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Adaptive Normative Modelling: a case study in the public-transport domain

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Abstract. Data analytics promises to detect behavioral patterns, which may be used to improve decision making. However, decisions need to motivated, and they are often motivated by models. In this paper we explore the interplay between data analytics and process modeling, specifically in normative settings. We look specifically at value nets, mathematical models of the flow of money and goods, as used in accounting. Such models can be used to analyze the proportions of various flows, such as resources consumed and products produced. Such analyses can be used in the planning and control cycle, for forecasting, setting a budget, testing and possibly adjusting the budget. In other words, it can be used for adaptive normative modeling. We look in particular at a case study of a provider of public transport services for school children. The case shows that the use of value nets for analysis of proportions is (i) feasible, and (ii) useful, in the sense that it provides valuable new insights about the revenue model.

Keywords: data analytics, business process modeling, normative systems

1 Introduction

Data analytics is heralded as the new big thing [5]. People, services, goods or processes leave traces, producing vast amounts of data, which can be analyzed. Some have claimed about 'Big Data' that the sheer volume of data and the power of the analytical tools would remove the need for scientific method [2]. Patterns in behavior emerge which can be made actionable, i.e., they can used to make better decisions. We disagree. Decisions that matter must be motivated. In many application domains, professional decisions are based on models. In particular, this is true in normative settings, where situations are compared to norms or standards, which can be either based on practices and conventions, or on formal rules and regulations. In such normative settings, the model has become a reference model or norm. For instance, in compliance checking, a 'de jure' model, based on the law , is compared with a 'de facto' model, based on data about the real situation [10].

In this paper we explore the interplay between computational models, on the one hand, and data analytics tools on the other hand for decision making in a very specific task and domain. In particular, we focus on the planning and control cycle, that involves forecasting, setting a target, and comparing results to the target [12]. Clearly, this setting is normative and the decisions are guarded by a set of expectations and norms. In this setting there is also a challenge: data analytics is essentially empirical; it is exploratory and starts from the available data. Analyzing models in a computational setting, on the other hand, is typically based on theory, generalizations and abstractions. The challenge is to combine the power of data analytics, with the justification of models for decision making. Moreover, as the world changes, it is likely that models need to be adjusted. How can models be adapted? In other words: we need *adaptive normative modeling*. In Section 2 we argue that value nets model the essential process flow ratios that underly the numerical relation between data analysis and process modeling.

Value nets can be viewed as instantiations of business process models useful for process diagnosis and norm analysis. Norm analysis is a foundational notion buttressing the planning and control cycles within organizations. Just as data analytics, business process modeling is interested in finding and describing structural patterns in data. In this sense structural means that the decision structure is explicitly captured. Value nets provides in a computational framework for the modeling and analysis of business processes that focuses on the normative ratios in enterprise behavior for the purpose of quality management, quality assurance, and measurement. Here do data-analytics and value nets modeling meet.

In Section 3 value nets will be modeled as extended Petri Nets that capture the flow of values in an enterprise. From this model an enterprise's normative cyclic behavior can be computed as so called tours. These tours contain the ratios of the process events can be factored into what Ijiri calls causal chains of events. Section 4 shows how the causal chains can be computed by factoring the value net's incidence matrices. In the case study in Section 5 it is shown how this is used to interpret the data for planning and control.

The case study focuses on the planning an control cycle of a public transport provider, providing schoolbus services to schoolchildren. The governance setting is similar to that described in [6]. This is a highly regulated domain; compliance plays an important role. As you can imagine, this involves a lot of data about transport movements, which may contain many hidden patterns of behavior. Some of these patterns are relevant to the auditing task: are the invoices legitimate? Other patterns are more relevant to management, when they want to improve efficiency. We have access to this data.

The research can be positioned as part of Business Process Management (BPM) [7,4]. Here, we observe the same contrast in attitude between modeling (forward looking; engineering) and data analytics (backward looking; empirical). Related to this, we made three observations about the way BPM has developed, which make it less suitable for the management accounting domain. First, much of the research effort in BPM has traditionally been focused on process models:

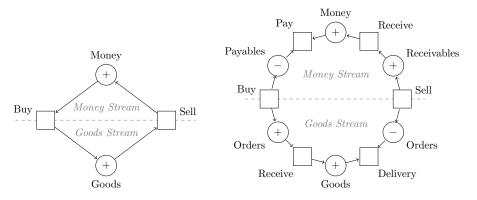


Fig. 1. Value Nets for trading companies. The horizontal dashed line splits the model into the *money flow* in the upper part and the *goods flow* in the lower part.

specification and verification of formal properties, such as termination or conformance to rules. This is done at design time. What has sometimes been lacking, is a look at real data, after execution, to see whether these generalizations apply in practice. Second, BPM models, in particular those based on work-flow, typically look at individual cases. However, in accounting and management, we are more interested in aggregated data: how many cases of a certain type per month? What was the use of resources? This aspect is also taken on by process mining. Third, BPM models tend to focus on the existence of dependencies between process steps or activities; we are just as much interested in the amount of value or the number of objects flowing along these links. In other words, we are not only interested in the control flow, but rather in the size of the flows.

The contribution of this paper is a computational framework for the modeling and analysis of business processes that focuses on the normative ratios in enterprise behavior for the purpose of quality management, quality assurance, and measurement. These normative ratios provide in the de jure models which can be compared with the de facto models extracted from the data using data analytics and vise versa

2 Business Process modeling and Data Analytics

The price mechanism ensures that a market price of a good and or service accurately summarizes the vast array of information held by market participants [3]. It needs no elaboration that the information aggregation characteristic of the pricing system buttresses many theories about the communicative function of prices and the decisions participants in the marketplace make to exploit business opportunities in the creation of value.

Management of enterprises decide upon a business model that depicts the transaction content, structure, and the governance designed so as to create value through the exploitation of business opportunities " [1]. The transaction content

refer to goods, services or the information exchanged, where the transaction structure defines the way parties i.e. agents participate in the exchange and how they are (inter)linked. Transaction governance refers to the legal form of organization, and to the incentives for the participants in transactions. Hence data, processes and governance are intertwined notions which are strongly related to each other.

More specifically a contract is an instantiation of a business model. Consequently a contract depicts the agreed upon content, structure, the incentives and the rules of conduct among parties involved in the contract. In the case a contract is executed the buyer receives goods or services from the vendor in the agreed upon quality and the buyer pays the vendor for the agreed upon price coined as value. Actually aforementioned paying mechanism is often referred to as the revenue model of the vendor. More specifically a revenue model refers to the specific modes in which a business model enables revenue generation.

Economic transactions as described above can be formally modeled as a value cycle which is interlinked to the business model of the enterprise i.e. the revenue model. This view is inspired by accounting models in the owner ordered accounting tradition [16] and value chain theory [13]. When used in computer science, often the purpose of these models is to analyze the representations of actions and events in a business process, and study their well-formedness. Consequently an intra-organizational workflow i.e. business processes can be modeled as a value net which provides a top-level view of an enterprise that focuses on the economic events equivalent to the value cycle of an enterprise. Figure 1 shows an elementary and a more elaborate example of a value cycle for a trading company. Each event is a transfer of value.

3 Value cycle and value Nets

A value net is modeled with a dimensioned Petri Net, extended with a place sign and a valuation, with some special structural characteristics that make it a good representation of the intra-organizational structure. Figure 2 shows the value cycle for the case study from the next sections.

Definition 1. A Value Net is a tuple (P, T, F, B, s, v) with

- -P a finite collection of places,
- -T a finite collection of transitions,
- $-F, B: P \times T \rightarrow \mathbb{N}$ the net's incidence matrices
- $-s: P \rightarrow \{\text{asset, liability}\}\$ the indicator for each place's sign, and
- $-v: P \rightarrow \mathbb{R} \ a \ valuation$

Additionally a value net has the following structural properties:

- The net is cyclic in the following sense. There is a special place labeled money from which it is possible to reach every other node in the model and that can be reached from every node in the model. Put differently, every node in the model is on a path from money to money.

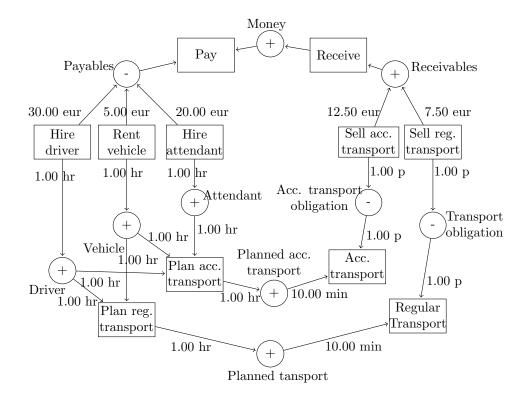


Fig. 2. Value Net for a transport provider.

 The net is divided in a money part and a production part. Place *money* is in the money part. The transitions in which the input places are in the money part and the output places in the production part are called *buy* transitions. The transitions in which the input places are in the money part and the output places in the production part are called *sell* transitions.

We use subscripts to differentiate the money and the production part. So, partitioning $P = P_{mon} \cup P_{prod}$ splits the places into the money part and the production part. Transitions in T_{mon} are between places the money part and transitions in T_{prod} between places in the production part. A T_{buy} transition consumes value from the money part and produces value in the production part. A T_{sell} transition consumes value from the production part and produces value in the money part.

A value net's semantics follows from the Petri Net semantics. A marking m is a $P \to \mathbb{N}$ vector that denotes the number of tokens in each place. A transition vector t is enabled when $F \cdot t \leq m$. When it fires it consumes $F \cdot t$ tokens and it produces $B \cdot t$ tokens. Let flow matrix G be defined by G = B - F. The effect of transition vector t is then the marker change computed as the matrix-vector product $G \cdot t$.

A value net extends a Petri net with a valuation, because for the computation of (normative) behavior in workflows it is essential to distinguish in monetary units and in product units. Just working with values leads to information loss, because value is the product of a valuation and a quantity.

$Value = Valuation \times Quantity$

A value net in monetary units has the advantage that everything is commensurable. It is even a requirement for the balancing of events in double bookkeeping [11,8] [14]. However, it is unsuitable for conservation laws because the value depends on a valuation that fluctuates. As soon as money is exchanged for a good this good gets exposed to price fluctuations. Only when the good is exchanged into money again later the effect on value becomes fixed. Exact connections in the product part is only feasible in product units. Therefore it is necessary to split values into valuations and quantities and model a business model in product units. A valuation is a linear property of a marking with a monetary unit of measurement. It gives the total economic value of the tokens in a dimensioned net. A weight vector v contains the value of a token for each place. Given a marking m the inner product $\langle v, m \rangle$ is the total value.

The other extension is the distinction between assets and liabilities. Each place in a value net represents either a positive or a negative value. In a value net the positive values flow anti-clockwise and the negative values flow clockwise. In many computations it is convenient to have all value flow in the same direction, and in these case we use an unsigned value net. Let matrix G be the flow matrix of a value net and let matrix S be a diagonal matrix with +1 for positive and -1 for negative places, then matrix $U = S \cdot G$ is the unsigned flow matrix, and the corresponding value net is called the unsigned value net. In an unsigned net the edges connected to a negative place are switched with respect to the signed net. An unsigned value net produces the same outcome but with signs switched. Multiplying with sign matrix S again gives the identical outcome.

4 Causal Chains of Events and Reconciliation Relations

A tour relates the behavior of an enterprise to the cyclic structure of its value net i.e. a process model as depicted in Figure 2. The cyclic structure is not directly apparent in behavior because transitions can fire at any point in the cycle. We want to group transactions that make up what Ijiri calls a causal chain of events [11]. In general terms such a chain of events is a variation on buying products and resources, producing an end product, and selling the product. The cycle starts with the consumption of money and ends with the production of money. Such cyclic behavior follows from the cyclic structure of a business process.

A formal notion of the tour concept was introduced for an audit context in [9]. It defines a tour as a constellation of events whose total effect is on money only. All other produced tokens are at some point consumed by another step in the process.

Place	$F_{\rm buy} \cdot t$	$B_{\rm buy} \cdot t$	$C \cdot t$	$F_{\text{sell}} \cdot t$	$B_{\text{sell}} \cdot t$
payables	$3500 \mathrm{~c}$	-	-	-	-
vehicle	-	$3600 \mathrm{\ s}$	-	-	-
driver	-	$3600 \mathrm{\ s}$	-	-	-
planned reg. transport	-	-	$3600 \mathrm{~s}$	-	-
reg. transport obligation	-	-		6 p	-
receivables	-	-	-	-	$4500~{\rm c}$
·					
		_			
Place	$F_{\mathrm{buy}} \cdot t$	$B_{\mathrm{buy}} \cdot t$	$C \cdot t$	$F_{\text{sell}} \cdot t$	$B_{\text{sell}} \cdot t$
Place payables	$\frac{F_{\rm buy} \cdot t}{5500 \ \rm c}$	$B_{\text{buy}} \cdot t$	$C \cdot t$	$F_{\text{sell}} \cdot t$ -	$B_{\text{sell}} \cdot t$
	b	$\frac{B_{\rm buy} \cdot t}{-}$ 3600 s	$C \cdot t$	$F_{\text{sell}} \cdot t$	$B_{\text{sell}} \cdot t$
payables	b	-	<u>C · t</u> - -	$F_{\text{sell}} \cdot t$	$B_{\text{sell}} \cdot t$
payables vehicle	b	- 3600 s	<u>C · t</u> - - -	$F_{\text{sell}} \cdot t$	$B_{\text{sell}} \cdot t$
payables vehicle driver	b	- 3600 s 3600 s	$\frac{C \cdot t}{\frac{1}{2}}$	$F_{\text{sell}} \cdot t$	$\frac{B_{\text{sell}} \cdot t}{-}$
payables vehicle driver attendant	b	- 3600 s 3600 s	- - -	<i>F</i> _{sell} · <i>t</i> - - - - 6 p	B _{sell} · t - - - -

Fig. 3. Factored tour for the transport case from Figure 2. From left to right the table shows the sacrificed value, the purchased products, the intermediate products, the produced products and the received value.

Definition 2. A transition vector t with firing result $m = G \cdot t$ is a tour when m[money] > 0 and m[x] = 0 for any place $x \neq \text{money}$.

Vector t is the number of times an events has to occur per tour. Every time such a tour has occurred the amount of money increases by m[money]. This increase is called the value jump. This formal tour concept captures the essence of the concept of value form a enterprise point of view.

With the unsigned value net we can reveal the various parts of a tour by factoring tour result $S \cdot U \cdot t$.

$$U = U_{\rm mon} + U_{\rm buy} + U_{\rm prod} + U_{\rm sell} \tag{1}$$

If we also split matrix U into B - F to see the difference between token production and token consumption then we have the factoring to express connections between the value cycle's parts. Let t be a tour. Unsigned tour result $U \cdot t$ is factored in a causal chain as follows

$$(B_{\rm mon} + B_{\rm buy} + B_{\rm prod} + B_{\rm sell} - F_{\rm mon} - F_{\rm buy} - F_{\rm prod} - F_{\rm sell}) \cdot t \qquad (2)$$

The various terms in this expression are the input and output of the different steps in the causal chain.

Grouping the money part and the production part immediately give the following equations:

$$(B_{\text{buy}} + B_{\text{prod}} - F_{\text{prod}} - F_{\text{sell}}) \cdot t = 0$$
(3)

$$(B_{\text{sell}} + B_{\text{mon}} - F_{\text{mon}} - F_{\text{buy}}) \cdot t = G \cdot t \tag{4}$$

The matrix in $(B_{\text{sell}} + B_{\text{mon}} - F_{\text{mon}} - F_{\text{buy}}) \cdot t$ is exactly the flow matrix filtered for the production places and does not contain the money place. Because t is a tour, the product must therefore be zero according to Definition 2. The second equation then follows immediately because the tour result $G \cdot t$ must result from the other parts of Equation 2.

The first equation is Kirchhof's law for the production process. It shows that in a tour the input and output of production cancel. The second equations shows the value jump. It is the difference between monetary value produced and consumed.

Figure 3 shows the chains of events for the transport service provider from the case study. In this case there are two tours; one for regular transport, and one for accompanied transport. The case study focuses on the tour for regular transport.

5 Case Study: Public Transport Services

In 2010 a transport provider has agreed to provide so called taxi-bus transport services for secondary school pupils, who need to attend special schools, which are at a distance. For transport of this group of passengers, many rules and regulations apply. In the request for proposals, the municipalities have laid down a number of requirements for the contract, concerning maximal waiting times, durations, and routing. Every month, the transport provider sends an invoice with a data file, detailing the number of trips, passenger details, routes, departure and arrival times, departure and arrival addresses, ordered and canceled trips etc. This data file provides evidence of the services delivered for the invoice to be paid by the municipalities.

The contract specifies the following revenue model. Hence a revenue model refers to the specific modes in which a business model enables revenue generation. Based on trip requests received from the municipality specifying travel specifications from A to B at time t, at week day w, the routing software package used by the transport provider calculates the daily routes at each time of day, entailing trip requests i.e. the number of school pupils per route. Parties agreed that the routing software package is used to predict the best route and duration, for the sake of the contract. This prediction is based on standard speeds (100 km/h for motorways, 70 km/h for main roads, 40 km/h in town etc.). For each route, this gives occupied vehicle hours, which is priced at a certain tariff: revenue per route = price per occupied hour * occupied vehicle hours. This completes the description of the revenue model.

The price per occupied hour is the result of a normative decision made by management of the transport provider offering their quote in competing for the contract in 2010. No need to say that the quotation is a best estimate of expected outcome for providing future transport services. Within the context of the planning and control cycle of the transport provider the actuals i.e. the realized data are gathered on a daily basis for monitoring purposes. Deviations to the norm are very interesting from an operational perspective because operational man-

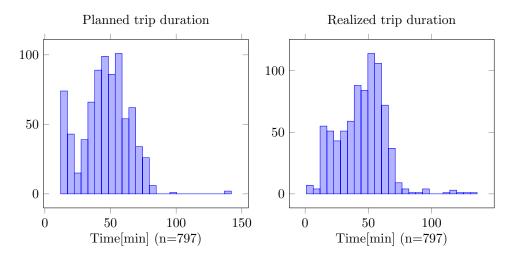


Fig. 4. Planned and realized trip durations. The average planned duration is 46.42 min, and the average realized duration is 45.10 min.

agement learns about the quality of planning operations and more over whether the norms used in calculating the price per occupied hour is correct. From a financial perspective the same information is very valuable in assessing whether the contract is profitable or not and learns the quality of the normative decision processes made by management.

In figure 2 the original value net buttressing the initial price bid is depicted. Factorization of the tour of the transport provider gives us the causal chain of events representing the normative representation of the inflows and outflows of business processes of the transport provider, revealing their normative ratios i.e. KPIs as input measures for analyzing deviations to the normative ratios. Hence on the right hand side we see that the sell of regular transport i.e. one trip from A to B at time t, at week day w, leads to a transport obligation. This transport obligation leads to planned transport measured in one hour. The value net shows that one trip equals 10minutes drive so the occupancy rate is easily derived from these ratios. On average six trips make up one planned vehicle route. In general the occupancy rate of a vehicle is a key performance indicator for analyzing operational planning effectiveness, analyzing profitability of the contract at hand, and for most whether norms actually used in calculating pricing bids are sound. We refer to figure 3 for detailed exposition of the factored tours. Note that the value net is the instantiation of the revenue model of the contract and reveals the value creation within organizations.

We have access to the data detailing the number of trips, passenger details, routes, departure and arrival times, departure and arrival addresses, ordered and canceled trips etc. The period we studied comprised one month period. In table 1 we give some preliminary figures of the data file en the computational results applying the value net logic as explained in section three and four on the planned

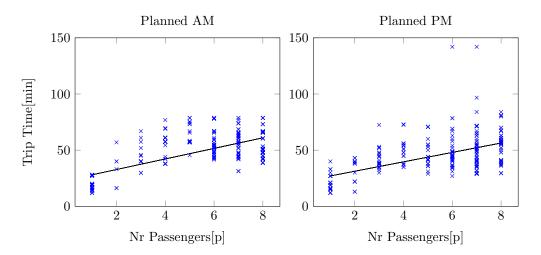


Fig. 5. Scatter plots for the planned trip durations in the morning (AM) and in the afternoon (PM).

data and realized data in that month. The occupancy rate is the reciprocal of the KIP, converted to hours.

	Planned	Planned	Realized	Realized
	AM	\mathbf{PM}	AM	\mathbf{PM}
Mean	48.46 min	47.36 min	43.88 min	42.28 min
Std Dev	$17.57 \min$	$19.06 \min$	$18.00 \min$	$16.83 \min$
KPI	9.07 min/p	$8.86 \min/p$	$8.73 \min/p$	8.41 min/p
Occupancy	6.62 p/hr	$6.77 \mathrm{\ p/hr}$	$6.87 \mathrm{~p/hr}$	$7.13 \mathrm{~p/hr}$

The occupied vehicle hour planned and realized are depicted in figure 4. The average per route differs only slightly from each other, here 1.32 min. When we combine this insight with the KPI and the occupancy rate then we may conclude from a financial and operational perspective that we have met our objective(s). Nothing has to be done. But if we look more closely to the data and make a more detailed analysis by splitting up the data in routes performed in the morning and in the afternoon than we must revise our opinion. The morning routes are more problematic as we thought. The overall performance in the afternoon is certainly better compared with the morning routes. The value net analysis gives us a different tour results. In this case financial management may still be content with the results but operations has now a motivation to look into the data asking why the overall performance in the afternoons is better than the morning routes. If we take a closer look at the planned data AM depicted in Fig 5 and compare them with the realized data AM depicted in Fig 6 than we see that the scatter plot of the KPI minutes per person per route is quiet informative. In the case we compare these data with the planned and realized data PM than there are two things that are noteworthy. First we see that the morning routes show similar

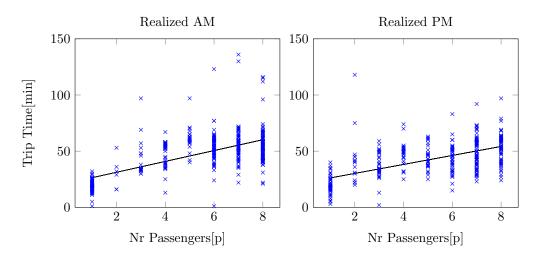


Fig. 6. Scatter plots for the realized trip durations in the morning (AM) and in the afternoon (PM).

patters as the afternoon. After analyzing these data it turned out that traffic jams caused delays in the morning. Secondly after analyzing daily patterns it turned out that only two days in the week caused real delays. The planning department did not optimize routes for each route per day part. Rationally we would expect that the schedules were altered. In practice the schedules were kept on a weekly (average) basis. The value net analysis revealed that when we analyze the tours that actually one trip contains the following information: a school pupil travels from A to B at time t, at week day w. So the unit of analysis is a weekday, at time t, per school pupil which gave the correct information.

6 Conclusion

In the introduction we mentioned three aspects of BPM, which make it less suitable for the financial domain: (i) little empirical verification of applicability of models, (ii) focus on individual cases, instead of aggregate flows, (iii) focus on causal links, neglecting how much value or how many goods flow over the links.

There is an analogy between our approach and process mining [15]. Process mining also combines the exploratory and empirical aspects of data mining, with the use of linear models of processes, to analyze their formal properties. Process mining can – to a certain extend – also deal with the size of flows and use of resources. However, what it can't do is express the amount of value or number of object that flow or should flow over a link. That means that in normative settings, also process mining is left to analyze whether a process follows the procedure; it can't identify deviations in the (financial) content of transactions.

In section 3 and 4 we elaborated on a computational approach for analysis of business processes models that focuses on the normative ratios in enterprise

behavior for the purpose of quality management, quality assurance, and measurement. As shown in the case analysis the value net approach provides in the de jure models which can be compared with the de facto models extracted from the data using data analytics and vice versa. This approach seems very fruitful for modeling adaptive mechanisms buttressing planning and control cycles within organizations.

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