# Convergence domain of image-based visual servoing with a line-scan camera

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**Keywords:** interval analysis, robotics, visual servoing, stability

## Introduction

Visual servoing (see [1] for an introduction to basic approaches) consists in controlling the motion of a robot by using computer vision data. Visual servoing schemes aim to minimize an error defined between a vector  $\mathbf{s}$  of visual features derived from image measurements, and the vector  $\mathbf{s}^*$  of the desired values of the features (which correspond to the reference position). A first classical visual servoing scheme is the *image-based visual servoing* (IBVS), that employs for  $\mathbf{s}$  a set of features that are directly available in the image data. Another is position-based visual servoing, where  $\mathbf{s}$  is a set of robot position parameters that have to be estimated from image data.

The classical IBVS approach is considered in the sequel. It consists in using the image coordinates of a set of points to define the feature vector **s**. They are compared to their coordinates in a reference image taken at the desired camera position to control the robot motion. Stability and convergence of IBVS has been studied but remains challenging [2]. Visual servoing will be done in the so-called eye-in-hand configuration, in which the camera is mounted on the robot.

An holonomic 3 degrees-of-freedom robot is considered. Its configuration is given by its coordinates (x, y) in the plane and its heading  $\theta$ . The robot is equipped with a line-scan camera (a camera that captures a single row of pixels, i.e an image line). For the sake of simplicity, the camera and the robot pose are assumed to be the same.

This work aims to compute the set of camera poses from which IBVS will converge to the reference pose (that corresponds to the reference image). Since classical IBVS is done by matching feature points between the current image and the reference image, we also need to check that the feature points always stay in the camera field of view.

## Line-scan image-based visual servo control

A 3-DOF robot evolves in a planar world. It is equipped with a line-scan camera (1-D sensor) with pose  $(x, y, \theta)$ . The coordinates of a point **X** of the world in the camera body-frame are  $\mathbf{X} = (X, Z)^{\top}$ .

The projection x of a point  $\mathbf{X}$  on the image line is given by the 1-D pinhole model (perspective projection):

$$x = \frac{X}{Z} = \frac{u - u_o}{f},$$

(where u is the point abscissa in pixel units,  $u_0$  the principal point, and f the focal length). For IBVS, we take as feature x, the image line abscissa of the point.

The 3 DOF of the robot are directly controlled at the camera center, through the camera velocity vector  $\mathbf{v}_c = (v_x, v_y, \omega)$ , where  $v_x, v_y$  are instantaneous linear velocities and  $\omega$  rotation speed. This leads to movement of points in the camera frame:

$$\dot{\mathbf{X}} = \begin{pmatrix} \dot{X} \\ \dot{Z} \end{pmatrix} = \begin{pmatrix} -v_x + \omega Z \\ -v_y - \omega X \end{pmatrix},$$

and thus to movement of their projection on the image line:  $\dot{x} = \frac{-v_x + xv_y}{Z} + (1 + x^2)\omega$ . We then define the *interaction matrix*  $\mathbf{L}_x$  such that  $\dot{x} = \mathbf{L}_x v_c$  with  $\mathbf{L}_x = \begin{pmatrix} -1 & x \\ Y & Y \end{pmatrix}$ . Since at least three points are required to control the 3-DOF. The feature vector is thus  $\mathbf{x} = (x_1, x_2, x_3)$ and the corresponding interaction matrix  $L_{\mathbf{x}}$  is obtained by stacking interaction matrices for three

Control law is given by  $\mathbf{v}_c = -\lambda \mathbf{L}_{x^*}^+(\mathbf{x} - \mathbf{x}^*)$ , where  $\mathbf{L}_{x^*}^+$  is the Moore-Penrose pseudo-inverse of the interaction matrix for the desired position.

## Convergence domain computation

In addition to the differential equation derived from control law, additional constraints are implemented and have to be verified along the flow. They define the acceptable configurations domain, ensuring the the feature points stay in the camera field of view, and robot does not leave the working space.

We compute a guaranteed approximation of the convergence domain for the 3DOF IBVS task in two steps:

- First, we compute an attraction domain of the desired position using Lyapunov theory. (see [3] to compute a domain around an asymptotical stable point).
- Then, we employ guaranteed integration to iteratively increase the proven convergence domain, using a similar approach to [4]. Using subpayings and a branch and bound method, an inner and outer approximations of the convergence domain are computed. Boxes whose flow at a finite time t is a subset of the already proven convergence domain, and whose flow over [0,t] stays in the acceptable configurations domain are added to the proven convergence domain. Boxes that are outside the acceptable configurations domain, or whose flow leave it, are eliminated.

#### References

- [1] F. Chaumette, S. Hutchinson., Visual servo control, Part I: Basic approaches, *IEEE Robotics* and Automation Magazine, 13 (4), pp.82-90, 2006
- [2] F. Chaumette. Potential problems of stability and convergence in image-based and positionbased visual servoing. In The Confluence of Vision and Control, D. Kriegman, G. Hager and, A. Morse (eds) LNCIS Series, No 237, Springer-Verlag, pp.66-78, 1998
- [3] N. Delanoue, L. Jaulin, B Cottenceau, An algorithm for computing a neighborhood included in the attraction domain of an asymptotically stable point, Commun Nonlinear Sci Numer Simulat 21:181–189,2015
- [4] M. LHOMMEAU, L. JAULIN, L. HARDOUIN, Capture basin approximation using interval analysis, Int. J. Adapt. Control Signal Process. 25:264–272, 2011.