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Modelling thin tissue compartments using the immersed FEM (continuous Galerkin)

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Biomag 2016

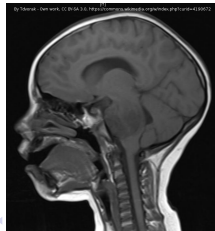
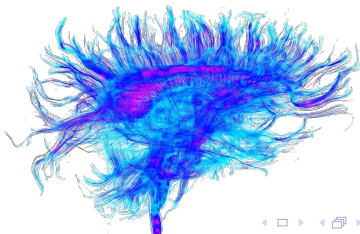
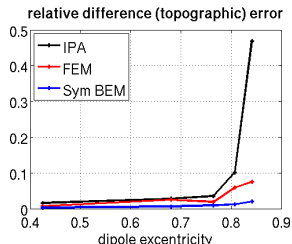
Motivations for Immersed FEM

Long term goal: Democratize EEG/MEG/sEEG FEM models.

Reasons: Competitive, handles anisotropy, volumic, ...

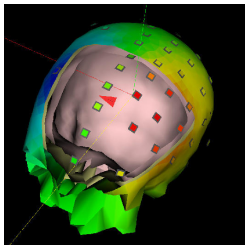
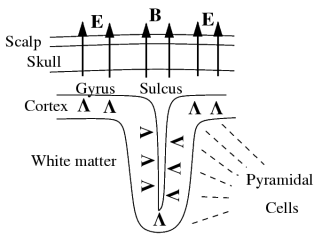
Challenges:

- Make software widely available [e.g. SimBio, NeuroFEM since 2013].
- Interface it with standard toolboxes [see Johannes talk].
- **Environment and tools to easily build FEM head models.**
 - Head tissue segmentation [FSL, Free Surfer, BrainSuite, SPM, Slicer, BESA, ...]
 - **Build the FEM model from segmentations.**
 - **Explore the solution.**



The EEG Forward Problem

Quasi-static electromagnetism



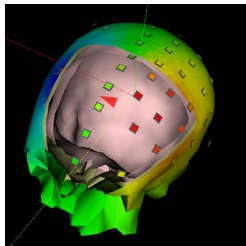
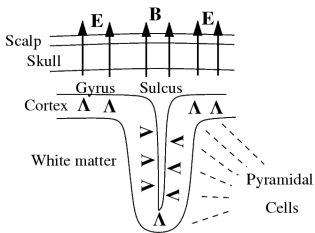
The potential V is related to the source currents \mathbf{J}^P by:

$$\begin{cases} \nabla \cdot (\sigma \nabla V) = \nabla \cdot \mathbf{J}^P & \text{in } \Omega \\ \sigma \nabla V \cdot \mathbf{n} = g & \text{on } \partial\Omega. \end{cases}$$

- Ω : head volume.
- σ : local conductivity.
- $[V] = [\sigma \nabla V] = 0$ across interfaces.

The EEG Forward Problem

Quasi-static electromagnetism



Weak formulation:

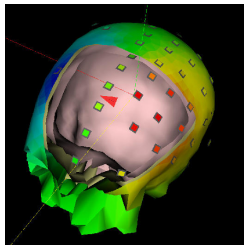
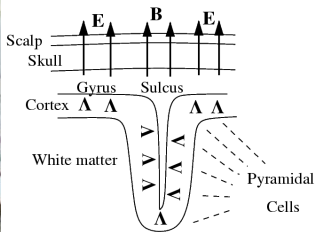
$$C(V) = \frac{1}{2} \int_{\Omega} \sigma(\mathbf{r}) \|\nabla V(\mathbf{r})\|^2 d\mathbf{r} + \int_{\Omega} \nabla \cdot \mathbf{J}^P(\mathbf{r}) V(\mathbf{r}) d\mathbf{r} - \int_{\partial\Omega} g V ds .$$

\implies after FEM discretization: $[\Delta_{\sigma}] \mathbf{V} = [\nabla]^T \mathbf{J}^P$

m

The EEG Forward Problem

Quasi-static electromagnetism

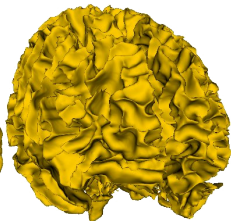
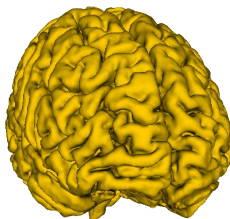
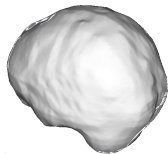
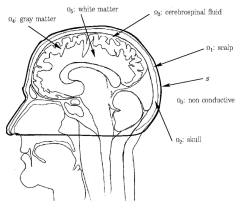
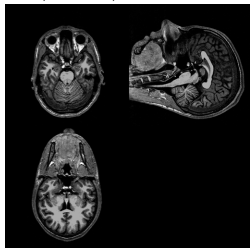


MEG can be put in the same framework (adjoint method).

EEG Forward Model (FEM)

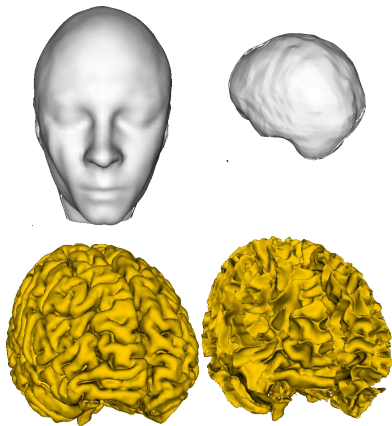
Head segmentation and mesh generation

MRIs, CT-scan, ...



EEG Forward Model (FEM)

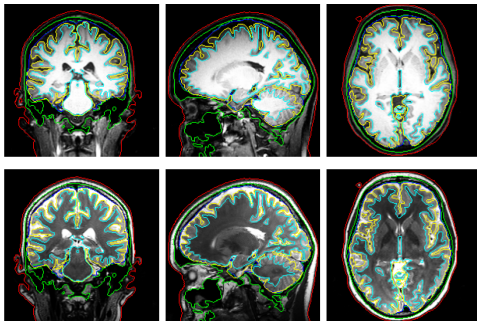
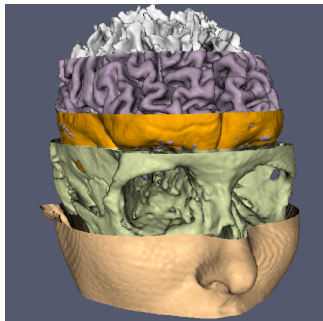
Head segmentation and mesh generation



- Very thin structure:
 - Esp. for infants.
 - Very resolved meshes.
 - Derefinement.
- Topology changes.
 - Self intersections.
 - Holes in the model.
 - Complicate head modelling.
- Lots of work.
- Huge meshes.
- Computational quality.

EEG Forward Model (FEM)

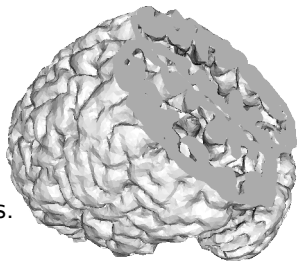
Potentially much more complex geometries...



- Complex skull geometry.
- Blood vessels.

From surfacic meshes to matrices $[\Delta_\sigma]$

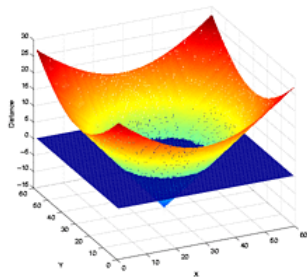
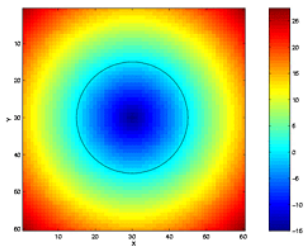
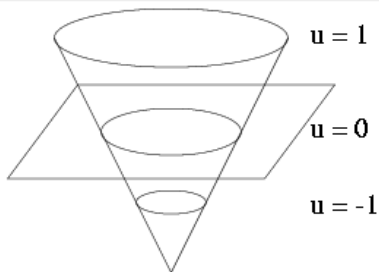
- Volumic meshing between surfaces.
 - Merge volumic meshes.
 - Matrix computation from base functions.
-
- For each subject.
 - Size/accuracy tradeoff.
 - Image data ignored during simplification.



Solution: Compute matrices directly from segmentations.

Direct Use of Segmentations

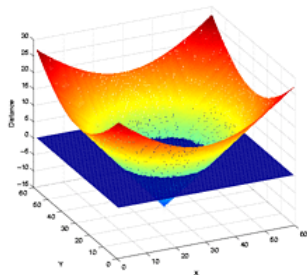
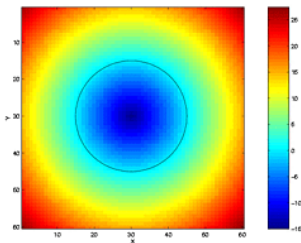
Level sets representation of surfaces



Direct Use of Segmentations

Level sets representation of surfaces

Most curve (2D) or surface (3D) manipulations can be expressed as levelset manipulations.
Segmentation.

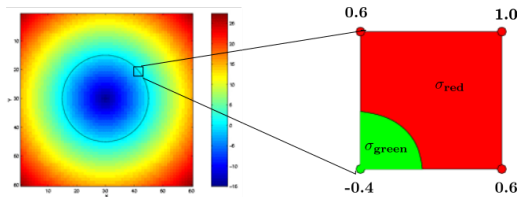
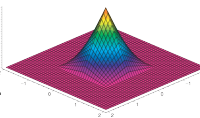


Direct Use of Segmentations

Direct computation of $[\Delta_\sigma]$ from levelsets

The MRI image space is the computation reference space.

- Use the "image mesh". Trilinear interpolation method (Q1).
 \implies The solution is an image of potential.
- Interfaces defined by implicit functions (levelsets).
 \implies Computation of partial integrals wrt interfaces.



Implicit boundary:

$$f(x, y) = c_{11}xy + c_{10}x + c_{01}y + c_{00} = 0.$$

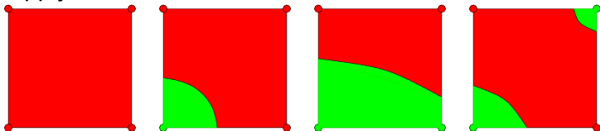
Sum green and
red contributions

\implies Closed form formula in 2D (integral of a rational fraction).

Direct Use of Segmentations

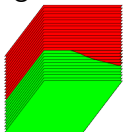
2D/3D algorithms

- 2D:
- Apply transform (rotations, symmetries, sign inversion) to reduce to:



- Compute the explicit 2D formulae.
- Correct for the transform action.

- 3D: Numerical integration of 2D values:

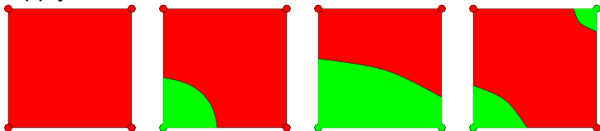


2D topological changes.

Direct Use of Segmentations

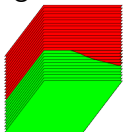
2D/3D algorithms

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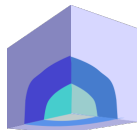


- Compute the explicit 2D formulae.
- Correct for the transform action.

3D: Numerical integration of 2D values:



2D topological changes.



Multiple interfaces per voxel can be handled.

Non-differentiable Elements

V and $\sigma \nabla V$ continuous over $\Omega \dots$

\dots but σ is discontinuous across interface boundaries

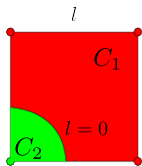
$\implies \nabla V$ discontinuous: **not representable with Q1 elements.**

\implies **Use non-differential continuous elements (piecewise Q1):**

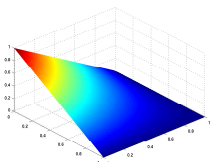
$$\begin{cases} \phi(x, y, z) = p_1(x, y, z) & \text{in } C_1, \\ \phi(x, y, z) = p_2(x, y, z) & \text{in } C_2, \end{cases} \quad p_1, p_2 \in Q1$$

$$p_1 - p_2 = \lambda l, \quad \lambda \in \mathbb{R}$$

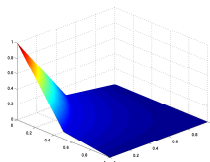
$$\int_{l=0} \sigma_1 \nabla p_1 \cdot \mathbf{n} - \sigma_2 \nabla p_2 \cdot \mathbf{n} = 0$$



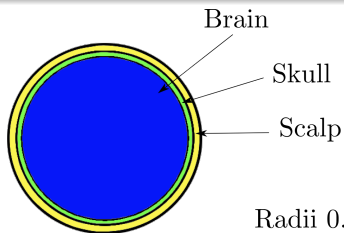
$$\begin{aligned} \sigma_1 &= 10 \\ \sigma_2 &= 1 \end{aligned}$$



\rightsquigarrow

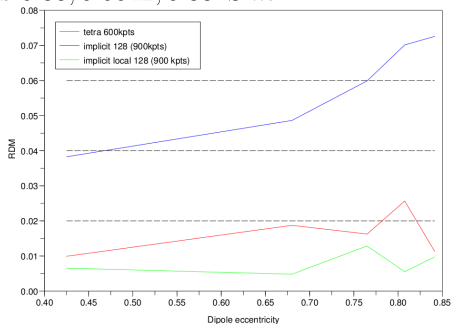
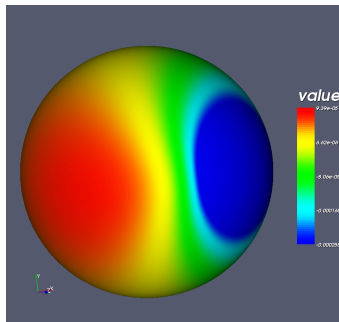


Results: Spherical Model

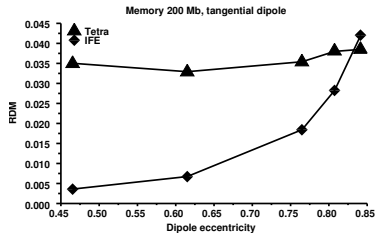
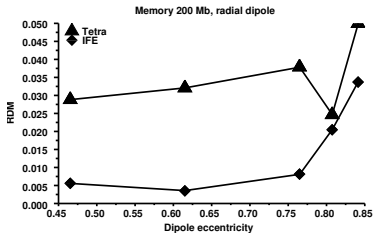
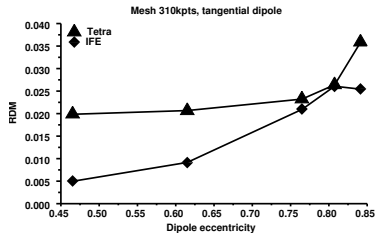
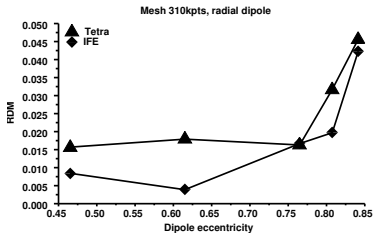


$$RDM = \left\| \frac{V_{num}}{\|V_{num}\|} - \frac{V_{anal}}{\|V_{anal}\|} \right\|$$

Radii 0.87, 0.92, 1.

Conductivities 0.33, 0.0042, 0.33 $S.m^{-1}$.

Results: Spherical Model



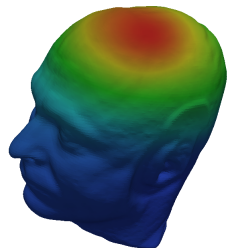
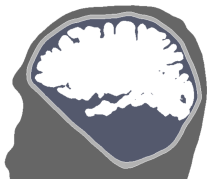
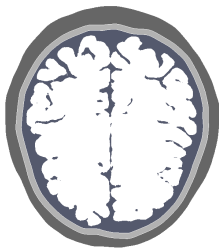
Results: Real Data

MRI image of size $256 \times 256 \times 160$

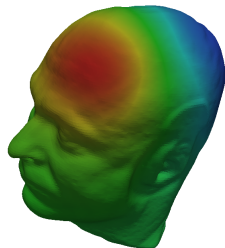
5 conductivity domains segmented using four levelsets.

Rescaled to $128 \times 128 \times 80$

\Rightarrow **many thin structures and double interface cells are present !**



Radial



Tangential

Conclusions

Take home messages

Immersed FEM:

- Use MRI as the computational space.
- Direct connexion Q1 Levelsets \longleftrightarrow FEM matrices.
- **Simple, fast and unsurprising: a real use case advantage.**
- **Thin structure easy to deal with.**
- **Accuracy similar to standard FEM.**
- **Easy visualization and exploration of the solution.**
- Multiscale and parallelisation are easy.
- Control of boundary condition of solution more complex.

Acknowledgments:

Sylvain Vallaghé, Jérôme Piovano, EADS foundation grant n°2118.

Details: *A trilinear immersed finite element method for solving the EEG forward problem*, Vallaghé, Papadopoulos.

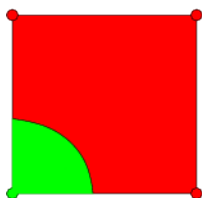
Thank you !!

Direct Use of Segmentations (2)

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⇒ Computation of partial integrals wrt interfaces.



Implicit boundary: $f(x, y) = c_{11}xy + c_{10}x + c_{01}y + c_{00} = 0$.

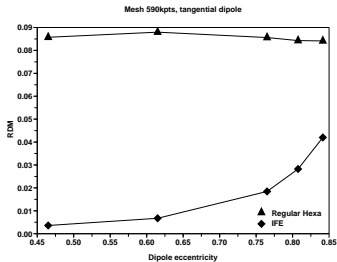
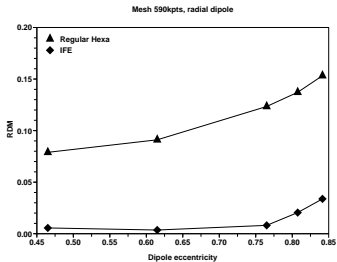
Green contribution:

$$\begin{aligned} \int \int_{Green} m(x, y) dy dx &= \int_0^{x_0} \int_0^{-\frac{c_{10}x + c_{00}}{c_{11}x + c_{01}}} m(x, y) dy dx \\ &= \int_0^{x_0} M \left(x, -\frac{c_{10}x + c_{00}}{c_{11}x + c_{01}} \right) dx . \end{aligned}$$

⇒ Analytic formula (integral of a rational fraction).

- Special cases (polynomial expressions): $c_{11}c_{00} - c_{10}c_{01} = 0$,
 $c_{11} \simeq 0$ (Taylor expansion).

Comparison with FD



More Comparisons with P1 Implementation

