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Alain Jean-Marie, Mabel Tidball

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# Extraction cost: before or after harvesting? Economic and environmental consequences

Alain Jean-Marie, Inria  
LIRMM, University of Montpellier/CNRS, France

Mabel Tidball, INRA  
Lameta, Montpellier, France

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## Context of the paper

- Groundwater extraction: marginal cost depends on the level of the aquifer
- In general, resources with accessibility problems: cost depends on scarcity
- The main ingredient: make the cost depend on the projected evolution of the resource: before or after the extraction or rainfall
- The goal: deduce its economical and environmental consequences
- The method: revisit the discrete time, infinite horizon dynamic game model of Provencher and Burt (JEEM 1993).

# Contributions

We analyze a variant of Provencher and Burt's model, in the linear-quadratic case.

- We characterize the existence of Nash equilibria in affine feedback
- We prove qualitative properties of equilibria as a function of the discount factor.
- We discuss the particular cases: myopic (zero discount factor) and "golden rule" (discount factor tending to one).
- We focus on the case of scarce resources and find that taking harvesting and rainfall into account in the cost is better than the standard situation.

# Criteria of evaluation

## Criteria for defining the “better” situation

- individual welfare of players
- state of the resource
  - ▶ asymptotic (steady state)
  - ▶ transient (not in this study)
- social welfare (not in this study)

## The model of Provencher and Burt

We consider the extraction of groundwater by two players.  
The dynamic of groundwater:

$$G_{t+1} = G_t + R - u_t^1 - u_t^2, \quad G_0, \quad \text{given.}$$

We suppose  $R$  is a constant.

The instantaneous profit:

$$\pi_i = F_i(u_t^i) - C_i(G_t) \times u_t^i.$$

The marginal extraction cost ( $C_i(\cdot)$ ) depends on the **current** level of the groundwater.

## The extended model

Introduce the more general instantaneous profit function:

$$\pi_i(u_t^i) = F_i(u_t^i) - C_i(G_t + mR - n(u_t^1 + u_t^2)) u_t^i$$

where  $n, m \in [0, 1]$ .

The extreme cases:

- $n = 0, m = 0$  (the standard case): cost based on current resource
- $n = 1, m = 1$ : cost based on the state of the resource in the following period.

When  $n \neq 0$  the profit function of player  $i$  depends on the action of the other player: strategic interaction not just through the dynamics.

## The dynamic game setting

We formulate a discrete time, infinite-horizon, discounted dynamic game: for player  $i \in \{1, 2\}$ ,

$$\max_{\{u_t^i\}_t} \sum_0^{\infty} \beta^t [F_i(u_t^i) - C_i(G_t + mR - n(u_t^1 + u_t^2)) u_t^i],$$

such that

$$G_{t+1} = G_t + R - u_t^1 - u_t^2, \quad G_0, \quad \text{given.}$$

The Bellman equation associated with a Nash Feedback equilibrium is:

$$\begin{aligned} V^i(G_0) = & \max_{u_t^i} [F_i(u^i) - C_i(G + mR - n(u_t^1 + u_t^2))u^i \\ & + \beta V^i(G_t + R - (u_t^1 + u_t^2))]. \end{aligned}$$



# Analysis of the model

- solution in the linear-quadratic case
- exercise in sensitivity analysis
- still work in progress!

# Solution in the Linear-Quadratic case

Assume:

$$F_i(u) = u - \frac{b_i}{2}u^2, \quad C_i(x) = z_i - c_i x > 0.$$

We propose as solution of the Bellman equation:

- a value function for Player  $i \in \{1, 2\}$  of the form

$$V^i(G) = \frac{A_i}{2}G^2 + B_iG + C_i,$$

- a feedback law of the form

$$u^i = \alpha_i G + \gamma_i.$$

The unknown  $A_i, B_i, C_i, \alpha_i, \gamma_i, i = 1, 2$ , are found identifying

- the coefficients of the quadratic function in the Bellman equation after optimization
- the optimal strategy for  $j$  and the one conjectured by  $i$ .

# Solution in the Linear-Quadratic case (ctd)

More precisely, a solution by stages:

- system of 3rd degree polynomial equations for  $\{\alpha_1, \alpha_2\}$
- $A_1$  and  $A_2$  as simple functions of  $\{\alpha_1, \alpha_2\}$

$$A_i = \frac{c_i(1 - \alpha_j n) - \alpha_i(b + 2c_i n)}{\beta(1 - \alpha_1 - \alpha_2)}$$

- linear system for  $\{B_1, B_2, \gamma_1, \gamma_2\}$
- simple formulas for  $\{C_1, C_2\}$ .

# Interior solutions

The existence of a useful solution is not granted because:

- the LQ problem is not concave

$$\pi^i(u) = u(1 - z_i - \frac{b_i}{2}u) + c_i u G$$

- there are physical constraints:  
positive harvesting (!)

$$u_t^i \geq 0, \quad \forall t$$

positive and bounded stock

$$\bar{G} \geq G_t \geq 0, \quad \forall t.$$

$\implies$  we find only **interior** solutions.

# Equilibrium trajectories

When affine feedback controls  $u^i = \alpha_i G + \gamma_i$  are implemented, the dynamics becomes:

$$G_t = (1 - \alpha_1 - \alpha_2)G_{t-1} + R - \gamma_1 - \gamma_2$$

with solution:

$$G_t = (1 - \alpha_1 - \alpha_2)^t G_0 + \frac{R - \gamma_1 - \gamma_2}{\alpha_1 + \alpha_2} (1 - (1 - \alpha_1 - \alpha_2)^t).$$

Necessary conditions for the trajectory to be valid are:

- $0 \leq \alpha_1 + \alpha_2 < 2$
- $\gamma_1 + \gamma_2 \leq R$ .

Otherwise, an interior solution of our problem does not exist.

# The symmetric linear quadratic case

Symmetric case: look for symmetric equilibria:  $\alpha_1 = \alpha_2 = \alpha$ .  
With the procedure described above we find that  $\alpha$  solves:

$$p(Z) := \beta \{2(b + 2cn)Z^3 - (3b + 8cn)Z^2 + 2cZ\} \\ + (1 - \beta) \{-(b + 3cn)Z + c\} = 0.$$

## Existence and uniqueness of solution

There is one **unique** root of  $p(Z)$  in  $(0, 1)$  if

$$c < \frac{b}{1 + \beta}.$$

If  $c < b/2$ , this root actually satisfies:

$$0 \leq \alpha \leq \frac{c}{b + 3cn} < \frac{1}{2}.$$

# Solution (end)

Once  $\alpha$  is determined, we can compute

$$A = \frac{c - \alpha(b + 3cn)}{\beta(1 - 2\alpha)}$$

$$\gamma = \frac{\alpha}{c} \left[ 1 - z + mcR - \frac{R[c - \alpha(b + 3cn)]}{(1 - 2\alpha)(1 - \beta + \alpha\beta)} \right]$$

$$B = \dots$$

$$C = \dots$$

Observations:

- $\alpha$  and  $A$  do not depend on  $z$ ,  $R$  or  $m$
- $\gamma$  and  $B$  depend linearly on  $m$  and  $R$

# Qualitative analysis

We investigate the variation of the equilibrium (feedback parameters, value)

- when the discount factor  $\beta$  varies
- when the cost adjustment parameters  $n$  (harvest) and  $m$  (rainfall) vary.



## Variation with respect to $\beta$

When agents are more shortsighted, they react more aggressively and the environment suffers, whatever the cost structure.

### Monotony in $\beta$

Under the assumption  $c < b/2$ , the function:

- $\beta \mapsto \alpha(\beta)$  is decreasing on  $[0, 1]$
- $\beta \mapsto \gamma(\beta)$  is decreasing on  $[0, 1]$  when it is positive
- $\beta \mapsto G_\infty(\beta)$  is increasing when  $\gamma$  is positive.

We conjecture that  $\gamma(\cdot)$  is actually always decreasing.

# The green golden rule, $\beta = 1$

Note that the limit when  $\beta$  goes to 1 can allow to select a solution for the static green golden rule

$$\max_{u^i} F_i(u^i) - C_i(\cdot)u^i, \quad \text{such that} \quad u^1 + u^2 = R.$$

As it is well know, this is a game with coupled constraints and there exists an infinite number of solutions.

In the symmetric case,  $\lim_{t \rightarrow \infty} u_t^i = R/2$  for  $i = 1, 2$  and all  $\beta$ . But in the asymmetric case this limit can allow to select one equilibrium.

Observe that  $\lim_{\beta \uparrow 1} \alpha(1) \neq 0$ .

# Comparing the cost situations

Lexicon for the cost mechanism:

tag	$n$	$m$	
00	0	0	before rainfall and harvest
H0	1	0	after harvest but before rainfall
0R	0	1	before harvest but after rainfall
HR	1	1	after harvest <i>and</i> rainfall

Since the equilibrium reaction rate  $\alpha$  and the leading coefficient  $A$  do not depend on  $R$ , nor on  $m$ :

$$\alpha_{0R} = \alpha_{00}$$

$$\alpha_{H0} = \alpha_{HR}$$

$$A_{0R} = A_{00}$$

$$A_{H0} = A_{HR}.$$

# Benchmark situation: the myopic case $\beta = 0$

When  $\beta = 0$ , we find:

$$u(G) = \underbrace{\frac{c}{b + 3cn}}_{\alpha(0)} G + \underbrace{\frac{1 - z + cmR}{b + 3cn}}_{\gamma(0)}.$$

The value function of both players is:

$$\pi(G_0) = \frac{(cG_0 + 1 - z + cmR)^2}{(b + 3cn)^2} \frac{b + 2cn}{2}.$$

And the asymptotic stock is:

$$G_\infty = \frac{R - 2\gamma(0)}{2\alpha(0)} = \frac{R(b + 3cm - 2cn) - 2(1 - z)}{2c}.$$

Clearly:

$$\frac{\partial \alpha(0)}{\partial n} < 0, \quad \frac{\partial \pi(G_0)}{\partial n} < 0, \quad \frac{\partial \gamma(0)}{\partial m} > 0, \quad \frac{\partial \pi(G_0)}{\partial m} > 0.$$

Accordingly:

### Ranking of controls, gains and steady states

- Controls are ordered as:  $u_{H0}(G) < u_{00}(G)$  and

$$u_{H0}(G) < u_{HR}(G), \quad u_{00}(G) < u_{0R}(G)$$

- Value functions are ordered as:  $\pi_{H0}(G) < \pi_{00}(G)$  and

$$\pi_{H0}(G) < \pi_{HR}(G), \quad \pi_{00}(G) < \pi_{0R}(G)$$

- Steady-state stocks are ordered as:

$$G_{H0}^{\infty} > G_{HR}^{\infty} > G_{00}^{\infty} > G_{0R}^{\infty}.$$

## Rankings (ctd)

Under additional conditions: if  $R$  small enough (scarce resource), controls and values can be totally ordered.

$$u_{H0}(G) < u_{HR}(G) < u_{00}(G) < u_{0R}(G)$$

$$\pi_{H0}(G) < \pi_{HR}(G) < \pi_{00}(G) < \pi_{0R}(G)$$

$$G_{H0}^{\infty} > G_{HR}^{\infty} > G_{00}^{\infty} > G_{0R}^{\infty}.$$

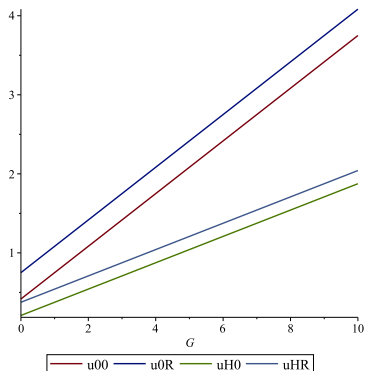
However, in order to have positive harvesting policy for all  $G$  and a positive steady state, we must impose

$$R > \frac{2(1-z)}{b-2c}$$

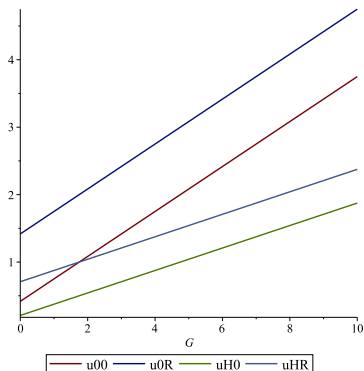
that is,  $R$  “not too small”.

# Numerical illustration

Case of interior feedback but different rankings for controls



$R$  small



$R$  large

# Variation of $\alpha$ and $\gamma$

Towards a generalization of these findings to general  $\beta \in [0, 1]$ .

## Variations of $\alpha$ and $\gamma$

- The function  $n \mapsto \alpha(n)$  is decreasing, so that,

$$\alpha_{0R} = \alpha_{00} > \alpha_{H0} = \alpha_{HR}.$$

- The function  $m \mapsto \gamma(m)$  is increasing, so that:

$$\gamma_{H0} < \gamma_{HR}, \quad \gamma_{00} < \gamma_{0R}$$

and consequently:

$$u_{H0}(G) < u_{HR}(G), \quad u_{00}(G) < u_{0R}(G).$$



# Constraints on the rainfall level $R$

According to the formula for  $\gamma$ :

$$\gamma = \frac{\alpha}{c} \left[ 1 - z + mcR - \frac{R[c - \alpha(b + 3cn)]}{(1 - 2\alpha)(1 - \beta + \alpha\beta)} \right]$$

the constraints

$$0 \leq \gamma \leq \frac{R}{2}$$

imply that  $R$  should be not too small and not too large...

## Other properties

We have:

- when  $\gamma > 0$ , and  $R$  small enough, the total ranking of controls hold:

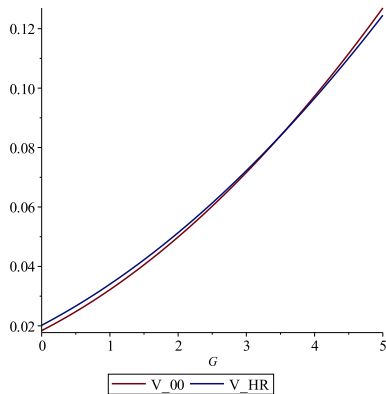
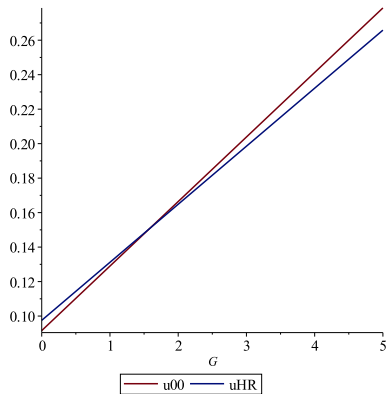
$$u_{H0}(G) < u_{HR}(G) < u_{00}(G) < u_{0R}(G).$$

- when  $\beta \sim 0$  and  $z < 1$ , then  $\gamma > 0$ : the equilibrium control  $u(G)$  is valid for all  $G$
- when  $\beta \sim 1$ ,  $\gamma < 0$ .

More analysis is needed for  $\beta$  large and  $R$  small.

# When determining cost after harvesting and rainfall is a good economic and environmental option

Assume:  $\beta = 0.3, b = 1, c = 0.04, z = 0.9, R = 1$ .



## Scarce resources, ctd

We have:

- $G_{00}^{\infty} < G_{HR}^{\infty}$ , as always;
- $u_{00}(G) > u_{HR}(G)$ ,  $V_{00}(G) > V_{HR}(G)$ , when  $G$  big, same ranking as in the myopic case: **conflict**;
- $u_{00}(G) < u_{HR}(G)$ ,  $V_{00}(G) < V_{HR}(G)$ , when  $G$  small, reversed ranking: **win-win**.

When the level of the groundwater is small, setting costs after harvesting and rainfall is better from the economic *and* environmental point of view than the standard literature case where the cost is announced before rain and harvesting, *even* for quite myopic agents.

# Conclusions and extensions

## Conclusions:

- We illustrate the interest to charge users in function of their behavior, not just in function of the level of resource
- Possibility of win-win situations
- More analysis to better explain the phenomenon

## Extensions:

- Stochastic case
- Stackelberg game with the regulator announcing the cost
- ...

## Stochastic extension

Assume the recharge is a i.i.d. sequence  $\{R_t; t = 0, 1, \dots\}$ .  
In the LQ case, the Bellman equation becomes:

$$\begin{aligned} V^i(G_0) &= \max_{u_t^i} \left[ F_i(u^i) - C_i(G + m\mathbb{E}R - n(u_t^1 + u_t^2))u^i \right. \\ &\quad \left. + \beta \mathbb{E} V^i(G_t + R - (u_t^1 + u_t^2)) \right] \\ &= \max_{u_t^i} \left[ F_i(u^i) - C_i(G + m\mathbb{E}R - n(u_t^1 + u_t^2))u^i \right. \\ &\quad \left. + \beta V^i(G_t + \mathbb{E}R - (u_t^1 + u_t^2)) + \frac{1}{2} \sigma_R^2 (V^i)'' \right] \end{aligned}$$

and has the same controls as solution.