PACO : Vers une meilleure paramétrisation de la côte et des conditions limites dans les modèles d'océan

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PACO
Vers une meilleure paramétrisation de la côte et des conditions limites dans les modèles d'océan

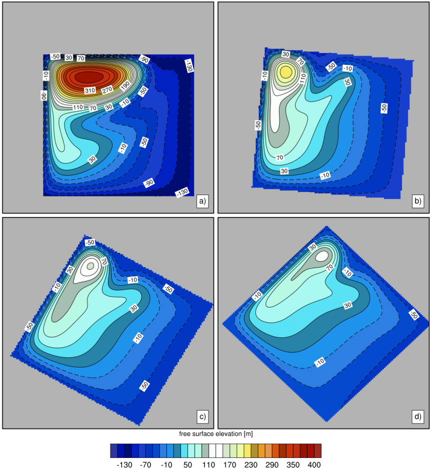
Eugene Kazantsev, Florian Lemarié, Eric Blayo

LJK/INRIA, AirSea, Grenoble

Contribuer à la compréhension fine des interactions entre représentation discrétisée de la côte, profils analytiques de couches limites latérales, et choix de discrétisation des conditions aux limites en s'appuiant sur :

- calculs analytiques (prenant en compte la forme continue de la ligne de côte dans la formulation des conditions aux limites)
- contrôle optimal des schémas numériques au voisinage de la côte.

LEFE-GMMC, LEFE-MANU



Solution instantanée après 8 ans d'intégration dans le cas free-slip pour un angle de

0° (a), 5° (b), 30° (c) et 45° (d)

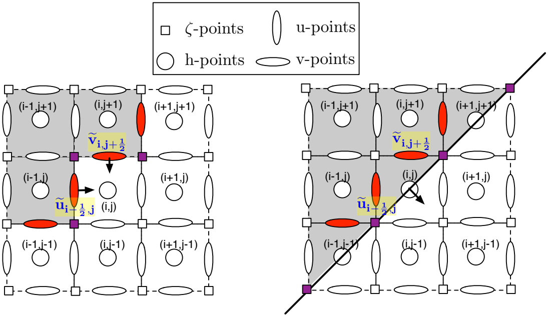
Code *shallow water* 2D

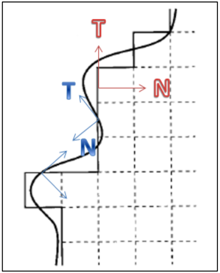
- Schéma en temps : RK4
- Advection : forme vecteur-invariant (enstrophy conserving)
- Viscosité : forme vorticité-divergence

$$\nu\nabla^2 u \quad \rightarrow \quad \nu\,(\partial_x D - \partial_y \zeta)$$
$$\nu\nabla^2 v \quad \rightarrow \quad \nu\,(\partial_x \zeta + \partial_y D)$$

Paramètres du problème

- 1750 km × 1750 km ($\Delta x = 10$ km)
- $\nu = 500$ m^2 s^{-1}
- $g = 0.01$ m s^{-2}
- $r_D = 10^{-7}$s^{-1}





Conditions limites : $\mathbf{u}\cdot\mathbf{n} = 0$, $\partial\mathbf{u}/\partial\mathbf{n} = 0$

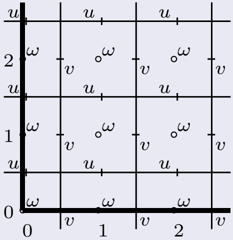
Imperméabilité $\Rightarrow \zeta = \left(\dfrac{\mathbf{u}}{R} - \dfrac{\partial\mathbf{u}}{\partial\mathbf{n}}\right)\cdot\boldsymbol{\tau} \Rightarrow$ glissement $+\ R \rightarrow \infty$: $\zeta = 0$

$$\left\{\begin{array}{lcl} \tilde{u}_{i+\frac{1}{2},j+1} + u_{i+\frac{1}{2},j} & = & \tilde{v}_{i,j+\frac{1}{2}} + v_{i+1,j+\frac{1}{2}} \\ \tilde{u}_{i+\frac{1}{2},j+1} + \tilde{v}_{i,j+\frac{1}{2}} & = & u_{i+\frac{1}{2},j} + v_{i+1,j+\frac{1}{2}} \end{array}\right. \Rightarrow \left\{\begin{array}{lcl} \tilde{u}_{i+\frac{1}{2},j+1} & = & v_{i+1,j+\frac{1}{2}} \\ \tilde{v}_{i,j+\frac{1}{2}} & = & u_{i+\frac{1}{2},j} \end{array}\right.$$

$$D_{i,j} = \frac{u_{i+\frac{1}{2},j}}{\Delta x} - \frac{v_{i,j-\frac{1}{2}}}{\Delta y} \rightarrow D_{i,j} = \left(\frac{u_{i+\frac{1}{2},j}}{\Delta x} + \frac{u_{i+\frac{1}{2},j}}{\Delta y}\right) - \left(\frac{v_{i,j-\frac{1}{2}}}{\Delta y} + \frac{v_{i,j-\frac{1}{2}}}{\Delta x}\right)$$

Layout:

- NEMO, Rectangular box, $1^\circ/4$ resolution, single-gyre or double-gyre wind stress;
- Artificially generated data by the same model on the aligned grid;
- Assimilation of these data during 50 days interval;
- Analysis of the model on the 800 days interval



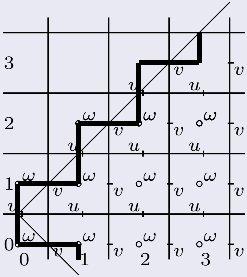


Figure: 45° rotated grid

Impermeability + free-slip boundary conditions: $(\vec{V},\vec{n}) = 0, \quad \dfrac{\partial(\vec{V},\vec{\tau})}{\partial \vec{n}} = 0$

Optimal Control of the Discrete Derivatives near the Boundary

Coefficients α are allowed to vary in order to find the best fit with requirements of the model and data (see [1, 2]).

$$\left.\frac{\partial v}{\partial x}\right|_{\omega_b} = \frac{\alpha_1 v_{1/2} + \alpha_2 v_{3/2}}{h}$$

$$\left.\frac{\partial u}{\partial y}\right|_{\omega_b} = \frac{\alpha_1 u_{1/2} + \alpha_2 u_{3/2}}{h}$$

$$\left.\frac{\partial v}{\partial x}\right|_{\omega_b} = \frac{\alpha_1 v_{-1/2} + \alpha_2 v_{1/2}}{h}$$

$$\left.\frac{\partial u}{\partial y}\right|_{\omega_b} = \frac{\alpha_1 u_{-1/2} + \alpha_2 u_{1/2}}{h}$$

$$\left.\frac{\partial \omega}{\partial x}\right|_{v} = \frac{\alpha_1 \omega_0 + \alpha_2 \omega_1}{h}$$

$$\left.\frac{\partial \omega}{\partial y}\right|_{u} = \frac{\alpha_1 \omega_0 + \alpha_2 \omega_1}{h}$$

with the Initial Guess

$$\alpha_1 = \alpha_2 = 0$$

$$\alpha_1 = -1, \quad \alpha_2 = 1$$

$$\alpha_1 = -1, \quad \alpha_2 = 1$$

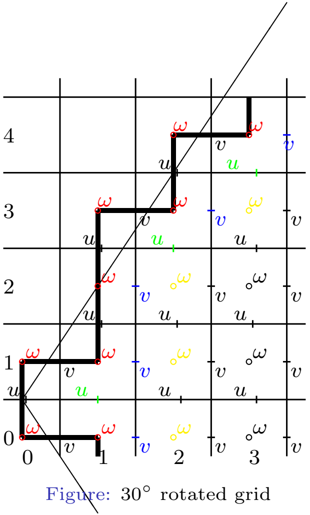


Figure: 30° rotated grid

The model: $x(t) = \mathcal{M}_{0,t}(x(0), \alpha)$ with $x = (u, v, T, S, ssh)^T$

Cost function J

$$J = 10^{-4}(\|x(0) - x_{bgr}\|^2 + \|\alpha - \alpha_{bgr}\|^2) + \\ + \int_{t=0}^{T} t \int\int (u - u_{\text{ref}})^2 + (v - v_{\text{ref}})^2 + (ssh - ssh_{\text{ref}})^2 dxdydt$$

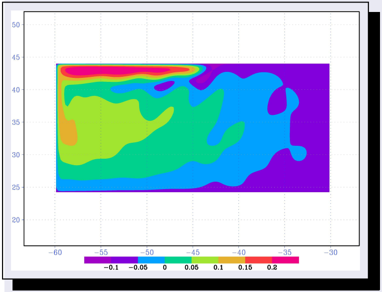
Layout:

- Joint control of the initial point $x(0)$ (interpolation errors) and the set of α;
- Artificially generated data by the same model on the aligned grid;
- Data Assimilation with the sequence of assimilation windows: 10, 30, 50 days with 30 iterations made in each window;
- Analysis of the solution on the 800 days interval.

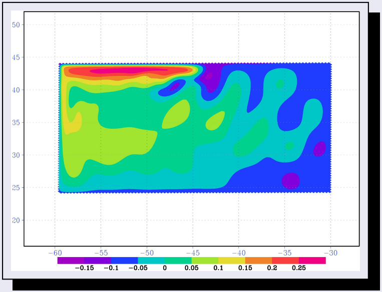
- Minimization is performed by M1QN3 (JC Gilbert, C.Lemarechal);
- Adjoint is generated by Tapenade (Ecuador team, INRIA).

Single Gyre Control: 45° rotation, SSH

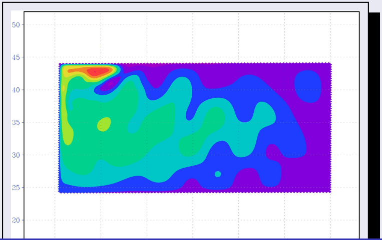
Reference, Optimal and Conventional BC 800 days later



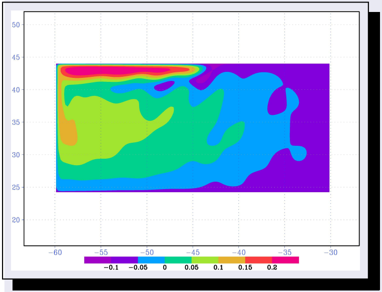
Reference SSH



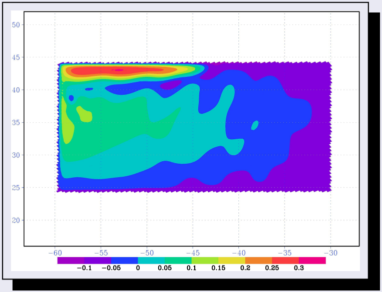
Rotated grid Optimal BC SSH



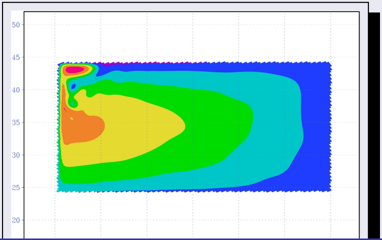
Reference, Optimal and Conventional BC 800 days later



Reference SSH



Rotated grid Optimal BC SSH



Optimized Coefficients: 45° rotation

$$\left.\frac{\partial v}{\partial x}\right|_{\omega_\circ} = \frac{\alpha_1 v_{-1/2} + \alpha_2 v_{1/2}}{h}$$

$$\left.\frac{\partial u}{\partial y}\right|_{\omega_\circ} = \frac{\alpha_1 u_{-1/2} + \alpha_2 u_{1/2}}{h}$$

$$\left.\frac{\partial v}{\partial x}\right|_{\omega_\circ} = \frac{\alpha_1 v_{1/2} + \alpha_2 v_{3/2}}{h}$$

$$\left.\frac{\partial u}{\partial y}\right|_{\omega_\circ} = \frac{\alpha_1 u_{1/2} + \alpha_2 u_{3/2}}{h}$$

$$\left.\frac{\partial \omega}{\partial x}\right|_{v} = \frac{\alpha_1 \omega_0 + \alpha_2 \omega_1}{h}$$

$$\left.\frac{\partial \omega}{\partial y}\right|_{u} = \frac{\alpha_1 \omega_0 + \alpha_2 \omega_1}{h}$$

$$\alpha_1 = 0, \quad \alpha_2 = 0$$

$$\alpha_1 = -1.5, \quad \alpha_2 = 0.5$$

$$\alpha_1 = -1, \quad \alpha_2 = 1$$

$$\alpha_1 = -1, \quad \alpha_2 = 1$$

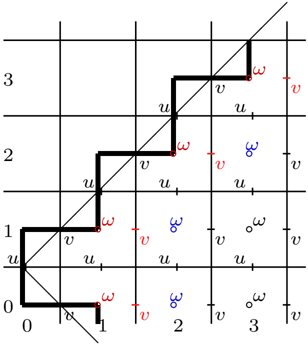


Figure: 45° rotated grid

That means the tangential velocity component is added to the vorticity formula:

$$\omega_\circ = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} - \frac{u+v}{h}$$

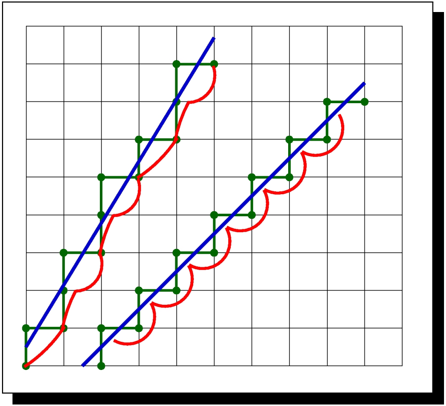
$$= \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} - \frac{\vec{V}.\vec{\tau}}{h/\sqrt{2}}$$

$$\omega_\circ = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} - \frac{u+v}{h}$$
$$= \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} - \frac{\vec{V}.\vec{\tau}}{h/\sqrt{2}}$$

- Free-slip condition on a curvilinear boundary (see [4]): $\omega|_{bnd} = \frac{\vec{V}.\vec{\tau}}{R}$

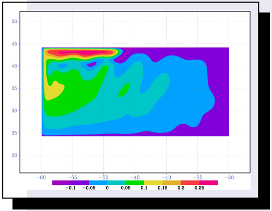
The optimized boundary is supposed to be a curvilinear one with:

- Constant $R_{45^\circ} = -h/\sqrt{2}$ for the 45° rotated grid,
- Variable R_{30° : $-h < R_{30^\circ} < 5h$ for the 30° rotated grid

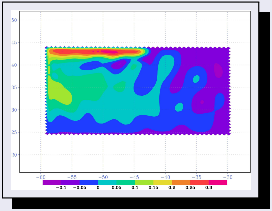


Physical boundary (blue), approximated by the grid (green) and optimized one (red).

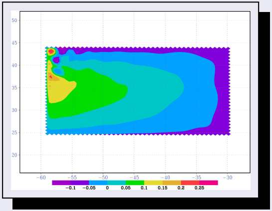
1°/2 resolution



Reference SSH

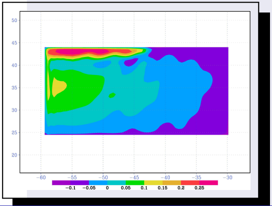


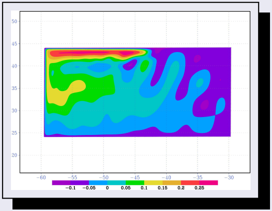
Rotated grid constant $R = -h/\sqrt{2}$ BC SSH

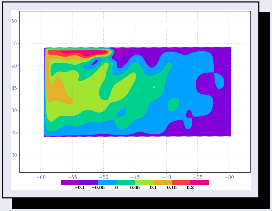


Rotated grid conventional BC SSH

1°/8 resolution







Boundary Conditions influence is important

- Optimal BCs allows **to correct errors** committed by the discretization,
- The model **is closer** to the reference one with optimal BC,
- Data assimilation allows to get the **optimized position and shape** of the boundary.

BUT

As well as for any adjoint parameter estimation

- The control may violate the model physics;
- The physical meaning of the optimal boundary is difficult to understand;
- The set of α is not unique;
- The problem of identifiability is not addressed yet;
- The problem of stability is not even posed.

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