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# Rotor speed, load torque and parameters estimations of a permanent magnet synchronous motor using extended observer forms

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**Abstract:** This paper presents a new approach to estimate states and parameters of the permanent magnet synchronous motor (PMSM) in the presence of unknown load torque disturbance. Indeed, it highlights an auxiliary dynamics that is added to the PMSM model. Thereby, it supplies a change of coordinates that transforms the extended model (PMSM model together with the auxiliary dynamics) into the so-called extended nonlinear observer forms. It turns out that the obtained form provides two observer normal forms. One of them supports the well-know high gain observer which is used to estimate the speed and the unknown load torque. The second one which is affine in the parameters enables us to estimate simultaneously the states and motor parameters by means of an adaptive observer. Performance of the proposed method is illustrated by the simulations results.

## 1. Introduction

Permanent magnet synchronous motor (PMSM) is increasingly used in several industrial applications, like renewable energy and Maglev (derived from magnetic levitation) due to some important features as stability, high power density and large torque to current ratio (see [56], [28]). However, the control of the PMSM is a complex task due to the presence of nonlinearities in its model, load torque perturbations and wear occurring in mechanical sensors. This leads to, inter alia, to the development of sensorless control method which can be represented by three main approaches: the first one is based on back-emf technique (see [37], [35], [17]) whose advantage is the simplicity of implementation. However, it should be avoided in low speed region because on one hand, the velocity and the flux estimation are sensitive to the stator resistance and on the other hand, the noise amplitude is higher than back-emf. The second one is the model reference adaptive system (MRAS) method in which the performance depends on the accuracy of the used model (see [29], [41]). Finally, the third one, that will be considered in

this paper, is based on the observer design as in ([54], [6]).

A high performance of a control requires measurement and identification of unknown parameters. Therefore, an accurate estimation of unavailable state variables is needed. In order to reach this goal, numerous observer strategies have been developed to estimate states and parameters of PMSM. Among these strategies, for example, we find exact linearization method (see [13], [55]), adaptive observer [3], sliding mode observer (see [18], [24], [40]) and extended Kalman filter (see [4], [21]).

Another interesting approach can be found in [19] where an adaptive interconnected observer has been proposed. Indeed, the dynamical system of PMSM was considered as an interaction between two sub dynamical systems: one contains the dynamics of current  $i_q$ , the stator resistance  $R_s$  and the second one is governed by the dynamics of current  $i_d$ , angular speed  $\Omega$  and load torque  $T_l$ .

In the literature, several works dealt with observer design for PMSM on the one hand, to avoid the use of tachometer which is expensive, on the other hand to ensure redundancy of information. However, few solutions have been proposed to estimate simultaneously the states, disturbances and all parameters which are required for example in diagnosis and fault-tolerant control.

When a nonlinear dynamical system contain unavailable states for measurements, then it becomes a difficult task to directly apply the standard estimation methods, such as the Luenberger's observer or Kalman's filter. To overcome this problem, Krener and Isidori [25] introduced the so-called linearization of a nonlinear dynamical system by means of a change of coordinates and output injection. This enables us to use the well-known Luenberger's observer for a class of nonlinear dynamical systems. Since, this concept has been used to the multiple outputs case in ([26] and [48],[10],[53]). Afterwards, several variations have been developed in ([23], [39], [43], [20], [33], [8], [31] and [30],[51],[50],[5]). Another interesting form is the so-called output depending observer normal form introduced by [42], and then improved in [52] and [47]. Recently, the so-called extended observer normal form was proposed in ([22], [36], [2], [7] and [9]). The idea behind this normal form is based on adding an auxiliary dynamics into the original system such that the augmented system fulfills conditions which guarantee the transformation of the nonlinear system into an extended observer normal form. Even now, the process of transforming nonlinear dynamical systems into nonlinear observer forms for a wide class of nonlinear systems keep going, as the extended output depending observer normal form is proposed in [44].

This paper presents a new approach to design nonlinear observer for PMSM by adding an auxiliary dynamics to the PMSM model such that the augmented dynamical system admits a change of coordinates that brings it into the so-called extended nonlinear observer form. This form enables us to effectively solve the problem of simultaneous estimation of PMSM states and parameters [32]. The present work improves our previous work in [45]. In fact, [45] is focused only on reaching the observer normal forms for PMSM model. Hence simulations were rather did for the mathematical model without worrying on the physical model. Therefore it contains many gaps and defects. The present study is more complete because it addresses the problem of estimation of rotor position, rotor speed, load torque, currents and the parameters. The quality of the estimation, despite the presence of a varying load torque, shows the robustness of this method to the disturbances.

The rest of this paper is organized as follows. Section 2 presents the dynamical model of PMSM and recalls the extended nonlinear observer form structure. Section 3 states an auxiliary dynamics and a change of coordinates that transforms the extended PMSM model into an extended nonlinear observer form. Furthermore, two cases will be considered. The first one assumes the knowledge of the parameters of PMSM and the second one does not. Section 4 provides simulation results that highlight the efficiency of the proposed approach.

## 2. Mathematical model of PMSM and Problem statement

In order to obtain a dynamical model that makes simpler the estimation and control procedures, the electrical and mechanical dynamics of PMSM are expressed in the direct and quadrature axis reference frame by transforming the original three-phase on two fictitious phase winding. The direct phase winding is parallel to the rotor magnet flux, and the quadrature phase winding is orthogonal to the direct phase winding. Based on states observer, the sensorless control system is shown in Figure 1. To avoid the flux weakening, a zero electric current is required in the direct phase, then the generated electromagnetic torque is given as product of constant permanent flux and electric current in direct axis. In this present study, we consider the cases of non-saliency rotor ( $L_d = L_q = L_s$ ). The dynamical model of PMSM in  $(d - q)$  reference frame is governed by the following set of differential equations (see [12], [38]).

$$\frac{d\theta}{dt} = \Omega \quad (1)$$

$$\frac{d\Omega}{dt} = -\frac{f_v}{J}\Omega + \frac{p}{J}\phi_f i_q - \frac{1}{J}T_l \quad (2)$$

$$\frac{di_d}{dt} = -\frac{R_s}{L_s}i_d + p\Omega i_q + \frac{1}{L_s}u_d \quad (3)$$

$$\frac{di_q}{dt} = -\frac{R_s}{L_s}i_q - p\Omega i_d - p\frac{1}{L_s}\phi_f\Omega + \frac{1}{L_s}u_q \quad (4)$$

where  $[\theta, \Omega, i_d, i_q]^T$  represents the state vector of the rotor angular position, the rotor mechanical speed and the output currents respectively.  $u_d$  and  $u_q$  are the  $d$ -axis and  $q$ -axis stator voltages, respectively.  $T_l$  is the load torque considered as an unknown exogenous input,  $R_s$  is the winding stator resistance,  $L_s$  is the stator winding inductance,  $\phi_f$  is the permanent magnet flux,  $p$  is the number of the pair poles,  $J$  is the rotor moment of inertia, and  $f_v$  is the viscous friction coefficient.

Within this work it is assumed that the available measurements are the currents in the  $(d - q)$  reference frame. Thus, the outputs of the above dynamical system are as follows

$$y_1 = h_1 = i_d \quad (5)$$

$$y_2 = h_2 = i_q \quad (6)$$

Motivated by the design of robust controllers and faults diagnosis strategies for PMSM applications, as it has been widely mentioned above in the introduction, many works had been conducted to estimate states and parameters of PMSM. Hereafter, we will present a new approach which enables us to:

- estimate the rotor mechanical speed  $\Omega$  and the load torque  $T_l$  when all motor parameters are known;
- estimate the parameters  $R_s$ ,  $L_s$  and  $\phi_f$ ;
- consequently, estimate the rotor angular position from the estimation of rotor mechanical speed.

In order to design an observer for PMSM model, it is necessary to study its observability property. This also enables us to effectively manage the loss of observability when it occurs. The rank observability conditions (see for example [25]) for system (2)-(4) together with the measurable outputs  $i_d$  and  $i_q$  is the rank of the following observability matrix

$$\mathcal{O} = \begin{bmatrix} dh_1 \\ dh_2 \\ dL_f h_1 \\ dL_f h_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ pi_q & -\frac{R_s}{L_s} & p\Omega \\ -\left(\frac{p}{L_s}\phi_f + pi_d\right) & -p\Omega & -\frac{R_s}{L_s} \end{bmatrix}$$

A straightforward calculation shows that the rank of the above matrix is

$$\text{rank}(\mathcal{O}) = 3$$

if and only if  $i_d \neq -\frac{\phi_f}{L_s}$  or  $i_q \neq 0$ .

As stated in the introduction, we will use the well-known estimation approach based on the extended nonlinear observer normal form. In order to do so, let us recall the basic idea and some properties of this concept. Consider the following nonlinear system

$$\begin{cases} \dot{x} = f(x) \\ y = h(x) \end{cases} \quad (7)$$

where  $x \in U \subseteq \mathbb{R}^n$  represents the state and  $y \in \mathbb{R}$  denotes the outputs. We assume that the vector field  $f$  and the output function  $h$  are sufficiently smooth. It is also assumed that the pair  $(h, f)$  satisfies the observability rank condition. Thus, the 1-forms  $\theta_i = dL_f^{i-1}h$ , for  $1 \leq i \leq n$ , are linearly independent where  $L_f^{i-1}h$  is the  $(i-1)^{th}$  Lie derivative of the output  $h$  along the vector field  $f$  and  $d$  the usual differential.

When system (7) cannot be transformed into an observer normal form by means of the so-called linearization by output injection [25], then we can overcome this limitation by using the immersion concept (dynamics extension). The idea is to seek for an auxiliary dynamics  $\dot{w} = \eta(y, w)$  with  $w \in \mathbb{R}$  so that the following extended dynamical system

$$\dot{x} = f(x) \quad (8)$$

$$\dot{w} = \eta(y, w) \quad (9)$$

$$y = h(x) \quad (10)$$

might be transformed by means of a change of coordinates  $(\xi^T, w)^T = \phi(x, w)$  to the following extended observer normal form

$$\dot{\xi} = A(y, w)\xi + B(y, w) \quad (11)$$

$$\dot{w} = B_{n+1}(y, w) \quad (12)$$

$$y = C\xi \quad (13)$$

where  $w \in \mathbb{R}$  is an auxiliary variable considered as an extra output,  $C = [0, \dots, 0, 1]$ , and

$$A(y, w) = \begin{pmatrix} 0 & & \dots & 0 \\ \alpha_2(y, w) & \ddots & & \vdots \\ 0 & \ddots & \ddots & \\ \vdots & & & \\ 0 & \dots & 0 & \alpha_n(y, w) & 0 \end{pmatrix} \quad (14)$$

where  $\alpha_i(y, w) \neq 0$  for  $2 \leq i \leq n$  are functions depending only on the output  $y$  and the extra output  $w$ .

This extended nonlinear observer normal form supports the high gain observer to estimate the state of nonlinear dynamical system [11]. Moreover, in the case of PMSM model we will see that  $B(\cdot)$  is affine on some parameters, therefore the adaptive observer can be used to estimate simultaneously the states and the parameters (see [32], [49]).

### 3. Change of coordinates and Extended dynamics

This section provides an auxiliary dynamics together with a change of coordinates that transforms the extended PMSM model (2)-(4) into an extended nonlinear observer normal form.

Let us consider the equations (2)-(4) together with an auxiliary dynamics presented as follows

$$\dot{x}_1 = -\frac{f_v}{J}x_1 + \frac{p}{J}\phi_f x_3 - \frac{1}{J}T_l \quad (15)$$

$$\dot{x}_2 = -\frac{R_s}{L_s}x_2 + px_1x_3 + \frac{1}{L_s}u_d \quad (16)$$

$$\dot{x}_3 = -\frac{R_s}{L_s}x_3 - px_1x_2 - p\frac{1}{L_s}\phi_f x_1 + \frac{1}{L_s}u_q \quad (17)$$

$$\dot{w} = \eta(y, w) \quad (18)$$

$$y_1 = x_2$$

$$y_2 = x_3$$

where  $[x_1, x_2, x_3] = [\Omega, i_d, i_q]$ ,  $y = [y_1, y_2]^T$  and  $w$  is an auxiliary variable considered as an extra output.

Then the following result provides the sought auxiliary dynamics  $\eta(y, w)$  and the change of coordinates.

**Theorem 1.** For any function  $\kappa(w) \neq 0$ , consider the following auxiliary dynamics

$$\dot{w} = -\frac{f_v}{J}\kappa(w) \quad (19)$$

Then, the following change of coordinates

$$z_1 = x_1 e^{-\int_0^w \frac{1}{\kappa(s)} ds} \quad (20)$$

$$z_2 = x_2 \quad (21)$$

$$z_3 = x_3 \quad (22)$$

transforms the nonlinear dynamical system (15)-(17) into following form

$$\dot{z}_1 = \alpha_1(w) T_l + \beta_1(y, w) \quad (23)$$

$$\dot{z}_2 = \alpha_2(y, w) z_1 + \beta_2(y, w) + \frac{1}{L_s} u_d \quad (24)$$

$$\dot{z}_3 = \alpha_3(y, w) z_2 + \beta_3(y, w) + \frac{1}{L_s} u_q \quad (25)$$

where

$$\alpha_1(y, w) = -\frac{1}{J} e^{-\int_0^w \frac{1}{\kappa(s)} ds}, \quad \alpha_2(y, w) = p y_2 e^{\int_0^w \frac{1}{\kappa(s)} ds}, \quad \alpha_3(y, w) = -p \left( y_1 + \frac{\phi_f}{L_s} \right) e^{\int_0^w \frac{1}{\kappa(s)} ds},$$

$$\beta_2(y, w) = -\frac{R_s}{L_s} y_1, \quad \beta_1(y, w) = \frac{p}{J} \phi_f y_2 e^{-\int_0^w \frac{1}{\kappa(s)} ds}, \quad \text{and} \quad \beta_3(y, w) = -\frac{R_s}{L_s} y_2$$

**Remark 1.** • Function  $\kappa(w) \neq 0$  is a free function. It has to be chosen in order to ensure the boundedness the auxiliary dynamics. For our goal, more details on this function are given in simulation section.

- As it can be seen, the PMSM outputs considered here are continuous-time measurements. In practice, with high frequency sampling device, the measurement can be considered as continuous. Even with a very low sampling sensor (rarely), thanks to our observer normal form, it is easy to adapt the observer proposed in [16] to work with sampled-outputs.

*Proof.* The time derivative of the state variable  $z_1$  given in equation (20) leads to

$$\dot{z}_1 = -\frac{1}{J} e^{-\int_0^w \frac{1}{\kappa(s)} ds} T_l + \frac{p}{J} \phi_f y_2 e^{-\int_0^w \frac{1}{\kappa(s)} ds}. \quad (26)$$

by setting

$$\alpha_1(y, w) = -\frac{1}{J} e^{-\int_0^w \frac{1}{\kappa(s)} ds}$$

$$\beta_1(y, w) = \frac{p}{J} \phi_f y_2 e^{-\int_0^w \frac{1}{\kappa(s)} ds}$$

hence equation (26) can be rewritten in the following form:

$$\dot{z}_1 = \alpha_1(y, w) T_l + \beta_1(y, w)$$

Now by replacing the variables  $x_2 = z_2$  and  $x_3 = z_3$  in equations (16) and (17), we obtain

$$\begin{aligned}\dot{z}_2 &= py_2 e^{\int_0^w \frac{1}{\kappa(s)} ds} z_1 - \frac{R_s}{L_s} y_1 + \frac{1}{L_s} u_d \\ &= \alpha_2(y, w) z_1 + \beta_2(y, w) + \frac{1}{L_s} u_d\end{aligned}\quad (27)$$

and

$$\begin{aligned}\dot{z}_3 &= -p \left( y_1 + \frac{\phi_f}{L_s} \right) e^{\int_0^w \frac{1}{\kappa(s)} ds} z_2 - \frac{R_s}{L_s} y_2 + \frac{1}{L_s} u_q \\ &= \alpha_3(y, w) z_2 + \beta_3(y, w).\end{aligned}\quad (28)$$

Putting together equations (26), (28) and (29) to meet the stated nonlinear extended normal form of the above theorem.  $\square$

Now, to design an observer for PMSM relying on the extended nonlinear observer normal form obtained in Theorem 1, two case studies are to be considered:

- estimation of rotor mechanical speed  $z_1 = \omega$  and the load torque  $T_l$  by using equations (23) and (25) where all parameters of the system are assumed known,
- estimation of  $z_1 = \omega$ , the load torque  $T_l$ , and the parameters  $R_s$ ,  $L_s$  and  $\phi_f$  by involving dynamics (23) and (24).

**Remark 2.** As proposed in [19], we will use the following observer to estimate the rotor position

$$\frac{d\hat{\theta}}{dt} = \hat{\Omega} + L_\theta (i_q - \hat{i}_q) \quad (29)$$

where  $L_\theta$  is a positive constant design parameter.

### 3.1. Estimation of motor speed and the load torque with the known parameters

In this subsection, it is assumed that all parameters of PMSM are known and the load torque  $T_l$  is a piece-wise constant function ( $\dot{T}_l = 0$ ). Now, in order to estimate the rotors speed and the load torque, we consider equation  $\dot{T}_l = 0$  together with dynamics  $\dot{z}_1$  and  $\dot{z}_3$  obtained from the extended nonlinear observer form given in Theorem 1. Thus, we consider the following nonlinear extended observer normal form:

$$\begin{aligned}\begin{bmatrix} \dot{T}_l \\ \dot{z}_1 \\ \dot{z}_3 \end{bmatrix} &= \begin{bmatrix} 0 & 0 & 0 \\ \alpha_1(y, w) & 0 & 0 \\ 0 & \alpha_3(y, w) & 0 \end{bmatrix} \begin{bmatrix} T_l \\ z_1 \\ z_3 \end{bmatrix} + \begin{pmatrix} 0 \\ \beta_1(y, w) \\ \beta_3(y, w, u) \end{pmatrix} \\ \dot{w} &= -\frac{f_v}{J} \kappa(w)\end{aligned}\quad (30)$$

where  $\alpha_1(y, w) = -\frac{e^{-\int_0^w \frac{1}{\kappa(s)} ds}}{J}$ ,  $\alpha_3(y, w) = -p \left( y_1 + \frac{\phi_f}{L_s} \right) e^{\int_0^w \frac{1}{\kappa(s)} ds}$ ,  $\beta_1(y, w) = \frac{p}{J} \phi_f y_2 e^{-\int_0^w \frac{1}{\kappa(s)} ds}$  and  $\beta_3(y, w, u) = -\frac{R_s}{L_s} y_2 + \frac{1}{L_s} u_q$

In order to design an observer for the above dynamical system, let us set:



$$A(y, w) = \begin{bmatrix} 0 & 0 & 0 \\ \alpha_1(y, w) & 0 & 0 \\ 0 & \alpha_3(y, w) & 0 \end{bmatrix} \text{ and } B(y, w, u) = \begin{pmatrix} 0 \\ \beta_1(y, w) \\ \beta_3(y, w, u) \end{pmatrix}.$$

Now, as in [11], let us consider this new dynamical system:

$$\begin{aligned} \dot{\hat{\xi}} &= A(y, w)\hat{\xi} + B(y, w, u) - \Gamma^{-1}(y) R_\rho^{-1} C^T (C\hat{\xi} - y) \\ 0 &= \rho R_\rho + G^T R_\rho + R_\rho G - C^T C \end{aligned} \quad (31)$$

where

$$G = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \Gamma(y, w) = \begin{pmatrix} \alpha_1 \alpha_3 & 0 & 0 \\ 0 & \alpha_3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and from the last algebraic equation we can deduce that:

$$R_\rho = \begin{bmatrix} \frac{6}{\rho^5} & \frac{-3}{\rho^4} & \frac{1}{\rho^3} \\ \frac{-3}{\rho^4} & \frac{2}{\rho^3} & \frac{-1}{\rho^2} \\ \frac{1}{\rho^3} & \frac{-1}{\rho^2} & \frac{1}{\rho} \end{bmatrix}$$

It is worth noting that the term  $-\Gamma^{-1}(y) R_\rho^{-1} C^T$  has the following simple expression:

$$\left[ -\frac{\rho^3}{\alpha_1 \alpha_3}, -3\frac{\rho^2}{\alpha_3}, -3\rho \right]^T.$$

It is well-known from [11] that if the output  $y$  and the extra-output  $w$ , functions  $\alpha_1(y, w)$  and  $\alpha_3(y, w)$  are bounded, then the dynamical system (31) is an observer for (26). Thus the error observation  $e = \hat{\xi} - \xi$  which is governed by the following dynamics:

$$\dot{e} = \dot{\hat{\xi}} - \dot{\xi} = (A(y, w) - \Gamma^{-1}(y, w) R_\rho^{-1} C^T C) e = \bar{A}(y, w) e$$

where the matrix  $\bar{A}(y, w)$  has the following expression  $\bar{A}(y, w) = \begin{pmatrix} 0 & 0 & -\frac{\rho^3}{\alpha_1 \alpha_3} \\ \alpha_1 & 0 & -3\frac{\rho^2}{\alpha_3} \\ 0 & \alpha_3 & -3\rho \end{pmatrix}$ , is

exponentially stable by an appropriate choice of  $\rho$ .

As in many works that deal with high gain observers, in our normal form we don't need to do any further assumption (as Lipschitz condition) on  $B(y, w, u)$  because it depends only on measurable quantities.

### 3.2. Estimation of motor speed, the load torque and identification parameters

In this subsection, it is assumed that parameters  $\phi_f$ ,  $L_s$  and  $R_s$  are unknown. Then, in order to deal with simultaneously estimation of states and parameters of dynamical system (2)-(4), we consider equation  $\dot{T}_l = 0$  together with dynamics  $\dot{z}_1$  and  $\dot{z}_2$  obtained from the extended nonlinear observer forms given in Theorem 1. Thus, we consider the following extended nonlinear observer forms

$$\begin{cases} \dot{T}_l = 0 \\ \dot{z}_1 = -\frac{1}{J}e^{-\int_0^w \frac{1}{\kappa(s)} ds} T_l + \frac{p}{J}\phi_f y_2 e^{-\int_0^w \frac{1}{\kappa(s)} ds} \\ \dot{z}_2 = p y_2 e^{\int_0^w \frac{1}{\kappa(s)} ds} z_1 - \frac{R_s}{L_s} y_1 + \frac{1}{L_s} u_d \\ \dot{w} = -\frac{f_v}{J}\kappa(w) \end{cases}$$

It is clear that this form is of the following adaptive observer form (affine on parameters)

$$\begin{cases} \dot{z} = A(w, y, u)z + \phi(w, u, y)\delta \\ y = Cz \end{cases} \quad (32)$$

where  $z = [T_l \quad z_1 \quad z_2]^T$ ,  $\delta = [\phi_f \quad \frac{R_s}{L_s} \quad \frac{1}{L_s}]^T$

$$A(w, y) = \begin{bmatrix} 0 & 0 & 0 \\ -\frac{1}{J}e^{-\int_0^w \frac{1}{\kappa(s)} ds} & 0 & 0 \\ 0 & p y_2 e^{\int_0^w \frac{1}{\kappa(s)} ds} & 0 \end{bmatrix} \quad (33)$$

and

$$\phi(w, u, y) = \begin{bmatrix} 0 & 0 & 0 \\ \frac{p}{J}y_2 e^{-\int_0^w \frac{1}{\kappa(s)} ds} & 0 & 0 \\ 0 & -y_1 & u_d \end{bmatrix}$$

Now, as in [32, 49], let us consider the following system:

$$\dot{\hat{z}} = A(y, w)\hat{z} + \phi(w, u, y)\hat{\delta} + \{\Lambda S_\delta^{-1} \Lambda^T C^T + S_z^{-1} C^T\} \Sigma (y - C\hat{z}) \quad (34)$$

$$\dot{\hat{\delta}} = S_\delta^{-1} \Lambda^T C^T \Sigma (y - C\hat{z}) \quad (35)$$

where  $\Lambda$ ,  $S_z$ ,  $S_\delta$  are respectively the solutions of the following equations:

$$\begin{aligned} \dot{\Lambda} &= \{A(y, w) - S_z^{-1} C^T C\} \Lambda + \phi(w, u, y) \\ \dot{S}_z &= -\rho_z S_z - A(y, w)^T S_z - S_z A(y, w) + C^T \Sigma C \\ \dot{S}_\delta &= -\rho_\delta S_\delta + \Lambda^T C^T \Sigma C \Lambda, \quad S_z(0), S_\delta(0) > 0 \end{aligned}$$

where  $\rho_z$  and  $\rho_\delta$  are positive constants.

Under the following persistence of excitation condition: there exist  $\lambda > 0$  and  $\sigma > 0$  such that

$$\lambda I \leq \int_t^{t+T} \Lambda(\tau)^T C^T \Sigma(\tau) C \Lambda(\tau) d\tau \leq \sigma I,$$

it has been proven in [32, 49] that dynamical system (34)- (35) is an observer for dynamical system (32). Thus, it enables us to estimate exponentially the state  $z$  and the parameters  $\delta$  for system (32). To validate the theoretical part of proposed approach, we will present the numerical results that reflect the good dynamics performance and fast estimation convergence to their real trajectories.

| Parameter                   | Value                    | Parameter                              | Value       |
|-----------------------------|--------------------------|--|-------------|
| Stator resistance ( $R_s$ ) | 2.875 $\Omega$           | Number of pole pairs ( $p$ )           | 3           |
| Stator inductance ( $L_s$ ) | 8.5 mH                   | Rotor magnetic flux ( $\phi_f$ )       | 0.175 Wb    |
| Moment of inertia ( $J$ )   | 0.00003 kgm <sup>2</sup> | Viscous friction coefficient ( $f_v$ ) | 0.0034 Nm/s |

**Table 1** TABLE MOTOR PARAMETERS

#### 4. Simulation results

The Table-I provides mechanical/electrical data for PMSM model used in simulation. The structure sensorless control system is depicted in Fig.1. The structure sensorless control system

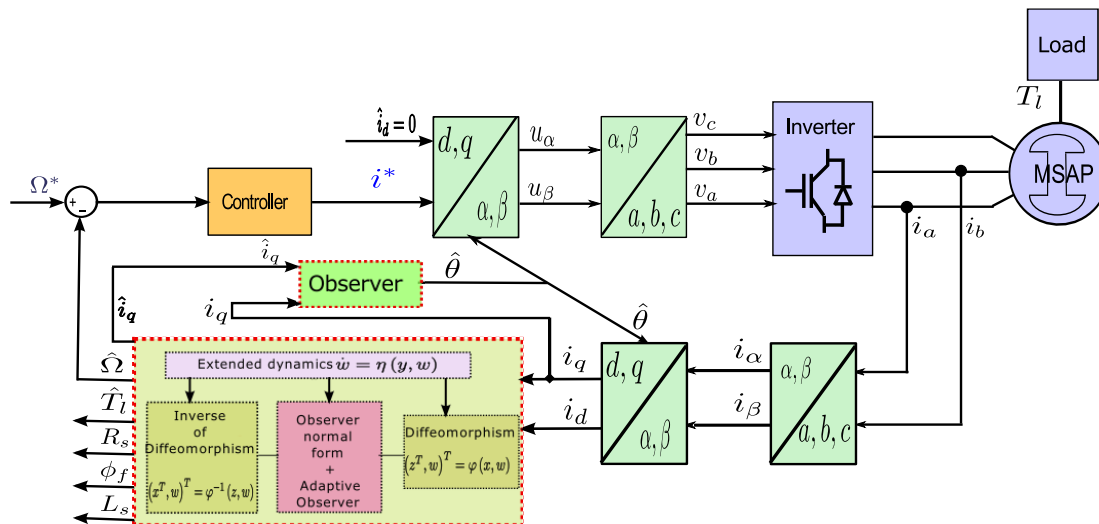


Fig. 1: Implementation of an Adaptive observer through a change of coordinates.

is depicted in Fig.1, where a zero electric current is setting in the direct phase  $i_d^*$  because the flux does not need to be controlled since it is constant in the case of non-saliency rotor and in order to optimize the torque-to-current ratio. In order to ensure the boundedness of extended dynamics (19), we take  $\kappa(w) = \frac{\sin^2 aw}{(aw)^2} \neq 0$  for  $|w| < \frac{\pi}{a}$ . The parameter  $a$  is chosen such that the simulation interval where  $w$  is bounded as wide as we want. In the present study, we take  $a = 0,0005$  to have an interval  $[-5, 5]$  (see Fig.2 below). In order to verify the efficiency of the proposed approaches, we use the input proposed in [15] to excite the motor under low and high transitions of rotor speed such that the observability and the PE conditions can be satisfied.

**Remark 3.** Fig.1 presents the general structure of observer-based control of motor. However, this paper deals only with the observer design problem and no controller has been investigated. Different kinds of controllers have already been proposed in the literature, such as observer-based sliding mode controller [46, 19], where the stability of the closed-loop system has been studied in the theoretical and practical situations. Interested reader can refer to the references therein.

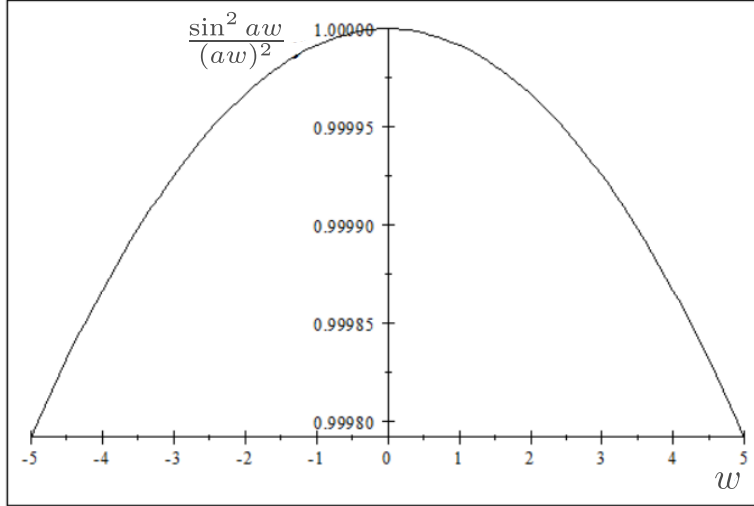


Fig. 2: Shape of  $\frac{\sin^2 aw}{(aw)^2}$ .

#### 4.1. Simulation results with high gain observer

When the parameters are assumed known, we consider the observer normal form (30) and the high gain observer (31) with  $\rho = 40$ . The efficiency of the proposed approach which estimates the rotor position, rotor speed and load torque is shown in Fig.3 - Fig.4.

To better highlight the role of the proposed observer normal form, we perform a comparison with sliding mode observer (SMO) [24], [40]. Although, for some classes of nonlinear systems particularly in presence of faults and uncertainties, the SMO is combined to the high gain observer to bring robustness in the estimation process. Concerning the famous chattering phenomenon, it is well known that several methods can be used to deduce it, like filter [27], sigmoid function, discrete SMO [1, 34] and so on. However, those modifications provide as well some disadvantages. For example, the introduction of low-pass filter can definitely reduce the chattering, but it will slightly introduce delay. Replacing signum function by sigmoid function can also reduce the chattering, but it will loss the robustness of SMO. More details can be found in [14]. However, in our work thanks to the obtained observer normal form (30), we reach a good estimation using only the high gain observer (HGO). The figures Fig.3 - Fig.4 show that for PMSM in d-q reference frame, the both methods (HGO, SMO) estimate successfully the states. Also, we note a less pronounced variation of estimation with the our method.

The load torque impact is noticeable on the figure Fig.3 across the appearance of-overshoot when it changes the value. However the quality of the estimation is not affected. Therefore, we deduce that this observer is robust to external disturbance. Moreover, despite the difference of initial conditions between the simulation and estimation, it can be seen that the rotor shaft position value is correctly estimated. In other words, whatever the initial position of the rotor, the states of PMSM can be estimated.

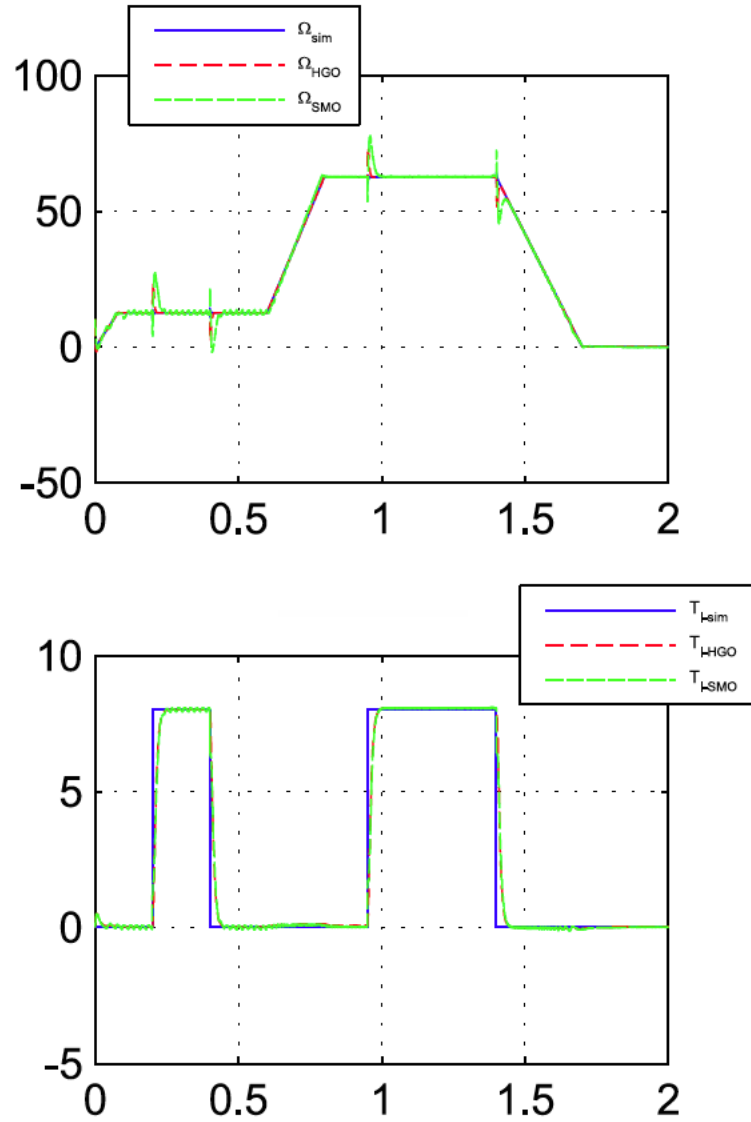


Fig. 3: Estimation of the rotor mechanical speed  $\Omega$  and the load torque  $T_l$ .

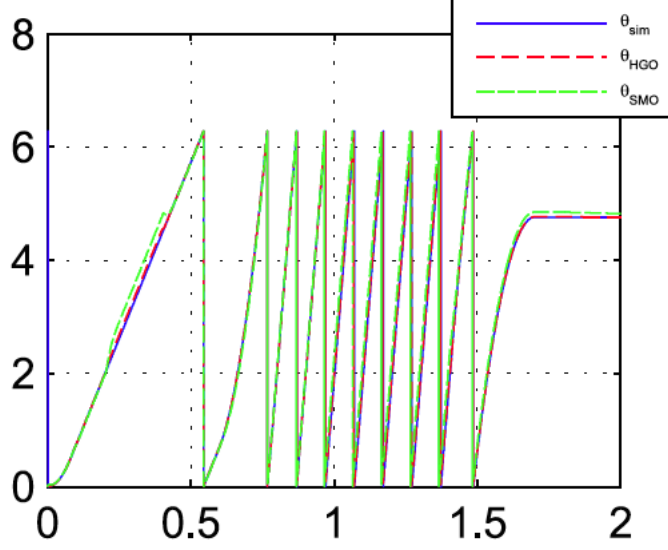


Fig. 4: Estimation of the rotor position  $\theta$ .

#### 4.2. Simulation results with adaptive observer

To simultaneously estimate the states and the parameters of PMSM, we consider the dynamical form (32) and the adaptive observer (34)-(35) with  $\Sigma = 0.05I_d$ ,  $\rho_z = 77$  and  $\rho_\delta = 60$ . The performances of this method are shown in Fig.5 - Fig.8.

Fig.5 - Fig.6 show a good agreement in starting up between the estimated and the real values despite the uncertainty of the parameters values and the difference of initial conditions. We also note in Fig. 5 that the estimated speed follows their references. Moreover, the results of parameter estimation are shown in Fig.7 and Fig.8. The obtained result proves that the extended adaptive observer form has a very satisfactory performance. The obtained result demonstrates that the proposed approach can be considered as a suitable alternative to other methods, particularly when the simultaneous stimulation of states and parameters is required.

However, as expected, the rapid transients of load torque (disturbance) affect momentarily the estimation process. This phenomena is depicted through the appearance of overshoot on the figures. In other words, a good converging performance of the estimated states and parameters can be observed, despite the appearance of overshoots due to the variation of load torque. It is worth noting that those over-shoots in estimation need to be carefully treated when designing observer-based controller. More precisely, if the designed controller depends on those estimations with over-shoot, the saturation function should be applied to mitigate the harmful influence of closed-loop system. This saturation can be normally realized since the states of system are always bounded in practice.

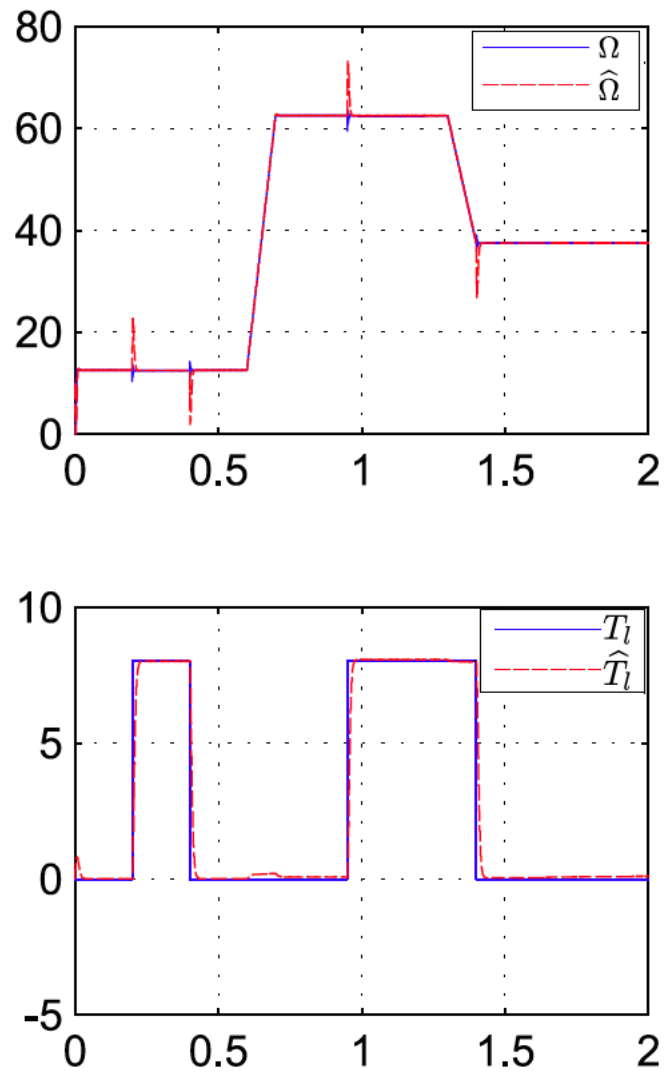


Fig. 5: Estimation of the rotor mechanical speed the load torque  $T_l$ .

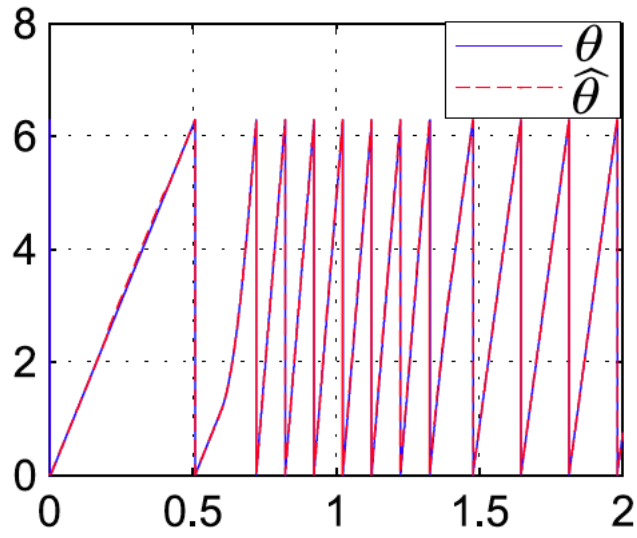


Fig. 6: Estimation of the rotor position  $\theta$ .

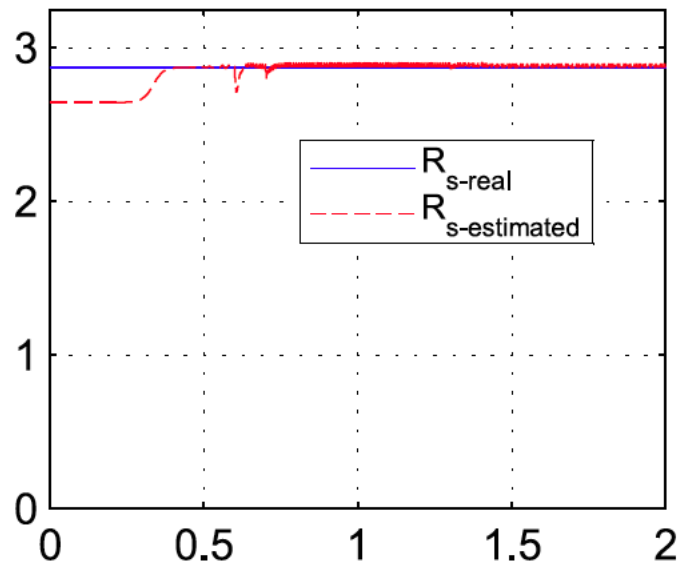


Fig. 7: Estimation of the stator resistor.



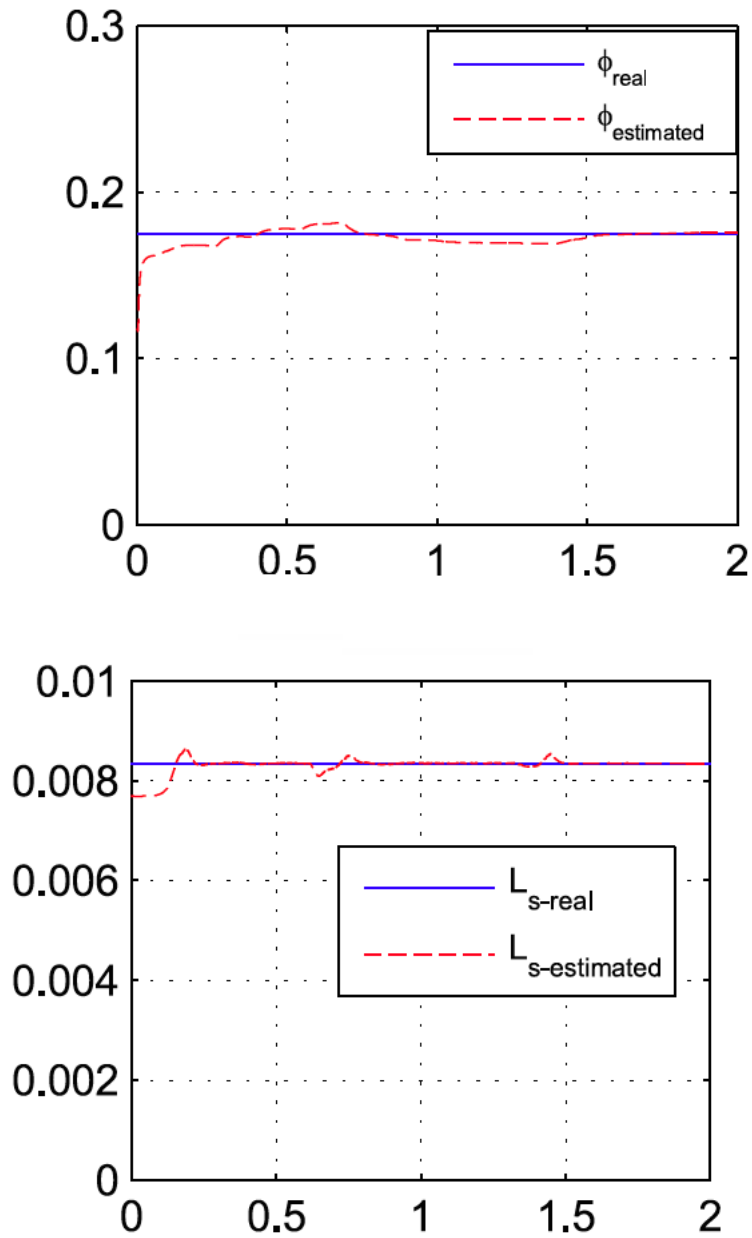


Fig. 8: Estimation of the magnet flux linkage and the stator inductance.

## 5. Conclusions

This work proposes a sensorless PMSM control based on the concept of extended nonlinear observer normal forms. This enables us to apply a high gain observer or an adaptive observer to estimate the state and the parameters of PMSM. Compared to other methods (back-emf and MRAS methods) where the parameters should be well known, the extended adaptive observer approach, is easier to be implemented, and shows good dynamic performances in all speed ranges. Furthermore, the simultaneous estimation of states and parameters can have a big interest in the faults diagnosis and fault tolerant control based on auto-tuning and real time adaptation. Several simulation results have been presented to show the effectiveness of the proposed approach.

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