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# Modelling thin tissue compartments using the immersed FEM (continuous Galerkin)

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Biomag 2016

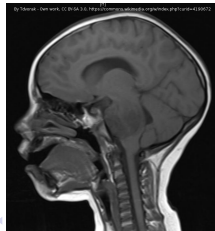
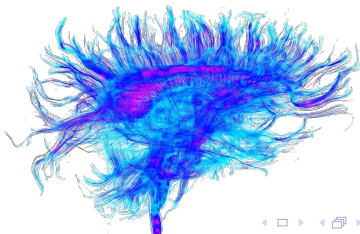
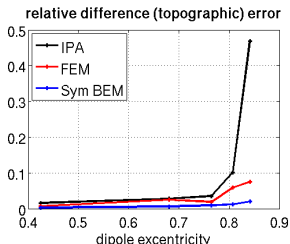
# Motivations for Immersed FEM

**Long term goal:** Democratize EEG/MEG/sEEG FEM models.

**Reasons:** Competitive, handles anisotropy, volumic, ...

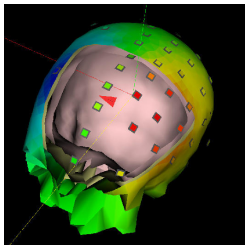
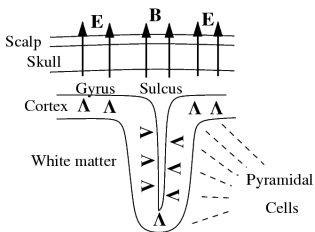
## Challenges:

- Make software widely available [e.g. SimBio, NeuroFEM since 2013].
- Interface it with standard toolboxes [see Johannes talk].
- **Environment and tools to easily build FEM head models.**
  - Head tissue segmentation [FSL, Free Surfer, BrainSuite, SPM, Slicer, BESA, ...]
  - **Build the FEM model from segmentations.**
  - **Explore the solution.**



# The EEG Forward Problem

Quasi-static electromagnetism



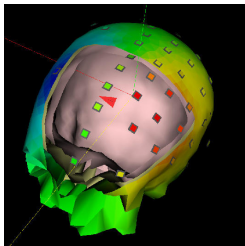
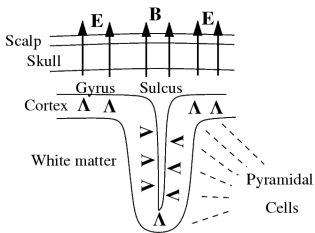
The potential  $V$  is related to the source currents  $\mathbf{J}^P$  by:

$$\begin{cases} \nabla \cdot (\sigma \nabla V) = \nabla \cdot \mathbf{J}^P & \text{in } \Omega \\ \sigma \nabla V \cdot \mathbf{n} = g & \text{on } \partial\Omega. \end{cases}$$

- $\Omega$ : head volume.
- $\sigma$ : local conductivity.
- $[V] = [\sigma \nabla V] = 0$  across interfaces.

# The EEG Forward Problem

Quasi-static electromagnetism



Weak formulation:

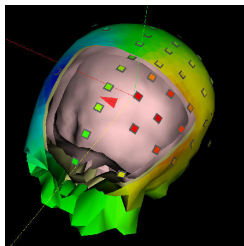
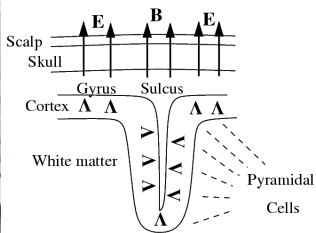
$$C(V) = \frac{1}{2} \int_{\Omega} \sigma(\mathbf{r}) \|\nabla V(\mathbf{r})\|^2 d\mathbf{r} + \int_{\Omega} \nabla \cdot \mathbf{J}^P(\mathbf{r}) V(\mathbf{r}) d\mathbf{r} - \int_{\partial\Omega} g V ds .$$

$\implies$  after FEM discretization:  $[\Delta_{\sigma}] \mathbf{V} = [\nabla]^T \mathbf{J}^P$

m

# The EEG Forward Problem

Quasi-static electromagnetism

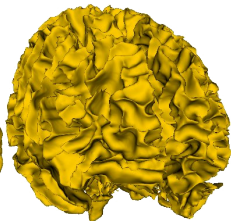
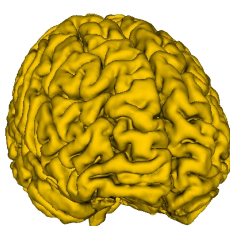
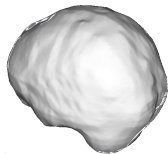
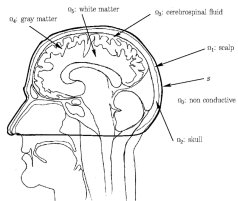
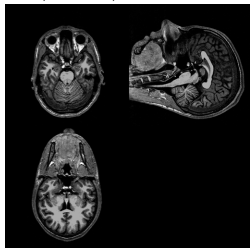


MEG can be put in the same framework (adjoint method).

# EEG Forward Model (FEM)

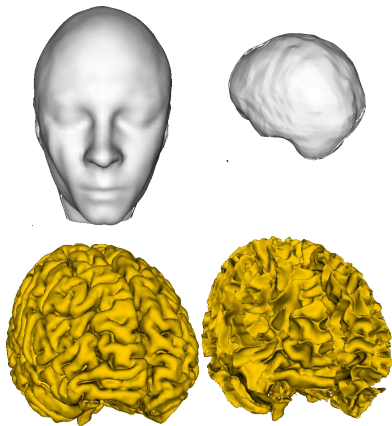
## Head segmentation and mesh generation

MRIs, CT-scan, ...



# EEG Forward Model (FEM)

## Head segmentation and mesh generation

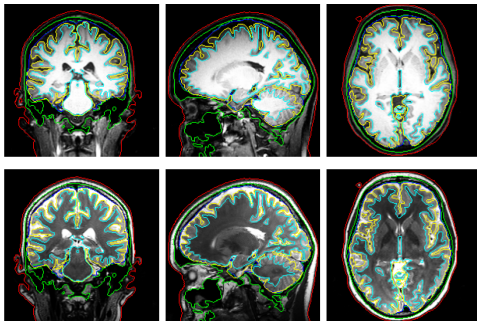
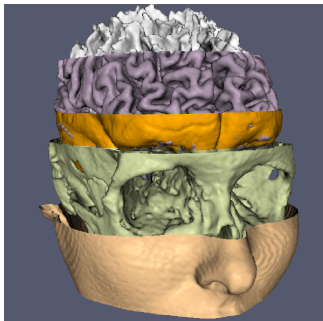


- Very thin structure:
  - Esp. for infants.
  - Very resolved meshes.
  - Derefinement.
- Topology changes.
  - Self intersections.
  - Holes in the model.
  - Complicate head modelling.
- Lots of work.
- Huge meshes.
- Computational quality.



# EEG Forward Model (FEM)

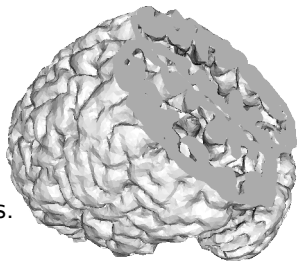
Potentially much more complex geometries. . .



- Complex skull geometry.
- Blood vessels.

# From surfacic meshes to matrices $[\Delta_\sigma]$

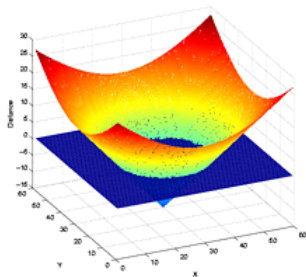
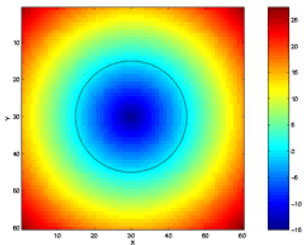
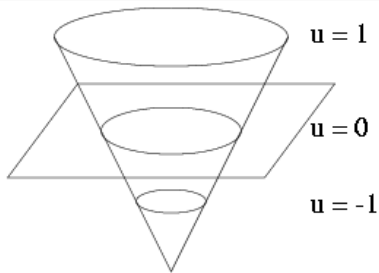
- Volumic meshing between surfaces.
  - Merge volumic meshes.
  - Matrix computation from base functions.
- 
- For each subject.
  - Size/accuracy tradeoff.
  - Image data ignored during simplification.



**Solution: Compute matrices directly from segmentations.**

# Direct Use of Segmentations

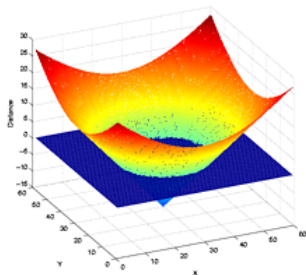
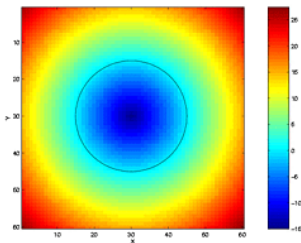
Level sets representation of surfaces



# Direct Use of Segmentations

Level sets representation of surfaces

**Most curve (2D) or surface (3D) manipulations can be expressed as levelset manipulations.**  
Segmentation.

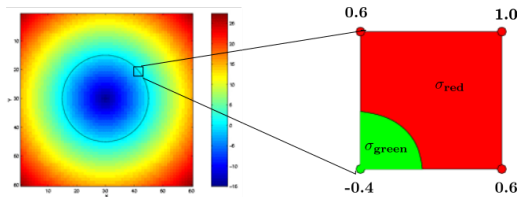
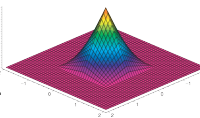


# Direct Use of Segmentations

Direct computation of  $[\Delta_\sigma]$  from levelsets

**The MRI image space is the computation reference space.**

- Use the "image mesh". Trilinear interpolation method (Q1).  
 $\implies$  The solution is an image of potential.
- Interfaces defined by implicit functions (levelsets).  
 $\implies$  Computation of partial integrals wrt interfaces.



Implicit boundary:

$$f(x, y) = c_{11}xy + c_{10}x + c_{01}y + c_{00} = 0.$$

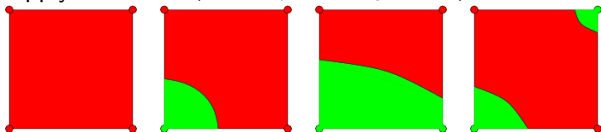
Sum green and  
red contributions

$\implies$  Closed form formula in 2D (integral of a rational fraction).

# Direct Use of Segmentations

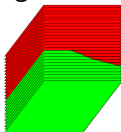
## 2D/3D algorithms

2D: • Apply transform (rotations, symmetries, sign inversion) to reduce to:



- Compute the explicit 2D formulae.
- Correct for the transform action.

3D: Numerical integration of 2D values:

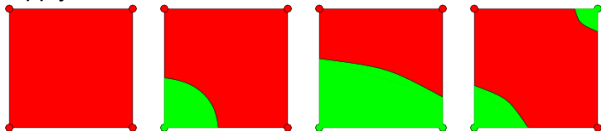


2D topological changes.

# Direct Use of Segmentations

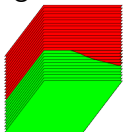
## 2D/3D algorithms

- 2D:
- Apply transform (rotations, symmetries, sign inversion) to reduce to:

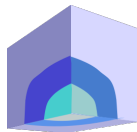


- Compute the explicit 2D formulae.
- Correct for the transform action.

- 3D: Numerical integration of 2D values:



2D topological changes.



Multiple interfaces per voxel can be handled.

# Non-differentiable Elements

$V$  and  $\sigma \nabla V$  continuous over  $\Omega \dots$

$\dots$  but  $\sigma$  is discontinuous across interface boundaries

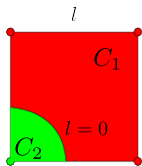
$\implies \nabla V$  discontinuous: **not representable with Q1 elements.**

$\implies$  **Use non-differential continuous elements (piecewise Q1):**

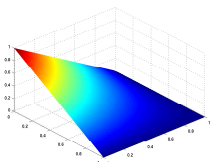
$$\begin{cases} \phi(x, y, z) = p_1(x, y, z) & \text{in } C_1, \\ \phi(x, y, z) = p_2(x, y, z) & \text{in } C_2, \end{cases} \quad p_1, p_2 \in Q1$$

$$p_1 - p_2 = \lambda l, \quad \lambda \in \mathbb{R}$$

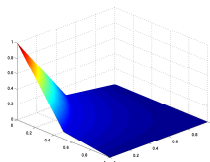
$$\int_{l=0} \sigma_1 \nabla p_1 \cdot \mathbf{n} - \sigma_2 \nabla p_2 \cdot \mathbf{n} = 0$$



$$\begin{aligned} \sigma_1 &= 10 \\ \sigma_2 &= 1 \end{aligned}$$

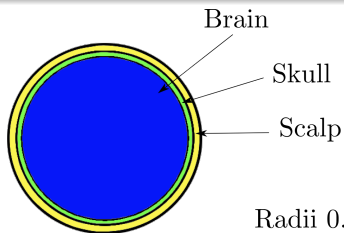


$\rightsquigarrow$



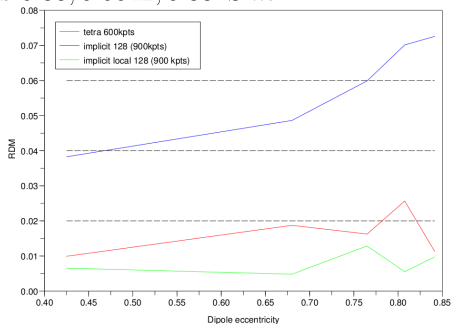
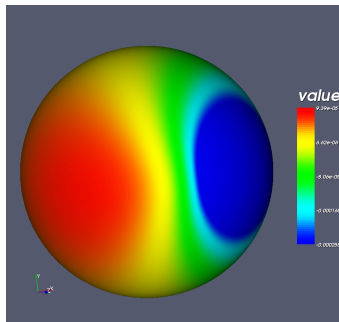


## Results: Spherical Model

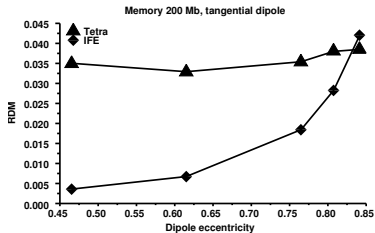
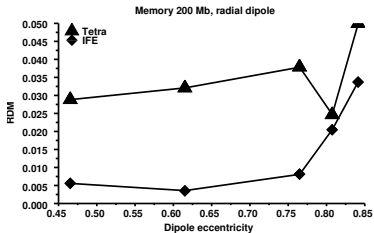
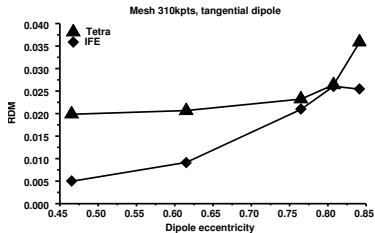
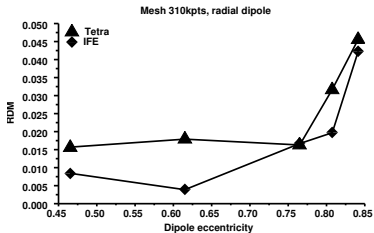


$$RDM = \left\| \frac{V_{num}}{\|V_{num}\|} - \frac{V_{anal}}{\|V_{anal}\|} \right\|$$

Radii 0.87, 0.92, 1.

Conductivities 0.33, 0.0042, 0.33  $S.m^{-1}$ .

## Results: Spherical Model



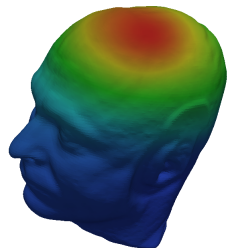
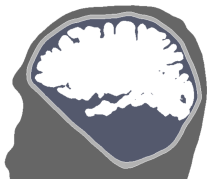
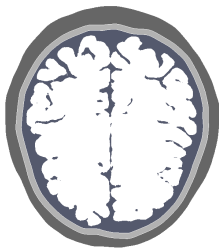
# Results: Real Data

MRI image of size  $256 \times 256 \times 160$

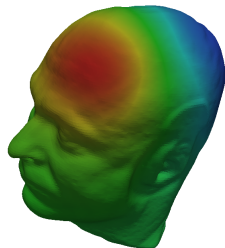
5 conductivity domains segmented using four levelsets.

Rescaled to  $128 \times 128 \times 80$

$\Rightarrow$  **many thin structures and double interface cells are present !**



Radial



Tangential

# Conclusions

## Take home messages

### Immersed FEM:

- Use MRI as the computational space.
- Direct connexion Q1 Levelsets  $\longleftrightarrow$  FEM matrices.
- **Simple, fast and unsurprising: a real use case advantage.**
- **Thin structure easy to deal with.**
- **Accuracy similar to standard FEM.**
- **Easy visualization and exploration of the solution.**
- Multiscale and parallelisation are easy.
- Control of boundary condition of solution more complex.

### Acknowledgments:

Sylvain Vallaghé, Jérôme Piovano, EADS foundation grant n°2118.

**Details:** *A trilinear immersed finite element method for solving the EEG forward problem*, Vallaghé, Papadopoulos.

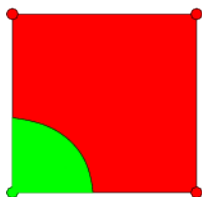
Thank you !!

# Direct Use of Segmentations (2)

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- Interfaces defined by implicit functions (levelsets).

⇒ Computation of partial integrals wrt interfaces.



Implicit boundary:  $f(x, y) = c_{11}xy + c_{10}x + c_{01}y + c_{00} = 0$ .

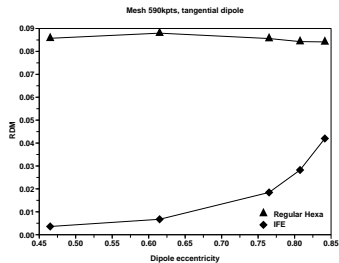
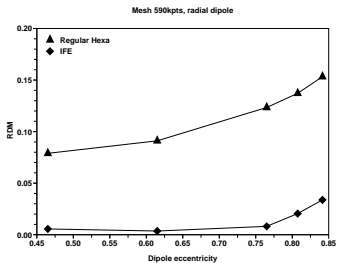
Green contribution:

$$\begin{aligned} \int \int_{Green} m(x, y) dy dx &= \int_0^{x_0} \int_0^{-\frac{c_{10}x + c_{00}}{c_{11}x + c_{01}}} m(x, y) dy dx \\ &= \int_0^{x_0} M\left(x, -\frac{c_{10}x + c_{00}}{c_{11}x + c_{01}}\right) dx. \end{aligned}$$

⇒ Analytic formula (integral of a rational fraction).

- Special cases (polynomial expressions):  $c_{11}c_{00} - c_{10}c_{01} = 0$ ,  
 $c_{11} \simeq 0$  (Taylor expansion).

## Comparison with FD



## More Comparisons with P1 Implementation

