



## PACO : Vers une meilleure paramétrisation de la côte et des conditions limites dans les modèles d'océan

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# PACO

Vers une meilleure paramétrisation de la côte  
et des conditions limites dans les modèles d'océan

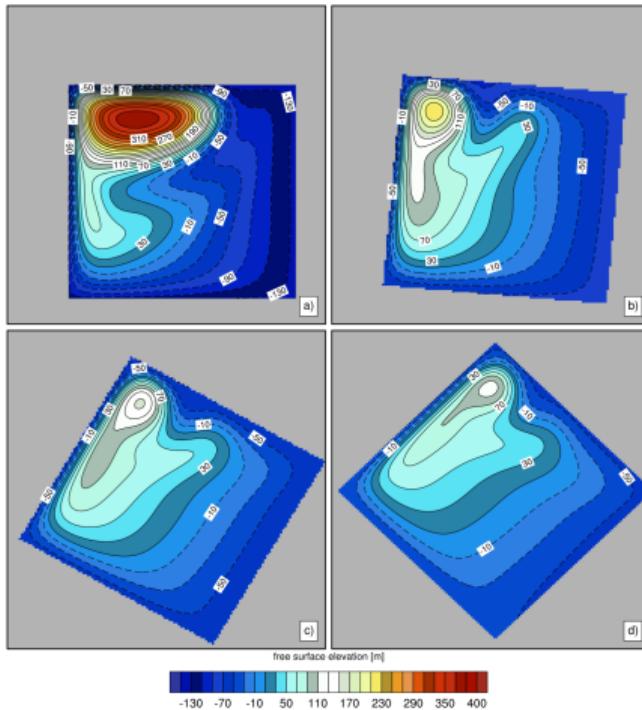
Eugene Kazantsev, Florian Lemarié, Eric Blayo

LJK/INRIA, AirSea, Grenoble

Contribuer à la compréhension fine des interactions entre représentation discrétisée de la côte, profils analytiques de couches limites latérales, et choix de discréttisation des conditions aux limites en s'appuyant sur :

- **calculs analytiques** (prenant en compte la forme continue de la ligne de côte dans la formulation des conditions aux limites)
- **contrôle optimal** des schémas numériques au voisinage de la côte.

LEFE-GMMC, LEFE-MANU



Solution instantanée après 8 ans d'intégration dans le cas free-slip pour un angle de

$0^\circ$  (a),  $5^\circ$  (b),  $30^\circ$  (c) et  $45^\circ$  (d)

## Code shallow water 2D

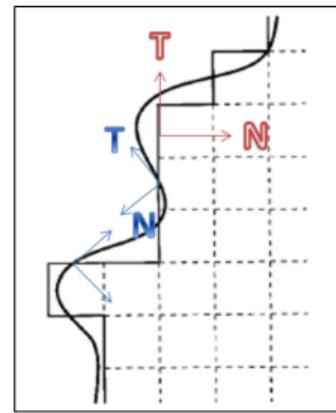
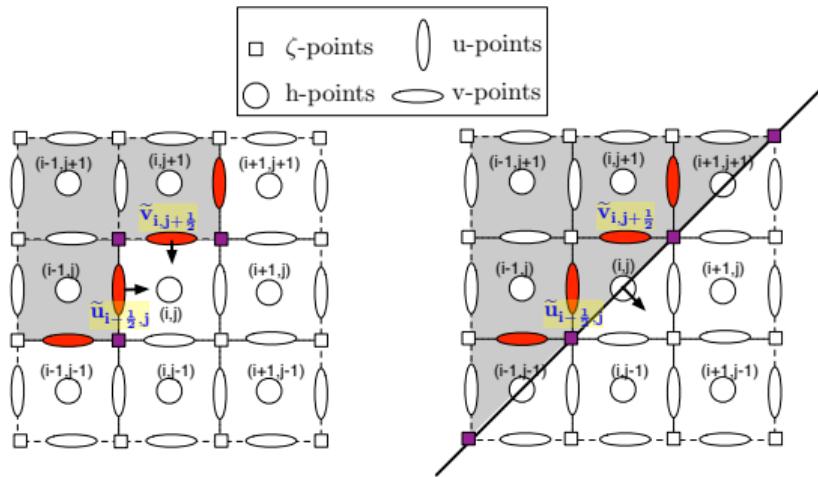
- Schéma en temps : RK4
- Advection : forme vecteur-invariant (enstrophy conserving)
- Viscosité : forme vorticité-divergence

$$\begin{aligned}\nu \nabla^2 u &\rightarrow \nu (\partial_x D - \partial_y \zeta) \\ \nu \nabla^2 v &\rightarrow \nu (\partial_x \zeta + \partial_y D)\end{aligned}$$

## Paramètres du problème

- $1750 \text{ km} \times 1750 \text{ km}$  ( $\Delta x = 10 \text{ km}$ )
- $\nu = 500 \text{ m}^2 \text{ s}^{-1}$
- $g = 0.01 \text{ m s}^{-2}$
- $r_D = 10^{-7} \text{ s}^{-1}$

# Cas particulier : rotation à 45° (see [3])



Conditions limites :  $\mathbf{u} \cdot \mathbf{n} = 0, \partial \mathbf{u} / \partial \mathbf{n} = 0$

Imperméabilité  $\Rightarrow \zeta = \left( \frac{\mathbf{u}}{R} - \frac{\partial \mathbf{u}}{\partial \mathbf{n}} \right) \cdot \boldsymbol{\tau} \Rightarrow$  glissement +  $R \rightarrow \infty : \zeta = 0$

$$\begin{cases} \tilde{u}_{i+\frac{1}{2},j+1} + u_{i+\frac{1}{2},j} &= \tilde{v}_{i,j+\frac{1}{2}} + v_{i+1,j+\frac{1}{2}} \\ \tilde{u}_{i+\frac{1}{2},j+1} + \tilde{v}_{i,j+\frac{1}{2}} &= u_{i+\frac{1}{2},j} + v_{i+1,j+\frac{1}{2}} \end{cases} \Rightarrow \begin{cases} \tilde{u}_{i+\frac{1}{2},j+1} &= v_{i+1,j+\frac{1}{2}} \\ \tilde{v}_{i,j+\frac{1}{2}} &= u_{i+\frac{1}{2},j} \end{cases}$$

$$D_{i,j} = \frac{u_{i+\frac{1}{2},j}}{\Delta x} - \frac{v_{i,j-\frac{1}{2}}}{\Delta y} \rightarrow D_{i,j} = \left( \frac{u_{i+\frac{1}{2},j}}{\Delta x} + \frac{u_{i+\frac{1}{2},j}}{\Delta y} \right) - \left( \frac{v_{i,j-\frac{1}{2}}}{\Delta y} + \frac{v_{i,j-\frac{1}{2}}}{\Delta x} \right)$$

# Optimal Control Approach: Staircase-shaped Boundary

Layout:

- NEMO, Rectangular box,  $1^\circ/4$  resolution, single-gyre or double-gyre wind stress;
- Artificially generated data by the same model on the aligned grid;
- Assimilation of these data during **50 days** interval;
- Analysis of the model on the **800 days** interval

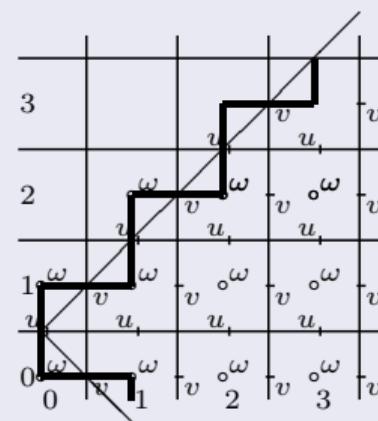
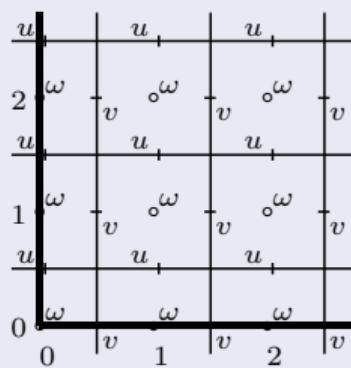


Figure:  $45^\circ$  rotated grid

Impermeability + free-slip boundary conditions:  $(\vec{V}, \vec{n}) = 0$ ,  $\frac{\partial(\vec{V}, \vec{r})}{\partial \vec{n}} = 0$

# Optimal Control of the Discrete Derivatives near the Boundary

Coefficients  $\alpha$  are allowed to vary in order to find the best fit with requirements of the model and data (see [1, 2]).

$$\begin{aligned}\frac{\partial v}{\partial x} \Big|_{\omega_b} &= \frac{\alpha_1 v_{1/2} + \alpha_2 v_{3/2}}{h} \\ \frac{\partial u}{\partial y} \Big|_{\omega_b} &= \frac{\alpha_1 u_{1/2} + \alpha_2 u_{3/2}}{h} \\ \frac{\partial v}{\partial x} \Big|_{\omega_b} &= \frac{\alpha_1 v_{-1/2} + \alpha_2 v_{1/2}}{h} \\ \frac{\partial u}{\partial y} \Big|_{\omega_b} &= \frac{\alpha_1 u_{-1/2} + \alpha_2 u_{1/2}}{h} \\ \frac{\partial \omega}{\partial x} \Big|_v &= \frac{\alpha_1 \omega_0 + \alpha_2 \omega_1}{h} \\ \frac{\partial \omega}{\partial y} \Big|_u &= \frac{\alpha_1 \omega_0 + \alpha_2 \omega_1}{h}\end{aligned}$$

with the Initial Guess

$$\alpha_1 = \alpha_2 = 0$$

$$\alpha_1 = -1, \quad \alpha_2 = 1$$

$$\alpha_1 = -1, \quad \alpha_2 = 1$$

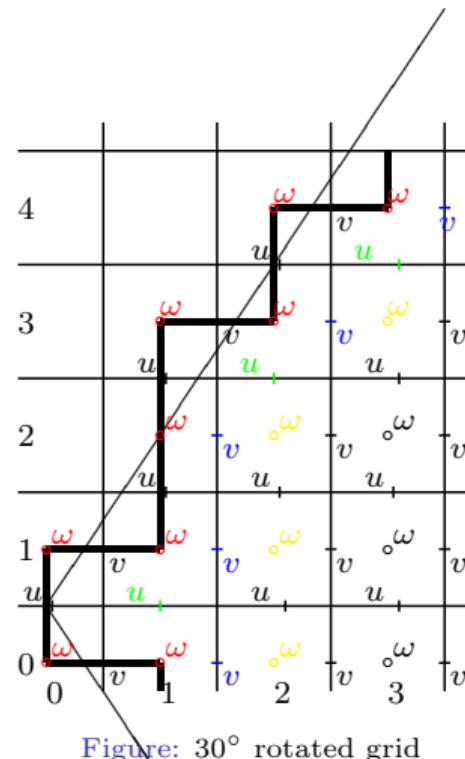


Figure:  $30^\circ$  rotated grid

The model:  $x(t) = \mathcal{M}_{0,t}(x(0), \alpha)$  with  $x = (u, v, T, S, ssh)^T$

## Cost function $J$

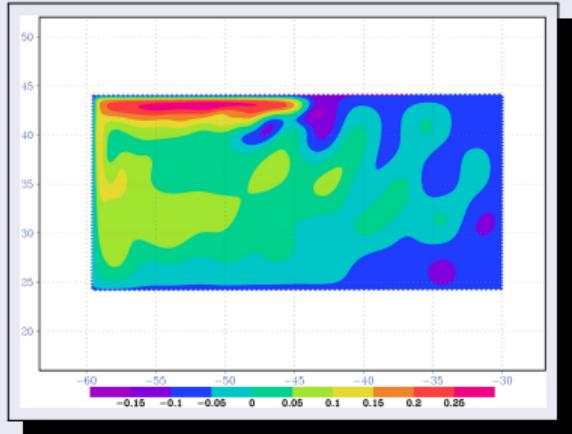
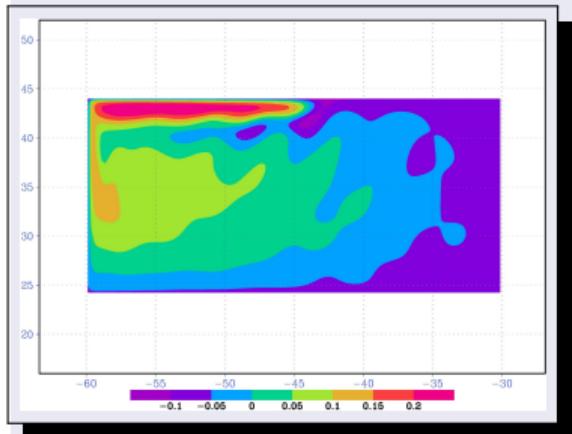
$$\begin{aligned} J &= 10^{-4}(\|x(0) - x_{bg}\|^2 + \|\alpha - \alpha_{bg}\|^2) + \\ &+ \int_{t=0}^T t \iint (u - u_{\text{ref}})^2 + (v - v_{\text{ref}})^2 + (ssh - ssh_{\text{ref}})^2 dx dy dt \end{aligned}$$

## Layout:

- **Joint control** of the initial point  $x(0)$  (interpolation errors) and the set of  $\alpha$ ;
  - Artificially generated data by the same model on the aligned grid;
  - Data Assimilation with the sequence of assimilation windows: **10, 30, 50 days** with 30 iterations made in each window;
  - Analysis of the solution on the **800 days** interval.
- 
- Minimization is performed by M1QN3 (JC Gilbert, C.Lemarechal);
  - Adjoint is generated by Tapenade (Ecuador team, INRIA).

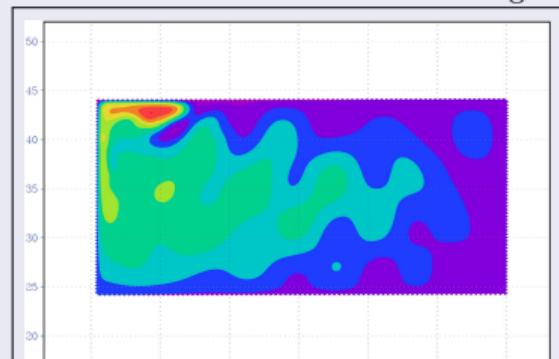
# Single Gyre Control: 45° rotation, SSH

Reference, Optimal and Conventional BC 800 days later



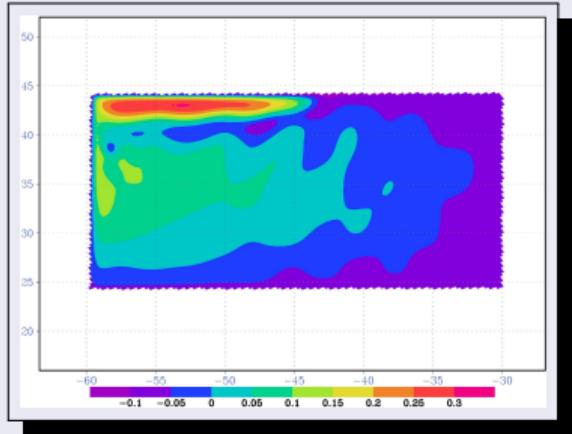
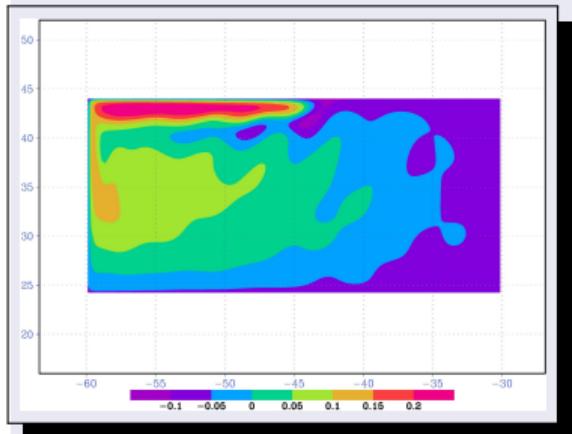
Reference SSH

Rotated grid Optimal BC SSH



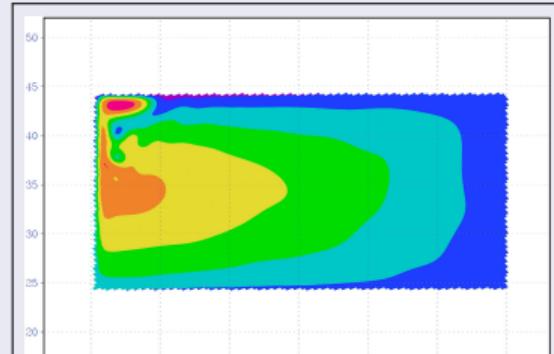
# Single Gyre Control: 30° rotation, SSH

Reference, Optimal and Conventional BC 800 days later



Reference SSH

Rotated grid Optimal BC SSH



# Optimized Coefficients: 45° rotation

$$\frac{\partial v}{\partial x} \Big|_{\omega_0} = \frac{\alpha_1 v_{-1/2} + \alpha_2 v_{1/2}}{h}$$

$$\frac{\partial u}{\partial y} \Big|_{\omega_0} = \frac{\alpha_1 u_{-1/2} + \alpha_2 u_{1/2}}{h}$$

$$\frac{\partial v}{\partial x} \Big|_{\omega_0} = \frac{\alpha_1 v_{1/2} + \alpha_2 v_{3/2}}{h}$$

$$\frac{\partial u}{\partial y} \Big|_{\omega_0} = \frac{\alpha_1 u_{1/2} + \alpha_2 u_{3/2}}{h}$$

$$\frac{\partial \omega}{\partial x} \Big|_v = \frac{\alpha_1 \omega_0 + \alpha_2 \omega_1}{h}$$

$$\frac{\partial \omega}{\partial y} \Big|_u = \frac{\alpha_1 \omega_0 + \alpha_2 \omega_1}{h}$$

$$\alpha_1 = 0, \quad \alpha_2 = 0$$

$$\alpha_1 = -1.5, \quad \alpha_2 = 0.5$$

$$\alpha_1 = -1, \quad \alpha_2 = 1$$

$$\alpha_1 = -1, \quad \alpha_2 = 1$$

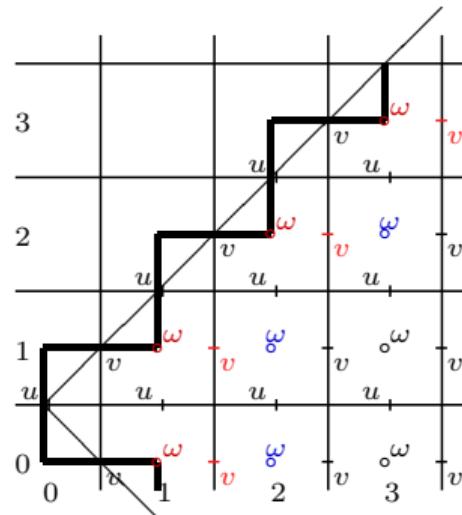


Figure: 45° rotated grid

That means the tangential velocity component is added to the vorticity formula:

$$\begin{aligned} \omega_0 &= \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} - \frac{u + v}{h} \\ &= \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} - \frac{\vec{V} \cdot \vec{r}}{h / \sqrt{2}} \end{aligned}$$

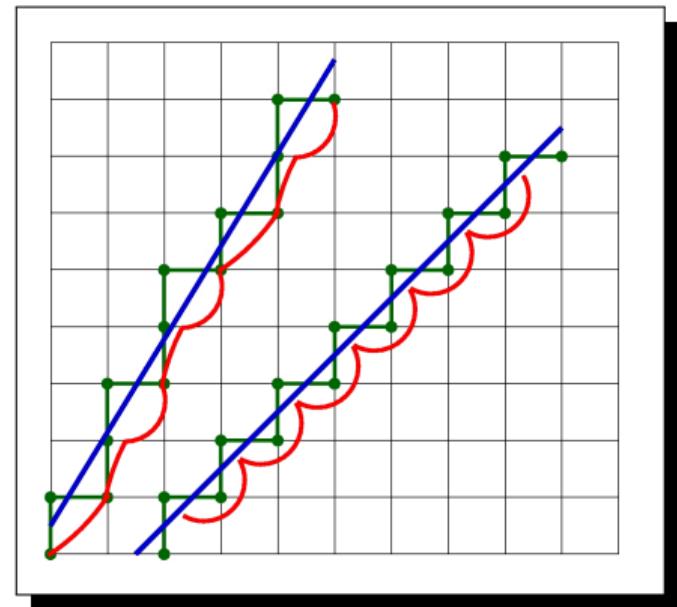
# Optimized boundary

$$\begin{aligned}\omega_o &= \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} - \frac{u + v}{h} \\ &= \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} - \frac{\vec{V} \cdot \vec{\tau}}{h/\sqrt{2}}\end{aligned}$$

- Free-slip condition on a curvilinear boundary (see [4]):  $\omega|_{bnd} = \frac{\vec{V} \cdot \vec{\tau}}{R}$

The optimized boundary is supposed to be a curvilinear one with:

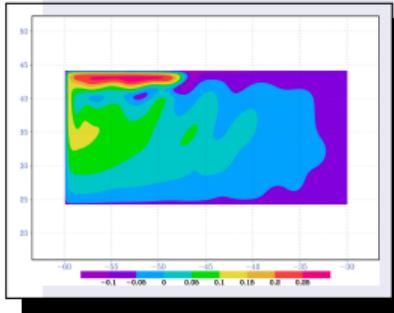
- Constant  $R_{45^\circ} = -h/\sqrt{2}$  for the  $45^\circ$  rotated grid,
- Variable  $R_{30^\circ} : -h < R_{30^\circ} < 5h$  for the  $30^\circ$  rotated grid



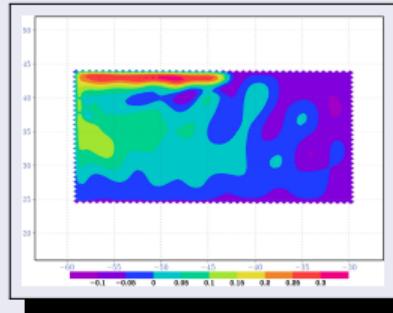
Physical boundary (blue), approximated by the grid (green) and optimized one (red).

# Different Resolutions, 45° rotation

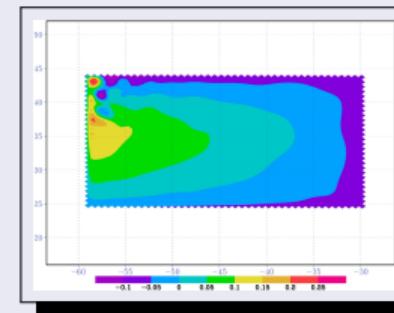
1°/2 resolution



Reference SSH

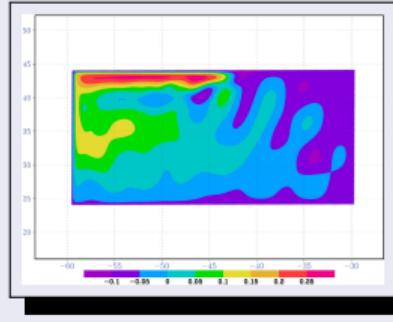
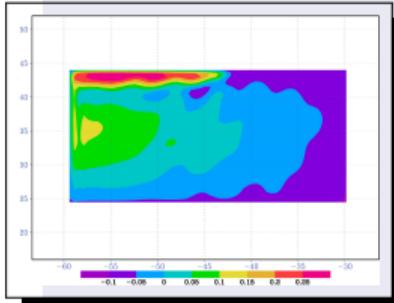


Rotated grid constant  
 $R = -h/\sqrt{2}$  BC SSH

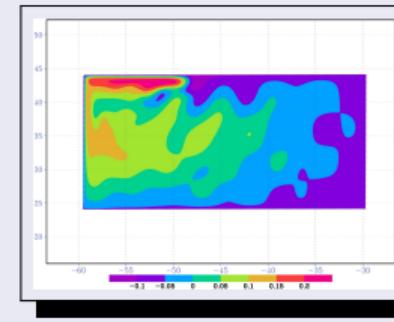


Rotated grid conventional BC  
SSH

1°/8 resolution



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# Conclusion

Boundary Conditions influence is **important**

- Optimal BCs allows **to correct errors** committed by the discretization,
- The model **is closer** to the reference one with optimal BC,
- Data assimilation allows to get the **optimized position and shape** of the boundary.

BUT

As well as for any adjoint parameter estimation

- The control may violate the model physics;
- The **physical meaning** of the optimal boundary is difficult to understand;
- The set of  $\alpha$  is **not unique**;
- The problem of **identifiability** is not addressed yet;
- The problem of **stability** is not even posed.

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