



Lateral Boundary Conditions at the staircase-like boundary of ocean models

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Lateral Boundary Conditions at the staircase-like boundary of ocean models.

Eugene Kazantsev, Florian Lemarié, Eric Blayo

LJK/INRIA, AirSea, Grenoble

PACO project: LEFE-GMMC, LEFE-MANU

Rectangular Box Configuration of the NEMO:

$30^\circ \times 20^\circ$ with $\frac{1}{4}^\circ$ resolution and 5 z levels.

$120 \times 80 \times 5$ nodes in (x, y, z) coordinates, 64 time steps per day.

$$\begin{aligned}
 \frac{\partial u}{\partial t} &= \underbrace{v(\omega + f)}_{\text{Enstr.cons.}} - \frac{\partial(u^2 + v^2)/2}{\partial x} - w \frac{\partial u}{\partial z} - \underbrace{\frac{\partial A_u^h \xi}{\partial x} + \frac{\partial A_u^h \omega}{\partial y}}_{\text{Vort-Div. form.}} + \\
 &+ \underbrace{\frac{\partial}{\partial z} A_u^z \frac{\partial u}{\partial z}}_{\text{Implicit}} + \underbrace{g \int_0^z \frac{\partial \rho(x, y, \zeta)}{\partial x} d\zeta}_{\text{Pressure gradient}} + \underbrace{g \frac{\partial(\eta + T_c \phi)}{\partial x}}_{\text{EGW filter}} \\
 \xi &= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}, \quad \omega = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \quad \text{Divergence, Vorticity} \\
 w &= \int_H^z \xi(x, y, \zeta) d\zeta; \quad w(x, y, H) = 0 \quad \text{Vertical velocity} \\
 A^z &= 1.2 \times 10^{-4} \frac{m^2}{s}; \quad A^h = 200 \frac{m^2}{s}, \quad f = \frac{4\pi}{86400} \sin(lat) \\
 \left. \frac{\partial u}{\partial z} \right|_{w_0} &= \frac{-0.1 \frac{N}{m^2} \cos\left(B\pi * \frac{lat - 24^\circ}{44^\circ - 24^\circ}\right)}{hz_1 \rho_0} \quad \begin{array}{l} B = 2, \quad \text{Double gyre} \\ B = 1, \quad \text{Single gyre} \end{array} \\
 (u_\perp, \omega)_{\text{Lateral Boundary}} &= 0 \quad (\text{Impermeability and Free-Slip conditions})
 \end{aligned}$$

45° rotated grid, Piecewise Shaved cells.

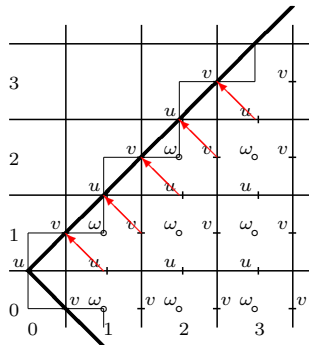


Figure: 45° rotated grid

Impermeability + free-slip boundary conditions:

$$(\vec{V}, \vec{n}) = 0, \quad \frac{\partial(\vec{V}, \vec{\tau})}{\partial \vec{n}} = 0$$

Impermeability: $u|_{bnd} = v|_{bnd},$

Free-slip:

$$v|_{bnd} = u|_{bnd} = u(x + h/2, y - h/2)$$

$$u|_{bnd} = v|_{bnd} = v(x + h/2, y - h/2)$$

Coriolis term on the 45° rotated grid.

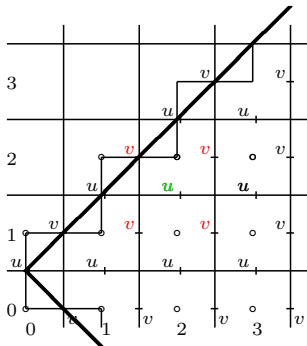


Figure: 45° rotated grid

$$\frac{\partial u}{\partial t} = f \bar{v}^{xy}$$

$$\frac{\partial v}{\partial t} = -f \bar{u}^{xy}$$

$$u(t) = u(0) \exp(ift)$$

$$\frac{\partial}{\partial t} \begin{pmatrix} u_1 \\ u_2 \\ \dots \\ u_n \\ v_1 \\ v_2 \\ \dots \\ v_n \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & f/4 & f/4 & 0 & 0 \\ 0 & 0 & 0 & 0 & f/4 & f/4 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & f/4 & f/4 \\ -f/4 & -f/4 & 0 & 0 & 0 & 0 & 0 & 0 \\ -f/4 & -f/4 & 0 & 0 & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & -f/4 & -f/4 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ \dots \\ u_n \\ v_1 \\ v_2 \\ \dots \\ v_n \end{pmatrix}$$

Coriolis term on the 45° rotated grid.

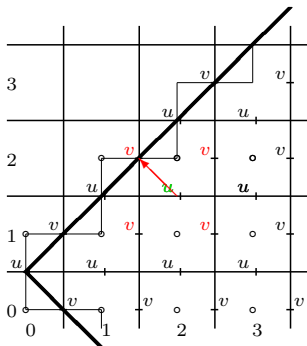


Figure: 45° rotated grid

$$\frac{\partial u}{\partial t} = f \bar{v}^{xy}$$

$$\frac{\partial v}{\partial t} = -f \bar{u}^{xy}$$

$$u(t) = u(0)e^{+ft/4}$$

$$\frac{\partial}{\partial t} \begin{pmatrix} u_1 \\ u_2 \\ \dots \\ u_n \\ v_1 \\ v_2 \\ \dots \\ v_n \end{pmatrix} = \begin{pmatrix} f/4 & 0 & 0 & 0 & f/4 & f/4 & 0 & 0 \\ 0 & f/4 & 0 & 0 & f/4 & f/4 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & f/4 & f/4 \\ -f/4 & -f/4 & 0 & -f/4 & 0 & 0 & 0 & 0 \\ -f/4 & -f/4 & 0 & 0 & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & -f/4 & -f/4 & 0 & 0 & 0 & -f/4 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ \dots \\ u_n \\ v_1 \\ v_2 \\ \dots \\ v_n \end{pmatrix}$$

Optimal Control of the Discrete Derivatives near the Boundary

Coefficients α are allowed to vary in order to find the best fit with requirements of the model and data (see [1, 2]).

$$\begin{aligned}\left.\frac{\partial v}{\partial x}\right|_{\omega_b} &= \frac{\alpha_1 v_{1/2} + \alpha_2 v_{3/2}}{h} \\ \left.\frac{\partial u}{\partial y}\right|_{\omega_b} &= \frac{\alpha_1 u_{1/2} + \alpha_2 u_{3/2}}{h} \\ \left.\frac{\partial v}{\partial x}\right|_{\omega_b} &= \frac{\alpha_1 v_{-1/2} + \alpha_2 v_{1/2}}{h} \\ \left.\frac{\partial u}{\partial y}\right|_{\omega_b} &= \frac{\alpha_1 u_{-1/2} + \alpha_2 u_{1/2}}{h} \\ \left.\frac{\partial \omega}{\partial x}\right|_v &= \frac{\alpha_1 \omega_0 + \alpha_2 \omega_1}{h} \\ \left.\frac{\partial \omega}{\partial y}\right|_u &= \frac{\alpha_1 \omega_0 + \alpha_2 \omega_1}{h}\end{aligned}$$

with the Initial Guess

$$\begin{aligned}\alpha_1 &= \alpha_2 = 0 \\ \alpha_1 &= -1, \quad \alpha_2 = 1 \\ \alpha_1 &= -1, \quad \alpha_2 = 1\end{aligned}$$

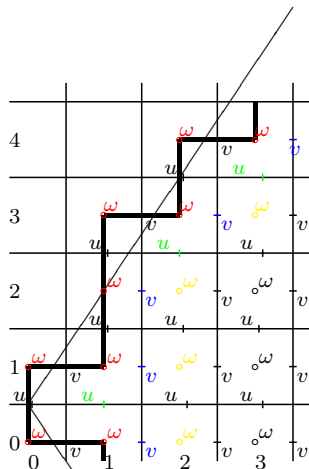


Figure: 30° rotated grid

The model: $x(t) = \mathcal{M}_{0,t}(x(0), \alpha)$ with $x = (u, v, T, S, ssh)^T$

Cost function J

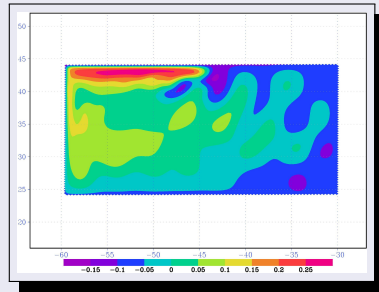
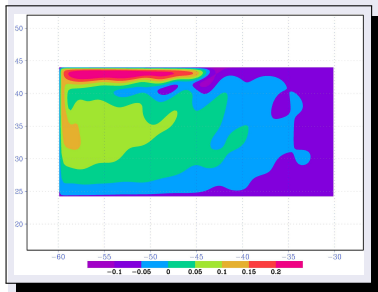
$$\begin{aligned} J &= 10^{-4}(\|x(0) - x_{bgr}\|^2 + \|\alpha - \alpha_{bgr}\|^2) + \\ &+ \int_{t=0}^T \int \int (u - u_{\text{ref}})^2 + (v - v_{\text{ref}})^2 + (ssh - ssh_{\text{ref}})^2 dx dy dt \end{aligned}$$

Layout:

- **Joint control** of the initial point $x(0)$ (interpolation errors) and the set of α ;
- Artificially generated data by the same model on the aligned grid;
- Data Assimilation with the sequence of assimilation windows: **10, 30, 50 days** with 30 iterations made in each window;
- Analysis of the solution on the **800 days** interval.

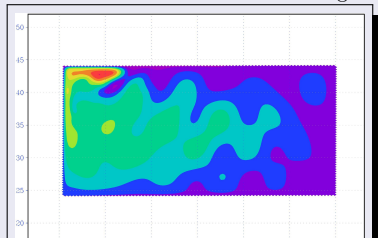
- Minimization is performed by M1QN3 (JC Gilbert, C.Lemarechal);
- Adjoint is generated by Tapenade (Ecuador team, INRIA).

Reference, Optimal and Conventional BC 800 days later



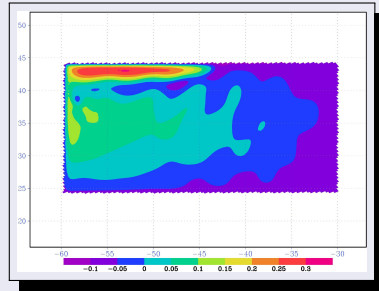
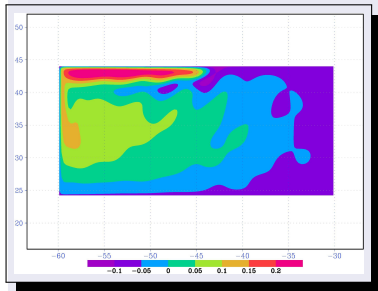
Reference SSH

Rotated grid Optimal BC SSH



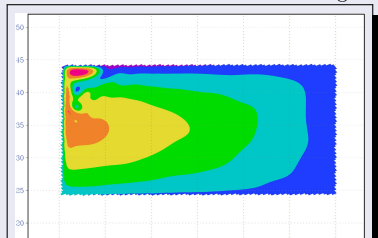
PACO

Reference, Optimal and Conventional BC 800 days later



Reference SSH

Rotated grid Optimal BC SSH



PACO

$$\left. \frac{\partial v}{\partial x} \right|_{\omega_b} = \frac{\alpha_1 v_{-1/2} + \alpha_2 v_{1/2}}{h}$$

$$\left. \frac{\partial u}{\partial y} \right|_{\omega_b} = \frac{\alpha_1 u_{-1/2} + \alpha_2 u_{1/2}}{h}$$

$$\left. \frac{\partial v}{\partial x} \right|_{\omega_b} = \frac{\alpha_1 v_{1/2} + \alpha_2 v_{3/2}}{h}$$

$$\left. \frac{\partial u}{\partial y} \right|_{\omega_b} = \frac{\alpha_1 u_{1/2} + \alpha_2 u_{3/2}}{h}$$

$$\left. \frac{\partial \omega}{\partial x} \right|_v = \frac{\alpha_1 \omega_0 + \alpha_2 \omega_1}{h}$$

$$\left. \frac{\partial \omega}{\partial y} \right|_u = \frac{\alpha_1 \omega_0 + \alpha_2 \omega_1}{h}$$

$$\alpha_1 = 0, \quad \alpha_2 = 0$$

$$\alpha_1 = -1.5, \quad \alpha_2 = 0.5$$

$$\alpha_1 = -1, \quad \alpha_2 = 1$$

$$\alpha_1 = -1, \quad \alpha_2 = 1$$

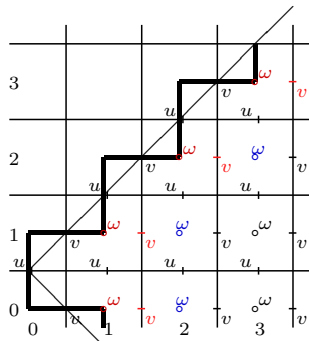


Figure: 45° rotated grid

That means the tangential velocity component is added to the vorticity formula:

$$\omega_o = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} - \frac{u + v}{h}$$

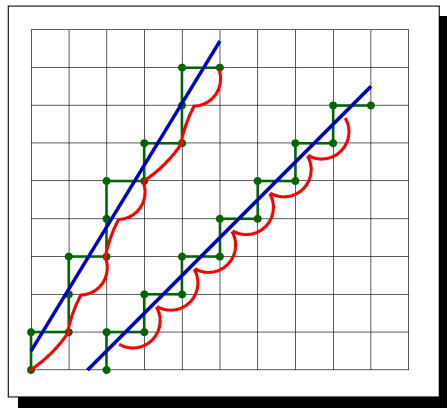
$$= \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} - \frac{\vec{V} \cdot \vec{\tau}}{h / \sqrt{2}}$$

$$\begin{aligned}\omega_o &= \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} - \frac{u + v}{h} \\ &= \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} - \frac{\vec{V} \cdot \vec{\tau}}{h/\sqrt{2}}\end{aligned}$$

- Free-slip condition on a curvilinear boundary (see [4]): $\omega|_{bnd} = \frac{\vec{V} \cdot \vec{\tau}}{R}$

The optimized boundary is supposed to be a curvilinear one with:

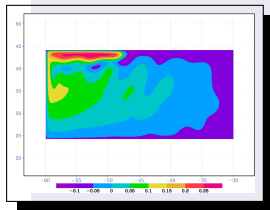
- Constant $R_{45^\circ} = -h/\sqrt{2}$ for the 45° rotated grid,
- Variable $R_{30^\circ} : -h < R_{30^\circ} < 5h$ for the 30° rotated grid



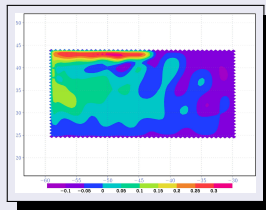
Physical boundary (blue), approximated by the grid (green) and optimized one (red).

Different Resolutions, 45° rotation

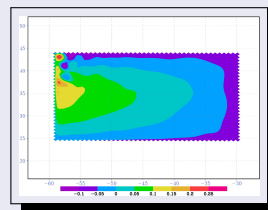
1°/2 resolution



Reference SSH

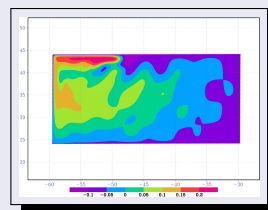
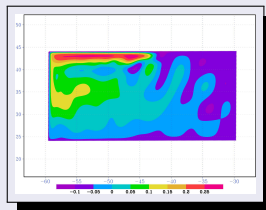
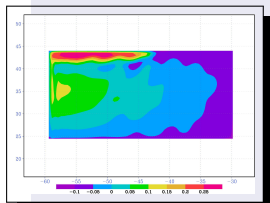


Rotated grid constant
 $R = -h/\sqrt{2}$ BC SSH

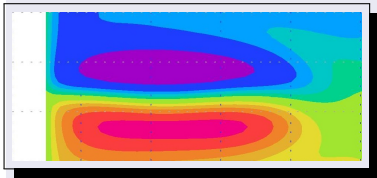


Rotated grid conventional BC
SSH

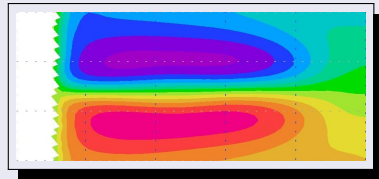
1°/8 resolution



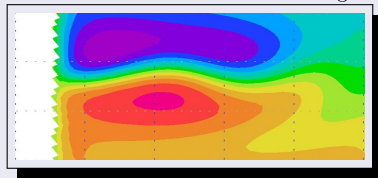
Reference, Optimal and Conventional BC. 10 years average.



Reference SSH



Rotated grid Optimal BC SSH



Rotated grid conventional BC SSH

Boundary Conditions influence is **important**

- Optimal BCs allows **to correct errors** committed by the discretization,
- The model **is closer** to the reference one with optimal BC,
- Data assimilation allows to get the **optimized position and shape** of the boundary.

BUT

As well as for any adjoint parameter estimation

- The control may violate the model physics;
- The **physical meaning** of the optimal boundary is difficult to understand;
- The set of α is **not unique**;
- The problem of **identifiability** is not addressed yet;
- The problem of **stability** is not even posed.

References:



1. E.Kazantsev. Optimized boundary conditions at staircase-shaped coastlines. *Ocean Dynamics*, 2015, vol.65, no.1, pp.49-63.



2. Ch.Kazantsev, E. Kazantsev and M.A. Tolstykh, Variational data assimilation for optimizing boundary conditions in ocean models. *Russian Meteorology and Hydrology*, 2015, vol.40, no.6, pp.383-391.



3. P. Marchand, Vers une meilleure prise en compte de la côte dans les modèles d'océan. Rapport de Stage réalisé au sein de Laboratoire Jean Kuntzmann, 2 Février - 31 Juillet 2015 sous la direction de E. Blayo et F. Lemarié



4. J. Verron and E. Blayo, The no-slip condition and separation western boundary currents. *J. Phys. Oceanogr.*, 1996, vol. 26, 1938-1951.



5. A. Adcroft and D. Marshall. How slippery are piecewise constant coastlines in numerical ocean models. *Tellus*, 1998, vol. 50A, 95-108.



6. F. Dupont, D. Straub and C. Lin, Influence of a step-like coastline on the basin scale vorticity budget of mid-latitude gyre models. *Tellus*, 2003, vol. 55A, 255-272.