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Lateral Boundary Conditions at the staircase-like boundary of ocean models.

Eugene Kazantsev, Florian Lemarié, Eric Blayo

LJK/INRIA, AirSea, Grenoble

PACO project: LEFE-GMMC, LEFE-MANU

The model (Dynamics)

Rectangular Box Configuration of the NEMO:

$30^\circ \times 20^\circ$ with $\frac{1}{4}^\circ$ resolution and 5 z levels.

$120 \times 80 \times 5$ nodes in (x, y, z) coordinates, 64 time steps per day.

$$\frac{\partial u}{\partial t} = \underbrace{v(\omega + f)}_{\text{Enstr.cons.}} - \frac{\partial(u^2 + v^2)/2}{\partial x} - w \frac{\partial u}{\partial z} - \underbrace{\frac{\partial A_u^h \xi}{\partial x}}_{\text{Vort-Div. form.}} + \underbrace{\frac{\partial A_u^h \omega}{\partial y}}_{\text{}}$$

$$+ \underbrace{\frac{\partial}{\partial z} A_u^z \frac{\partial u}{\partial z}}_{\text{Implicit}} + g \underbrace{\int_0^z \frac{\partial \rho(x, y, \zeta)}{\partial x} d\zeta}_{\text{Pressure gradient}} + g \underbrace{\frac{\partial(\eta + T_c \phi)}{\partial x}}_{\text{EGW filter}}$$

$$\xi = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}, \quad \omega = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \quad \text{Divergence, Vorticity}$$

$$w = \int_H^z \xi(x, y, \zeta) d\zeta; \quad w(x, y, H) = 0 \quad \text{Vertical velocity}$$

$$A^z = 1.2 \times 10^{-4} \frac{m^2}{s}; \quad A^h = 200 \frac{m^2}{s}, \quad f = \frac{4\pi}{86400} \sin(lat)$$

$$\left. \frac{\partial u}{\partial z} \right|_{w_0} = \frac{-0.1 \frac{N}{m^2} \cos\left(B \pi * \frac{lat - 24^\circ}{44^\circ - 24^\circ}\right)}{hz_1 \rho_0} \quad \begin{array}{ll} B = 2, & \text{Double gyre} \\ B = 1, & \text{Single gyre} \end{array}$$

$$(u_\perp, \omega)_{\text{Lateral Boundary}} = 0 \quad (\text{Impermeability and Free-Slip conditions})$$

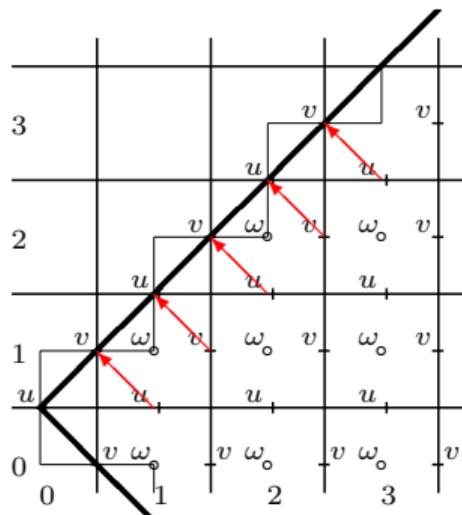


Figure: 45° rotated grid

Impermeability + free-slip boundary conditions:

$$(\vec{V}, \vec{n}) = 0, \quad \frac{\partial(\vec{V}, \vec{\tau})}{\partial \vec{n}} = 0$$

Impermeability: $u|_{bnd} = v|_{bnd}$,

Free-slip:

$$\begin{aligned} v|_{bnd} &= u|_{bnd} = u(x + h/2, y - h/2) \\ u|_{bnd} &= v|_{bnd} = v(x + h/2, y - h/2) \end{aligned}$$

Coriolis term on the 45° rotated grid.

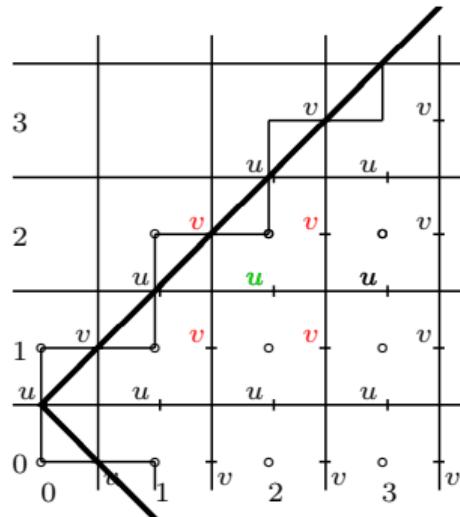


Figure: 45° rotated grid

$$\begin{aligned}\frac{\partial u}{\partial t} &= f \bar{v}^{xy} \\ \frac{\partial v}{\partial t} &= -f \bar{u}^{xy} \\ u(t) &= u(0) \exp(if t)\end{aligned}$$

$$\frac{\partial}{\partial t} \begin{pmatrix} u_1 \\ u_2 \\ \dots \\ u_n \\ v_1 \\ v_2 \\ \dots \\ v_n \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & f/4 & f/4 & 0 & 0 \\ 0 & 0 & 0 & 0 & f/4 & f/4 & 0 & 0 \\ \dots & & & & \dots & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & f/4 & f/4 \\ -f/4 & -f/4 & 0 & 0 & 0 & 0 & 0 & 0 \\ -f/4 & -f/4 & 0 & 0 & 0 & 0 & 0 & 0 \\ \dots & & & \dots & & & & \\ 0 & 0 & -f/4 & -f/4 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ \dots \\ u_n \\ v_1 \\ v_2 \\ \dots \\ v_n \end{pmatrix}$$

Coriolis term on the 45° rotated grid.

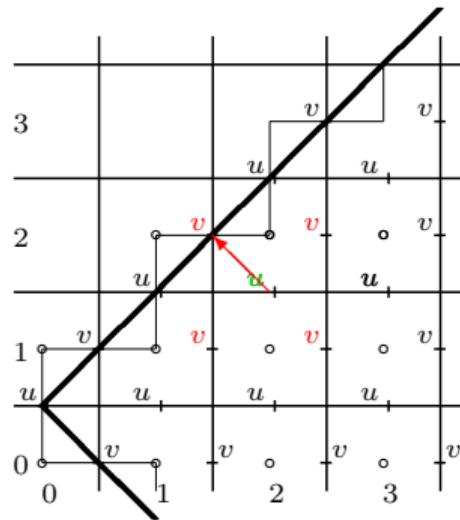


Figure: 45° rotated grid

$$\begin{aligned}\frac{\partial u}{\partial t} &= f \bar{v}^{xy} \\ \frac{\partial v}{\partial t} &= -f \bar{u}^{xy} \\ u(t) &= u(0)e^{+ft/4}\end{aligned}$$

$$\frac{\partial}{\partial t} \begin{pmatrix} u_1 \\ u_2 \\ \dots \\ u_n \\ v_1 \\ v_2 \\ \dots \\ v_n \end{pmatrix} = \begin{pmatrix} f/4 & 0 & 0 & 0 & f/4 & f/4 & 0 & 0 \\ 0 & f/4 & 0 & 0 & f/4 & f/4 & 0 & 0 \\ & & & & \dots & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & f/4 & f/4 \\ -f/4 & -f/4 & 0 & -f/4 & 0 & 0 & 0 & 0 \\ -f/4 & -f/4 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & \dots & & & & \\ 0 & 0 & -f/4 & -f/4 & 0 & 0 & 0 & -f/4 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ \dots \\ u_n \\ v_1 \\ v_2 \\ \dots \\ v_n \end{pmatrix}$$

Optimal Control of the Discrete Derivatives near the Boundary

Coefficients α are allowed to vary in order to find the best fit with requirements of the model and data (see [1, 2]).

$$\begin{aligned}\frac{\partial v}{\partial x} \Big|_{\omega_b} &= \frac{\alpha_1 v_{1/2} + \alpha_2 v_{3/2}}{h} \\ \frac{\partial u}{\partial y} \Big|_{\omega_b} &= \frac{\alpha_1 u_{1/2} + \alpha_2 u_{3/2}}{h} \\ \frac{\partial v}{\partial x} \Big|_{\omega_b} &= \frac{\alpha_1 v_{-1/2} + \alpha_2 v_{1/2}}{h} \\ \frac{\partial u}{\partial y} \Big|_{\omega_b} &= \frac{\alpha_1 u_{-1/2} + \alpha_2 u_{1/2}}{h} \\ \frac{\partial \omega}{\partial x} \Big|_v &= \frac{\alpha_1 \omega_0 + \alpha_2 \omega_1}{h} \\ \frac{\partial \omega}{\partial y} \Big|_u &= \frac{\alpha_1 \omega_0 + \alpha_2 \omega_1}{h}\end{aligned}$$

with the Initial Guess

$$\alpha_1 = \alpha_2 = 0$$

$$\alpha_1 = -1, \quad \alpha_2 = 1$$

$$\alpha_1 = -1, \quad \alpha_2 = 1$$

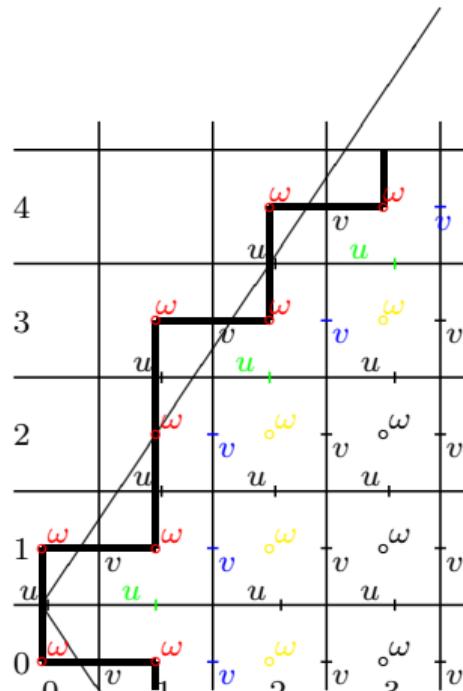


Figure: 30° rotated grid

The model: $x(t) = \mathcal{M}_{0,t}(x(0), \alpha)$ with $x = (u, v, T, S, ssh)^T$

Cost function J

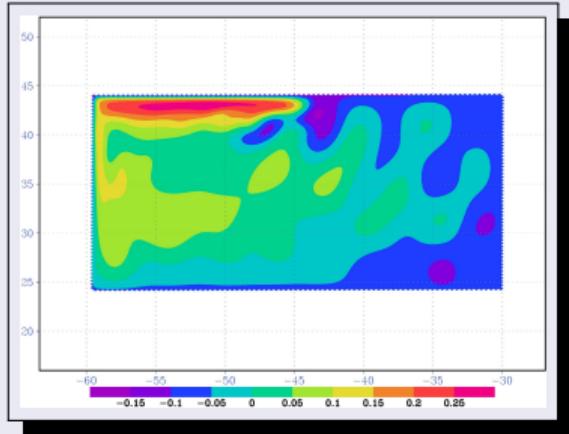
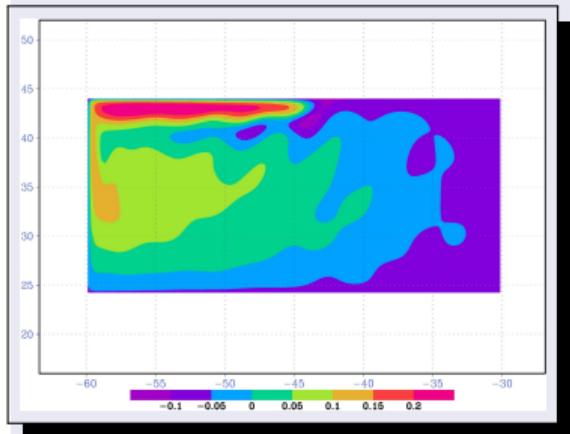
$$\begin{aligned} J &= 10^{-4}(\|x(0) - x_{bg}\|^2 + \|\alpha - \alpha_{bg}\|^2) + \\ &+ \int_{t=0}^T t \iint (u - u_{\text{ref}})^2 + (v - v_{\text{ref}})^2 + (ssh - ssh_{\text{ref}})^2 dx dy dt \end{aligned}$$

Layout:

- **Joint control** of the initial point $x(0)$ (interpolation errors) and the set of α ;
 - Artificially generated data by the same model on the aligned grid;
 - Data Assimilation with the sequence of assimilation windows: **10, 30, 50 days** with 30 iterations made in each window;
 - Analysis of the solution on the **800 days** interval.
-
- Minimization is performed by M1QN3 (JC Gilbert, C.Lemarechal);
 - Adjoint is generated by Tapenade (Ecuador team, INRIA).

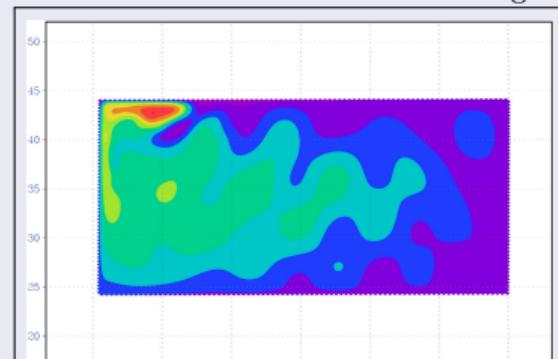
Single Gyre Control: 45° rotation, SSH

Reference, Optimal and Conventional BC 800 days later



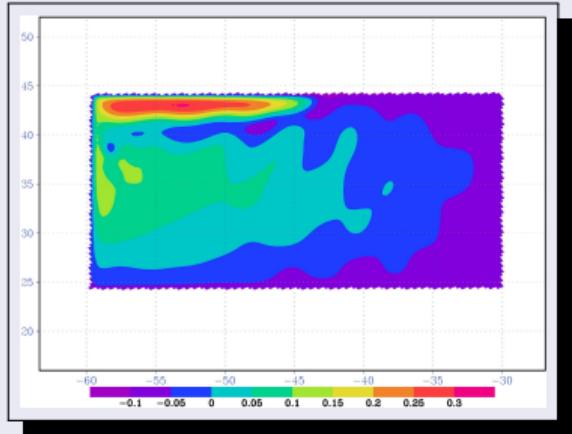
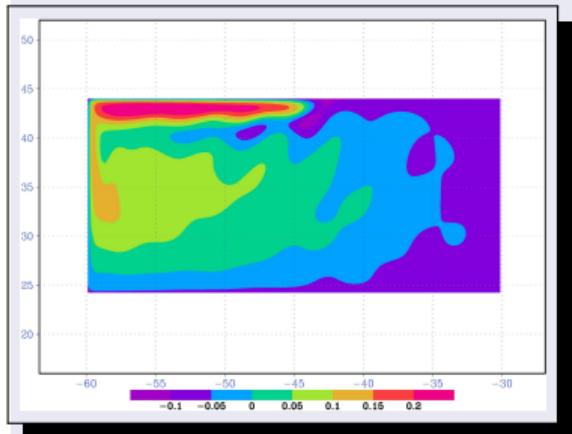
Reference SSH

Rotated grid Optimal BC SSH



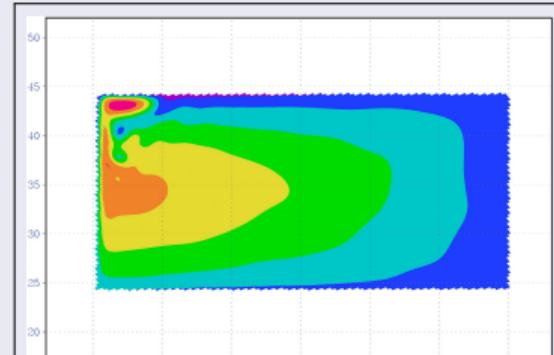
Single Gyre Control: 30° rotation, SSH

Reference, Optimal and Conventional BC 800 days later



Reference SSH

Rotated grid Optimal BC SSH



Optimized Coefficients: 45° rotation

$$\frac{\partial v}{\partial x} \Big|_{\omega_0} = \frac{\alpha_1 v_{-1/2} + \alpha_2 v_{1/2}}{h}$$

$$\frac{\partial u}{\partial y} \Big|_{\omega_0} = \frac{\alpha_1 u_{-1/2} + \alpha_2 u_{1/2}}{h}$$

$$\frac{\partial v}{\partial x} \Big|_{\omega_0} = \frac{\alpha_1 v_{1/2} + \alpha_2 v_{3/2}}{h}$$

$$\frac{\partial u}{\partial y} \Big|_{\omega_0} = \frac{\alpha_1 u_{1/2} + \alpha_2 u_{3/2}}{h}$$

$$\frac{\partial \omega}{\partial x} \Big|_v = \frac{\alpha_1 \omega_0 + \alpha_2 \omega_1}{h}$$

$$\frac{\partial \omega}{\partial y} \Big|_u = \frac{\alpha_1 \omega_0 + \alpha_2 \omega_1}{h}$$

$$\alpha_1 = 0, \quad \alpha_2 = 0$$

$$\alpha_1 = -1.5, \quad \alpha_2 = 0.5$$

$$\alpha_1 = -1, \quad \alpha_2 = 1$$

$$\alpha_1 = -1, \quad \alpha_2 = 1$$

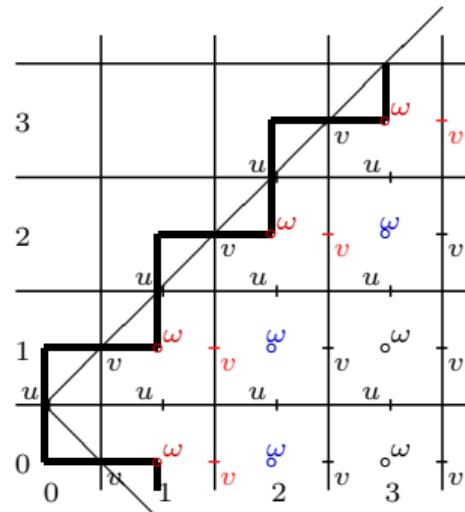


Figure: 45° rotated grid

That means the tangential velocity component is added to the vorticity formula:

$$\begin{aligned} \omega_0 &= \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} - \frac{u + v}{h} \\ &= \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} - \frac{\vec{V} \cdot \vec{r}}{h / \sqrt{2}} \end{aligned}$$

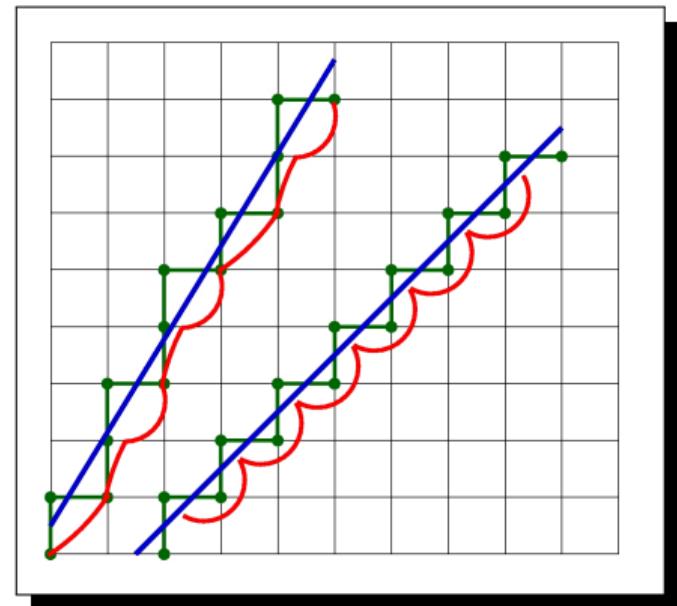
Optimized boundary

$$\begin{aligned}\omega_o &= \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} - \frac{u + v}{h} \\ &= \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} - \frac{\vec{V} \cdot \vec{\tau}}{h/\sqrt{2}}\end{aligned}$$

- Free-slip condition on a curvilinear boundary (see [4]): $\omega|_{bnd} = \frac{\vec{V} \cdot \vec{\tau}}{R}$

The optimized boundary is supposed to be a curvilinear one with:

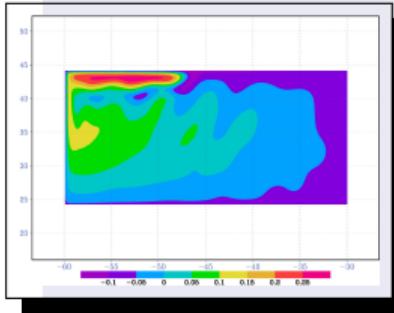
- Constant $R_{45^\circ} = -h/\sqrt{2}$ for the 45° rotated grid,
- Variable $R_{30^\circ} : -h < R_{30^\circ} < 5h$ for the 30° rotated grid



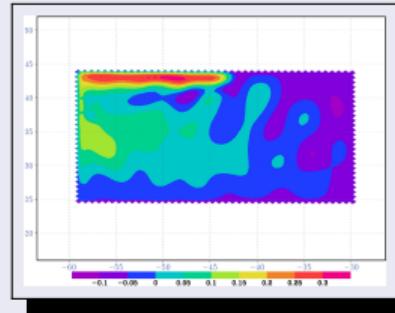
Physical boundary (blue), approximated by the grid (green) and optimized one (red).

Different Resolutions, 45° rotation

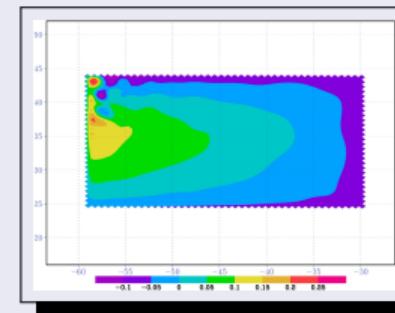
1°/2 resolution



Reference SSH

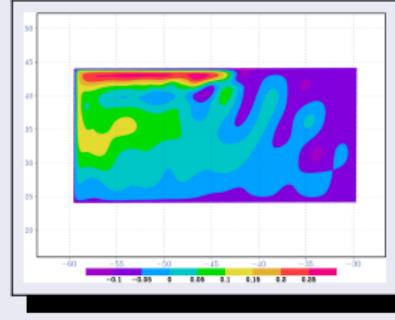
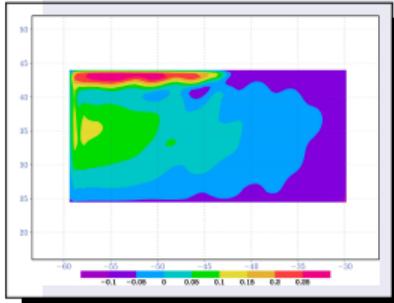


Rotated grid constant
 $R = -h/\sqrt{2}$ BC SSH

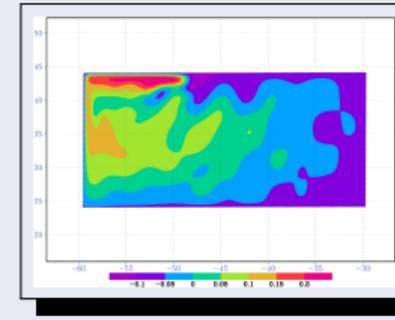


Rotated grid conventional BC
SSH

1°/8 resolution

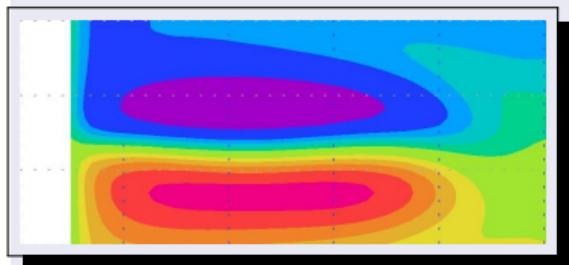


PACO

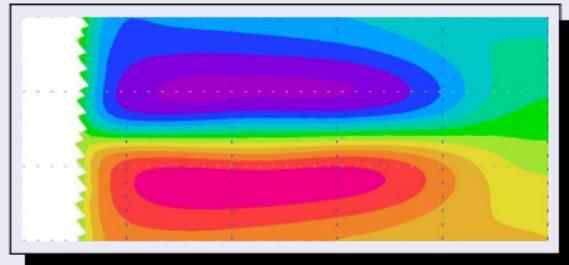


Double Gyre Control: 30° rotation, SSH

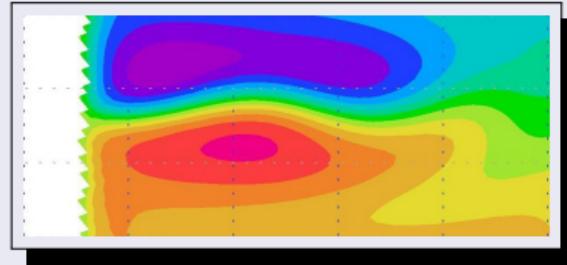
Reference, Optimal and Conventional BC. 10 years average.



Reference SSH



Rotated grid Optimal BC SSH



Rotated grid conventional BC SSH

Conclusion

Boundary Conditions influence is **important**

- Optimal BCs allows **to correct errors** committed by the discretization,
- The model **is closer** to the reference one with optimal BC,
- Data assimilation allows to get the **optimized position and shape** of the boundary.

BUT

As well as for any adjoint parameter estimation

- The control may violate the model physics;
- The **physical meaning** of the optimal boundary is difficult to understand;
- The set of α is **not unique**;
- The problem of **identifiability** is not addressed yet;
- The problem of **stability** is not even posed.

References:

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