2}}{1 + C_X^2}, \mu = \frac{k - p}{E[X]}, \frac{1}{k} \le C_X^2 < \frac{1}{k - 1}, and C_X^2 = \frac{Var[X]}{E^2[X]} = \frac{E[X^2] - E^2[X]}{E^2[X]}$$
(3)

where Var[X] and E[X] are the variance and mean of observed data samples. More details of GED are found in [12], [15], and literature therein.

4 Simulation and Analysis

Simulation is performed to see how precisely the distribution fitting method can estimate P_N and P_L . Important parameters and assumptions are as follows:

- Scenario 1: Two machines in a row
- Scenario 2: Typical COMS manufacturing fab. [16]
- Utilization of Machine i ($\rho_i = \lambda_i/\mu_i$): 0.5 or 0.8
- Cutting, Setup, and Tool Change Rates: Table 1
- Arrival Rate λ_i of Machine i: Number of arrivals per a time unit
- Processing Rate μ_i : Reciprocal of total sum of cutting, setup, and tool change time
- Arrival Distribution: Normal distribution with parameters in Table 2 and 3
- Processing Distribution: Exponential for Cutting, Setup, and Tool Change
- Queueing Model: Gi/M/1 as approximation
- Power Consumption (kVA) : W_S , W_T , W_C , W_N , W_L from Table 1
- Simulation Software: Simio V4.68

4.1 Scenario 1: Two Machines in Serial Line

Table 2 gives good estimations of P_N and P_L by fitting. Since the method is based on first two moments, normal arrivals would have been ideal subjects with symmetry and unimodal in their density. This also explains the smaller difference (F-S) of Machine

1 than of Machine 2 since arrivals to Machine 1 appear more normal than Machine 2's. It seems that there is no apparent relationship between F-S and utilization ρ .

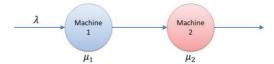


Fig. 2. Scenario 1 (Two Machines in Serial Line)

Table 2. Probabilities of Nominal / Low Power Idling States (Scenario 1)

| | | $\rho = 0.5$, interarrival mean = 2.1, $\sigma = 1.05$ | | | | | | | $\rho = 0.8$, interarrival mean = 1.31, $\sigma = 0.66$ | | | | | |
|---|-----|---|------|-------|------|------------|-------|------|--|-------|------|------|-------|--|
| | | Prob.(Low) Prob.(Nominal) | | | | Prob.(Low) | | | Prob.(Nominal) | | | | | |
| M | τ | Fit | Sim. | F-S | Fit | Sim. | F-S | Fit | Sim. | F-S | Fit | Sim. | F-S | |
| | 0.7 | 0.48 | 0.47 | 0.01 | 0.02 | 0.03 | -0.01 | 0.19 | 0.18 | 0.01 | 0.01 | 0.02 | -0.01 | |
| | 1.4 | 0.37 | 0.37 | 0.00 | 0.13 | 0.13 | 0.00 | 0.15 | 0.14 | 0.01 | 0.05 | 0.06 | -0.01 | |
| 1 | 2.1 | 0.22 | 0.23 | 0.00 | 0.28 | 0.27 | 0.01 | 0.09 | 0.08 | 0.01 | 0.11 | 0.12 | -0.01 | |
| | 2.8 | 0.11 | 0.11 | 0.00 | 0.39 | 0.39 | 0.00 | 0.05 | 0.04 | 0.01 | 0.15 | 0.16 | -0.01 | |
| | 3.5 | 0.05 | 0.03 | 0.01 | 0.45 | 0.47 | -0.01 | 0.02 | 0.01 | 0.01 | 0.18 | 0.19 | -0.01 | |
| | 0.7 | 0.43 | 0.47 | -0.04 | 0.06 | 0.03 | 0.04 | 0.15 | 0.20 | -0.05 | 0.04 | 0.01 | 0.03 | |
| | 1.4 | 0.31 | 0.40 | -0.09 | 0.18 | 0.10 | 0.08 | 0.11 | 0.17 | -0.06 | 0.08 | 0.04 | 0.04 | |
| 2 | 2.1 | 0.21 | 0.30 | -0.09 | 0.29 | 0.20 | 0.09 | 0.08 | 0.13 | -0.06 | 0.11 | 0.08 | 0.03 | |
| | 2.8 | 0.13 | 0.20 | -0.07 | 0.36 | 0.29 | 0.07 | 0.05 | 0.10 | -0.05 | 0.14 | 0.11 | 0.03 | |
| | 3.5 | 0.08 | 0.13 | -0.05 | 0.42 | 0.37 | 0.05 | 0.03 | 0.08 | -0.04 | 0.15 | 0.13 | 0.02 | |

4.2 Scenario 2: CMOS Manufacturing Fab.

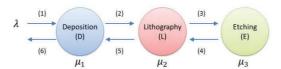


Fig. 3. Typical CMOS Manufacturing System

Table 3. Probabilities of Nominal / Low Power Idling States (Scenario 2)

| | | $\rho=0.5$, interarrival mean = 4.2, $\sigma=1.4$ | | | | | | $\rho = 0.8$, interarrival mean = 2.63, $\sigma = 0.88$ | | | | | | |
|---|------|--|----------|-------|------|----------------|------|--|------------|-------|------|----------------|------|--|
| | | P | rob.(Lov | v) | Pro | Prob.(Nominal) | | | Prob.(Low) | | | Prob.(Nominal) | | |
| M | τ | Fit | Sim. | F-S | Fit | Sim. | F-S | Fit | Sim. | F-S | Fit | Sim. | F-S | |
| | 0.44 | 0.43 | 0.48 | -0.04 | 0.07 | 0.03 | 0.04 | 0.17 | 0.19 | -0.02 | 0.03 | 0.01 | 0.02 | |
| | 0.88 | 0.31 | 0.40 | -0.08 | 0.19 | 0.10 | 0.08 | 0.12 | 0.16 | -0.04 | 0.08 | 0.05 | 0.03 | |
| D | 1.31 | 0.21 | 0.29 | -0.09 | 0.29 | 0.21 | 0.08 | 0.08 | 0.12 | -0.04 | 0.12 | 0.09 | 0.03 | |
| | 1.75 | 0.13 | 0.19 | -0.06 | 0.37 | 0.31 | 0.06 | 0.05 | 0.08 | -0.03 | 0.15 | 0.13 | 0.02 | |
| | 2.19 | 0.08 | 0.10 | -0.02 | 0.42 | 0.41 | 0.02 | 0.03 | 0.05 | -0.02 | 0.17 | 0.16 | 0.01 | |
| | 0.44 | 0.41 | 0.44 | -0.03 | 0.08 | 0.04 | 0.04 | 0.15 | 0.18 | -0.03 | 0.04 | 0.02 | 0.02 | |
| | 0.88 | 0.30 | 0.36 | -0.06 | 0.19 | 0.12 | 0.07 | 0.11 | 0.15 | -0.04 | 0.08 | 0.05 | 0.03 | |
| L | 1.31 | 0.20 | 0.29 | -0.09 | 0.29 | 0.19 | 0.10 | 0.07 | 0.11 | -0.04 | 0.11 | 0.08 | 0.03 | |
| | 1.75 | 0.13 | 0.22 | -0.09 | 0.36 | 0.27 | 0.10 | 0.05 | 0.09 | -0.04 | 0.14 | 0.11 | 0.03 | |
| | 2.19 | 0.08 | 0.16 | -0.08 | 0.41 | 0.33 | 0.08 | 0.03 | 0.07 | -0.03 | 0.15 | 0.13 | 0.02 | |
| | 0.88 | 0.69 | 0.72 | -0.03 | 0.05 | 0.03 | 0.03 | 0.48 | 0.56 | -0.08 | 0.11 | 0.03 | 0.08 | |
| | 1.75 | 0.51 | 0.61 | -0.10 | 0.23 | 0.13 | 0.10 | 0.35 | 0.50 | -0.15 | 0.24 | 0.09 | 0.15 | |
| E | 2.63 | 0.32 | 0.44 | -0.12 | 0.42 | 0.30 | 0.12 | 0.24 | 0.40 | -0.16 | 0.35 | 0.19 | 0.16 | |
| | 3.50 | 0.18 | 0.25 | -0.07 | 0.57 | 0.49 | 0.07 | 0.16 | 0.30 | -0.15 | 0.43 | 0.28 | 0.15 | |
| | 4.38 | 0.09 | 0.13 | -0.04 | 0.65 | 0.62 | 0.03 | 0.10 | 0.22 | -0.12 | 0.49 | 0.37 | 0.12 | |

Contrary to the previous result in Table 2, P_N and P_L of Etching in Table 3 show slightly larger difference with the max F-S of 0.16 than those of Deposition and Lithography. In the next subsection, it is shown and analyzed how different energy estimations are made between simulation and proposed models based on probabilities in Table 2 and 3.

4.3 Analysis and Discussion

From simulated parameters, expected energy demands for a unit time are calculated in this section by (1). Since these demands are the product of power consumption levels (W_X : X is any machine state) and the time proportion at each state (P_X), the resulting energy consumption between fitting and simulation depends on P_N and P_L as given in Table 4 and 5.

Table 4. Expected Energy Consumption for a Unit Time (Scenario 1)

| ρ — | N | fachine 1 (kVA* | min) | Machine 2 (kVA*min) | | | | | |
|-----|-------|-----------------|----------|---------------------|-------|----------|--|--|--|
| | Fit | Sim. | Diff.(%) | Fit | Sim. | Diff.(%) | | | |
| | 2.793 | 2.805 | -0.41 | 1.714 | 1.692 | 1.30 | | | |
| | 2.832 | 2.841 | -0.30 | 1.755 | 1.717 | 2.20 | | | |
| 0.5 | 2.886 | 2.892 | -0.21 | 1.793 | 1.753 | 2.30 | | | |
| | 2.926 | 2.933 | -0.25 | 1.821 | 1.788 | 1.88 | | | |
| | 2.949 | 2.962 | -0.44 | 1.840 | 1.813 | 1.47 | | | |
| | 4.084 | 4.100 | -0.40 | 2.342 | 2.283 | 2.56 | | | |
| | 4.099 | 4.117 | -0.42 | 2.356 | 2.294 | 2.70 | | | |
| 0.8 | 4.121 | 4.137 | -0.39 | 2.368 | 2.307 | 2.66 | | | |
| | 4.137 | 4.153 | -0.38 | 2.377 | 2.317 | 2.56 | | | |
| | 4.146 | 4.162 | -0.37 | 2.383 | 2.327 | 2.40 | | | |

Table 5. Expected Energy Consumption for a Unit Time (Scenario 2)

| ρ | Deposition (kVA*min) | | | Litho | graphy (kV | A*min) | Etching (kVA*min) | | |
|------------------------|----------------------|-------|----------|-------|------------|----------|-------------------|-------|----------|
| | Fit | Sim. | Diff.(%) | Fit | Sim. | Diff.(%) | Fit | Sim. | Diff.(%) |
| 0.5 (D, L) 0.25 (E) | 2.811 | 2.790 | 0.75 | 1.720 | 1.722 | -0.16 | 1.429 | 1.413 | 1.11 |
| | 2.853 | 2.817 | 1.27 | 1.760 | 1.750 | 0.58 | 1.492 | 1.451 | 2.74 |
| | 2.892 | 2.855 | 1.27 | 1.795 | 1.776 | 1.07 | 1.561 | 1.511 | 3.17 |
| | 2.920 | 2.893 | 0.95 | 1.821 | 1.803 | 1.01 | 1.612 | 1.581 | 1.93 |
| | 2.939 | 2.926 | 0.44 | 1.838 | 1.824 | 0.77 | 1.643 | 1.625 | 1.08 |
| | 4.092 | 4.058 | 0.85 | 2.343 | 2.313 | 1.29 | 1.917 | 1.885 | 1.66 |
| 0.8 (D, L) 0.4 (E) | 4.109 | 4.070 | 0.97 | 2.357 | 2.325 | 1.39 | 1.963 | 1.909 | 2.76 |
| | 4.124 | 4.084 | 0.97 | 2.368 | 2.337 | 1.36 | 2.002 | 1.942 | 3.00 |
| | 4.135 | 4.099 | 0.87 | 2.377 | 2.346 | 1.33 | 2.031 | 1.977 | 2.68 |
| | 4.142 | 4.110 | 0.77 | 2.383 | 2.353 | 1.26 | 2.052 | 2.008 | 2.11 |

Energy demands of Scenario 1 are shown in Table 4. As P_N and P_L of fitting and simulation show quite close values, the largest difference is less than 3% in energy consumption. This agreement is also observed in Table 5 of Scenario 2. Although the largest difference 3.17% between simulated and estimated values is seen with Etching process, this value does not seem to be very large in that we just used the first and second moments E[X] and $E[X^2]$ of observed arrival times to estimate the arrival flow. Since the difference of P_N and P_L in Table 3 between simulated and estimated values is relatively greater than others, this is also expected difference. As a consequence, both tables are suggesting that the proposed method provides quite accurate estima-

tions about the total spent energy on the machine network. In order to show the good fits between observed and fitted interarrival times, Figure 4 and 5 are added below. Both figures illustrate the histogram of observed data and pdf (probability density function) for Deposition and Etching processes respectively when the machine utilization 0.8 is considered.

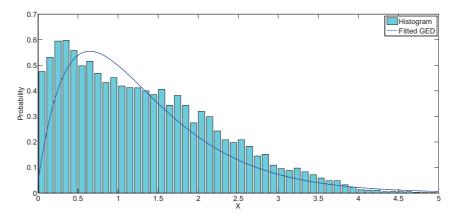


Fig. 4. Simulated Interarrival Histogram and pdf of Deposition (Scenario 2, Rho = 0.8)

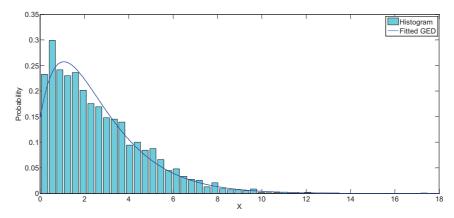


Fig. 5. Simulated Interarrival Histogram and pdf of Etching (Scenario 2, Rho = 0.8)

5 Conclusion and Future Research

This research proposes the method of estimating the energy amount of a manufacturing facility with machines in a network under the energy policy. Especially this approach aims at Gi/M/1 machines rather than M/M/1 systems for allowing more generality. To validate estimated values by the method, simulation is conducted, and the simulated result is compared with that of our approach. Consequently, it is shown that the proposed method gives, even at the worst case, only 4% different estimation. For

further research, other queueing models such as the Gi/G/1 need to be investigated with the percentile fitting method for considering more general distributions, since the proposed strategy of fitting is based on exponential service times.

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